Mass Predictions of Open-Flavour Hybrid Mesons from QCD Sum Rules

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Within QCD, colourless states may be constructed corresponding to exotic matter outside of the traditional quark model. Experiments have recently observed tetraquark and pentaquark states, but no definitive hybrid meson signals have been observed. With the construction of the PANDA experiment at FAIR, and with full commissioning of the GlueX experiment at JLab expected to be completed this year, the opportunity for the observation of hybrid mesons has greatly increased. However, theoretical calculations are necessary to ascertain the identity of any experimental resonances that may be observed. We present selected QCD sum rule results from a full range of quantum numbers for open-flavour hybrid mesons with heavy valence quark content, including non-perturbative condensate contributions up to six-dimensions.

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1. Correlation Functions of Open-Flavour Hybrid Mesons

The seminal application of QCD sum-rules to heavy-light hybrids was performed by Govaerts, Reinders, and Weyers [1] (hereafter GRW). Therein, they considered four distinct currents covering $J \in \{0, 1\}$ in an effort to compute a comprehensive collection of ground state hybrid masses. For all heavy-light hybrids, the square of the ground state hybrid mass was uncomfortably close to the continuum threshold (with a typical separation of roughly 10–15 MeV), and it was noted that even a modest hadron width would result in the resonance essentially merging with the continuum.

In this paper, we briefly review [2] where we extended the work of GRW [1] by including both 5d mixed and 6d gluon condensate contributions in our correlator calculations. As noted in GRW, for heavy-light hybrids, condensates involving light quarks are multiplied by a heavy quark mass allowing for the possibility of a numerically significant contribution to the correlator and to the sum-rules. By this reasoning, the 5d mixed condensate could be a significant component of a QCD sum-rules application to the hybrid systems under consideration. As well, recent sum-rules analyses of closed, heavy hybrids [3] have demonstrated that the 6d gluon condensate can have an important stabilizing effect on what were, in the pioneering work [1, 4, 5], unstable analyses.

Following GRW, we define open hybrid interpolating currents, $j_\mu = \frac{g_s}{2} \overline{Q} \, \Gamma^\rho \lambda^a q \, \mathscr{G}^a_{\mu\rho}$, where g_s is the strong coupling and λ^a are the Gell-Mann matrices. The field Q represents a heavy charm or bottom quark with mass M_Q whereas q represents a light up, down, or strange quark with mass m_q. The Dirac matrix Γ^ρ satisfies $\Gamma^\rho \in \{\gamma^\rho, \gamma^\rho \gamma_5\}$ and the tensor $\mathscr{G}^a_{\mu\rho}$, the portion of j_μ containing the gluonic degrees of freedom, satisfies $\mathscr{G}^a_{\mu\rho} \in \{G^a_{\mu\rho}, \tilde{G}^a_{\mu\rho} = \frac{1}{2}\varepsilon_{\mu\rho\nu\sigma}G^a_{\nu\sigma}\}$ where $G^a_{\mu\rho}$ is the gluon field strength and $\tilde{G}^a_{\mu\rho}$ is its dual defined using the totally antisymmetric Levi-Civita symbol $\varepsilon_{\mu\rho\nu\sigma}$.

For each of the four currents defined through j_μ above, we consider a corresponding diagonal, two-point correlation function

$$\Pi_{\mu\nu}(q) = i \int \mathrm{d}^4 x e^{iq \cdot x} \langle \Omega | \tau j_\mu(x) j^\dagger_\nu(0) | \Omega \rangle = \frac{q_\mu q_\nu}{q^2} \Pi^{(0)}(q^2) + \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) \Pi^{(1)}(q^2). \quad (1.1)$$

The tensor decomposition in (1.1) is such that $\Pi^{(0)}$ probes spin-0 states while $\Pi^{(1)}$ probes spin-1 states. We will reference each of the $\Pi^{(0)}$ and $\Pi^{(1)}$ according to the J^{PC} combination it would have were we investigating closed rather than open hybrids; however, to stress that the C-value can not be taken literally, we will enclose it in brackets.

Table 1: The $J^{P(C)}$ combinations probed through different choices of Γ^ρ and $\mathscr{G}^a_{\mu\rho}$.

Γ^ρ	$\mathscr{G}^a_{\mu\rho}$	$J^{P(C)}$
γ^ρ	$G^a_{\mu\rho}$	$0^{+(+)}, 1^{-(+)}$
γ^ρ	$\tilde{G}^a_{\mu\rho}$	$0^{-(+)}, 1^{+(+)}$
$\gamma^\rho \gamma_5$	$G^a_{\mu\rho}$	$0^{-(-)}, 1^{+(-)}$
$\gamma^\rho \gamma_5$	$\tilde{G}^a_{\mu\rho}$	$0^{+(-)}, 1^{-(-)}$

We calculate the correlators (1.1) within the operator product expansion (OPE) in which perturbation theory is supplemented by a collection of non-perturbative terms, each of which is the product of a perturbatively computed Wilson coefficient and a non-zero vacuum expectation value

