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Resumming large collinear logarithms in the non-linear QCD evolution at high energy

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The high-energy evolution in perturbative QCD suffers from a severe lack-of-convergence problem, due to higher order corrections enhanced by double and single transverse logarithms. We resum double logarithms to all orders within the non-linear Balitsky-Kovchegov equation, by taking into account successive soft gluon emissions strongly ordered in lifetime. We further resum single logarithms generated by the first non-singular part of the splitting functions and by the one-loop running of the coupling. The resummed BK equation admits stable solutions, which are used to successfully fit the HERA data at small x for physically acceptable initial conditions and reasonable values of the fit parameters.

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1. Introduction

It is by now well established that the Balitsky-JIMWLK hierarchy [1, 2, 3, 4, 5] and its mean field approximation known as the Balitsky-Kovchegov (BK) equation [1, 6] govern the high-energy evolution of scattering amplitudes in presence of non-linear effects (multiple scattering and gluon saturation) responsible for unitarization. Some of the most remarkable recent developments in that context refer to the first calculations of the next-to-leading order (NLO) corrections [7, 8, 9] to the B-JIMWLK and BK equations. However, the NLO version of the BK equation appeared to be unstable [10], and hence pretty useless in practice, due to large NLO corrections enhanced by single or double *transverse* logarithms.

A similar problem was originally identified and solved at the level of the NLO BFKL equation [11, 12] — the linearized version of the NLO BK equation which applies so long as the scattering remains weak. In that context, the solution involved all-order resummations of the perturbative corrections enhanced by collinear logs, which have restored the convergence of perturbation theory [13, 14, 15, 16, 17]. In turn, those resummations relied in an essential way on the existence of a Mellin representation for the (NLO) BFKL kernel and its 'collinear improvement'. However, this strategy cannot be simply adapted to the non-linear BK or B-JIMWLK equations, which do not admit a Mellin representation.

In what follows, we shall describe a different strategy, put forward in Refs. [18, 19] (see also [20] for another strategy), where the collinear resummations are performed directly in the transverse coordinate space — the most convenient representation for dealing with multiple scattering in the eikonal approximation.

2. The BK equation: from LO to NLO

The Balitsky-Kovchegov (BK) equation [1, 6] describes the pQCD evolution with increasing energy of the forward scattering amplitude for the scattering between a quark-antiquark dipole and a generic hadronic target (another dipole, a proton, or a nucleus), in the limit where the number of colors is large ($N_c \to \infty$). The leading order (LO) version of this equation defines the 'leading logarithmic approximation' (LLA): it resums all radiative corrections in which each power of the QCD coupling $\bar{\alpha} \equiv \alpha_s N_c/\pi$, assumed to be fixed and small, is accompanied by the energy logarithm $Y \equiv \ln(s/Q_0^2)$ (the 'rapidity'), with s the center-of-mass energy squared and Q_0 the characteristic transverse scale of the target. The LO BK equation reads

$$\frac{\partial T_{\boldsymbol{xy}}(Y)}{\partial Y} = \frac{\bar{\alpha}}{2\pi} \int \mathrm{d}^2\boldsymbol{z}\, \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2} \left[-T_{\boldsymbol{xy}}(Y) + T_{\boldsymbol{xz}}(Y) + T_{\boldsymbol{zy}}(Y) - T_{\boldsymbol{xz}}(Y)T_{\boldsymbol{zy}}(Y)\right]. \tag{2.1}$$

where the subscripts $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$ denote the transverse coordinates of the original quark-antiquark pair and, respectively, the soft gluon emitted in one step of the evolution. The first term in the r.h.s., which is negative, is a 'virtual' correction where the soft gluon has no overlap with the target. The other 3 terms describe 'real' corrections where the virtual gluon exists at the time of scattering (see the left diagrams in Fig. 1). In particular, the term quadratic in T, which is negative, describes unitarity corrections associated with multiple scattering; these become important when the target looks dense on the resolution scales of the projectile. For what follows though, we shall be mostly