(VEV) or condensate. We include condensates up to and including those of dimension (d) six:

$$\langle \overline{q}q \rangle = \langle \overline{q}_i^{\alpha} q_i^{\alpha} \rangle \tag{1.2}$$

$$\langle \alpha G^2 \rangle = \langle \alpha_s G_{\mu\nu}^a G_{\mu\nu}^a \rangle \tag{1.3}$$

$$\langle g\overline{q}\sigma Gq \rangle = \langle g_s \overline{q}_i^{\alpha} \sigma_{ij}^{\mu\nu} \lambda_{\alpha\beta}^a G_{\mu\nu}^a q_j^{\beta} \rangle \tag{1.4}$$

$$\langle g^3 G^3 \rangle = \langle g_s^3 f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c \rangle. \tag{1.5}$$

where the VEVs (1.2)–(1.5) are respectively referred to as the 3d quark condensate, the 4d gluon condensate, the 5d mixed condensate, and the 6d gluon condensate.

The Wilson coefficients are computed to leading-order (LO) in g_s using coordinate-space, fixed-point gauge techniques (see [6]). Light quark masses are included in perturbation theory through a light quark mass expansion, but have been set to zero in all other OPE terms. The contributing Feynman diagrams are depicted in Figure 1. Divergent integrals are handled using dimensional regularization in $D = 4 + 2\varepsilon$ spacetime dimensions at a renormalization scale μ^2. We use the program TARCER [7] to reduce complicated, two-loop integrals to a small collection of simple basic integrals, all of which are well-known for the diagrams under consideration.
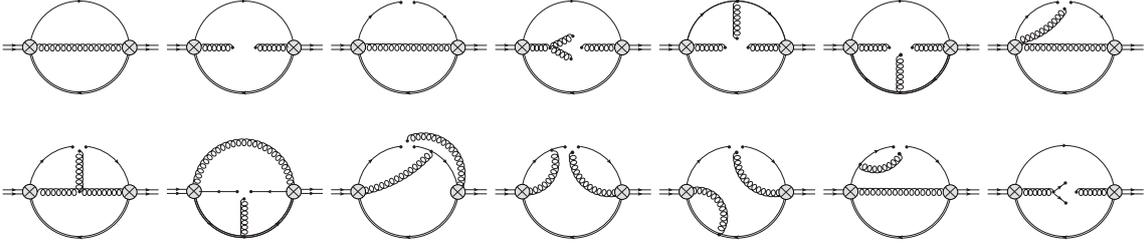

Figure 1: The Feynman diagrams calculated for the correlator (1.1). Single solid lines correspond to light quark propagators whereas double solid lines correspond to heavy quark propagators. All Feynman diagrams are drawn using JaxoDraw [8]

2. Results of Laplace Sum-Rules Analysis

Performing a Laplace sum-rules analysis of all eight distinct $J^{P(C)}$ combinations defined according to Table 1, we present the mass predictions and estimated uncertainties in Figure 2. For a more in-depth discussion, please refer to [2].

3. Conclusions

Predictions for the masses of heavy-strange and heavy-nonstrange hybrid mesons for $J^P \in \{0^{\pm}, 1^{\pm}\}$ are briefly presented, utilizing QCD sum-rules and improving upon the calculations of [1] by updating the non-perturbative parameters in the calculation, and including higher dimensional condensates in the OPE that have been shown important to sum-rule stability. A complete discussion of the analysis and results may be found in [2]. A degeneracy is observed in the heavy-light and heavy-strange states, and stabilization in the previously unstable $0^{-(-)}$ and $1^{-(+)}$ channels [1] driven by the addition of the higher dimensional mixed and 6d gluon condensate contributions.
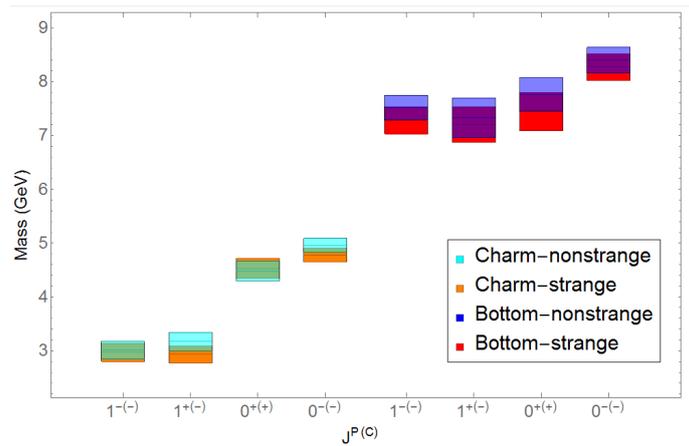
Figure 2: Summary of mass predictions with uncertainties for charm and bottom hybrid systems for the stabilizing $J^{P(C)}$ channels; channels which have been omitted do not stabilize.

As a consequence of these higher dimensional contributions, the $1^{+(+)}$ channel is destabilized from the original analysis of [1].

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