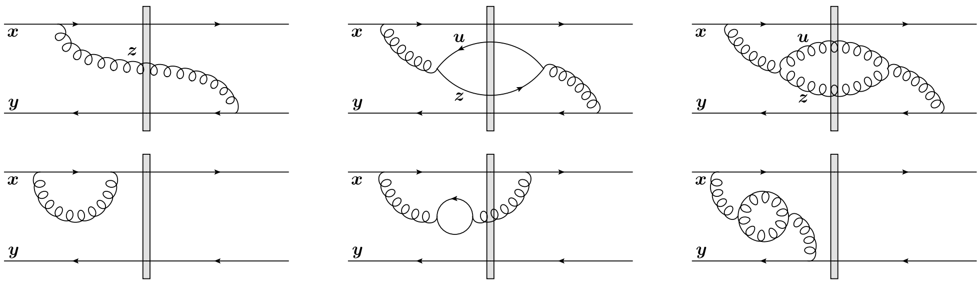

Figure 1: Typical diagrams contributing to the BK equation. The thick vertical line stands for the hadronic target, depicted as a shockwave. Left: LO terms. Middle: NLO terms involving a quark loop. Right: NLO terms involving a gluon loop.

interested in the dilute target, or weak scattering, regime, where this quadratic term is negligible and Eq. (2.1) can be well approximated by its linearized version, the celebrated BFKL equation. We shall moreover focus on the situation where the dipole looks very small on the transverse scale of the target: $rQ_0 \ll 1$, or $Q^2 \gg Q_0^2$, with $r \equiv |\boldsymbol{x}-\boldsymbol{y}| \equiv 1/Q$. Indeed, this regime is characterized by the existence of large radiative corrections, enhanced by the *transverse* (or 'collinear') logarithm $\rho \equiv \ln(Q^2/Q_0^2)$. These corrections come from gluons emissions which occur far outside the original dipole, such that $r \ll |\boldsymbol{x}-\boldsymbol{z}| \simeq |\boldsymbol{z}-\boldsymbol{y}| \ll 1/Q_0$. Such gluons look soft compared to their parent dipole but still hard compared to the target, so they scatter only weakly: $T(z) \ll 1$. In this regime, $T(z) \sim z^2$, hence the (linear) 'real' terms in Eq. (2.1) dominate over the 'virtual' one:

$$\frac{\partial}{\partial Y}\frac{T(r,Y)}{r^2} \simeq \bar{\alpha}\int_{r^2}^{1/Q_0^2} \mathrm{d}z^2\,\frac{r^2}{z^2}\,\frac{T(z,Y)}{z^2}\,. \tag{2.2}$$

The solution to this equation resums powers of $\bar{\alpha}Y\rho$ to all orders. This double logarithmic enhancement — an energy logarithm and a collinear one — reflects the soft and collinear singularities of bremsstrahlung. But Eq. (2.2) is not yet the correct double-logarithmic approximation in QCD at high energy, as we shall see.

The next-to-leading order (NLO) corrections to Eq. (2.1) arise from 2-loop diagrams which involve at least one soft gluon (see the middle and right diagrams in Fig. 1). The maximal contribution *a priori* expected for such a diagram (after subtracting the respective LLA piece, if any) is of order $(\bar{\alpha}Y\rho)\times(\bar{\alpha}\rho) = \bar{\alpha}^2 Y\rho^2$; such a contribution would provide a NLO correction $\sim \bar{\alpha}\rho$ to the BFKL kernel which is enhanced by a collinear log. Yet, the explicit calculation of all such 2-loop graphs in Ref. [7] reveals the existence of even larger corrections, of relative order $\bar{\alpha}\rho^2$, which are enhanced by a *double* collinear logarithm. The complete result at NLO appears to be extremely complicated [7], but it drastically simplifies if one keeps only the terms which are enhanced by at least one transverse logarithm in the regime where $Q^2 \gg Q_0^2$. Then it reads (at large N_c)

$$\frac{\partial T(r,Y)}{\partial Y} \simeq \bar{\alpha}\int_{r^2}^{1/Q_0^2} \mathrm{d}z^2\,\frac{r^2}{z^4}\left\{1-\bar{\alpha}\left(\frac{1}{2}\ln^2\frac{z^2}{r^2}+A_1\ln\frac{z^2}{r^2}-\bar{b}\ln r^2\mu^2\right)\right\}T(z,Y)\,, \tag{2.3}$$

which exhibits 3 types of NLO terms: the double-collinear log previously mentioned, a single collinear log with coefficient $A_1 \equiv 11/12+N_\mathrm{f}/6N_\mathrm{c}^3$, which can be recognized as part of the DGLAP evolution (see below), and the one-loop running coupling. (In the equation above, $\bar{b} = (11N_\mathrm{c} -$

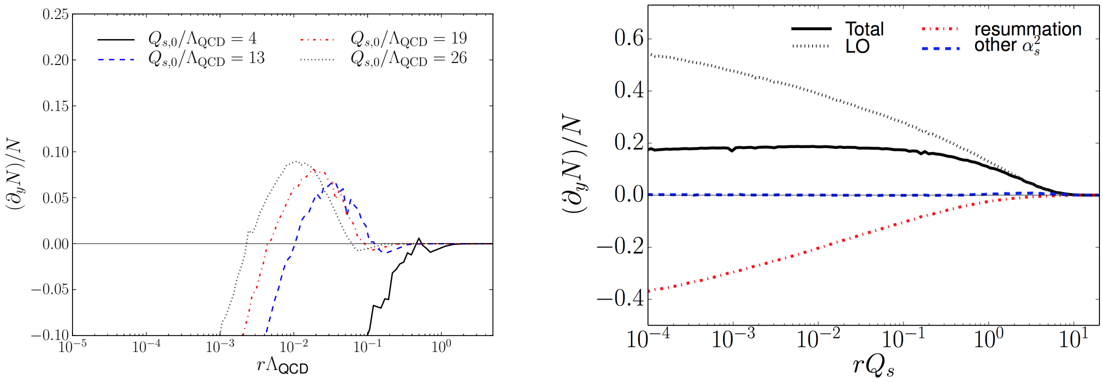

Figure 2: *The rate $(\partial_Y T)/T$ (at $Y = 0$) for the evolution of the dipole amplitude as obtained via numerical solutions [10, 22] to the NLO BK equation. Left [10]: the strict NLO version of the BK equation [7]. (The different curves corresponds to different choices of the saturation momentum $Q_{s,0}$ in the initial condition.) Right [22]: the 'collinearly-improved' version of the NLO BK equation, as constructed in [18, 19].*

$2N_{\rm f})/12N_{\rm c}$ is the first coefficient of the QCD β-function, and μ is a renormalization scale at which the coupling is evaluated.) The NLO corrections enhanced by collinear logs are negative and large and lead to numerical instabilities which render the NLO BK equation void of any predictive power [10, 18]. This is illustrated in Fig. 2 (left) and also in Fig. 3 (middle).

The left plot in Fig. 2 exhibits the rate $(\partial_Y T)/T$ (evaluated at $Y = 0$) for the evolution of the dipole amplitude as numerically obtained [10] from the NLO BK equation [7]. This rate appears to be negative, which signals an instability. (The right plot in Fig. 2 is obtained [22] after appropriate resummations [18, 19] to be described later.)

The main source of this difficulty is the double-collinear logarithm (DCL) $\bar{\alpha}\rho^2$, whose origin and resummation will be discussed in the next sections. This is illustrated in the plots in Fig. 3, which show the dipole amplitude $T(r,Y)$ as a function of $\rho \equiv \ln(1/r^2Q_0^2)$ as obtained by solving different versions of the BK equation with initial condition $T(r,Y=0) = \exp\{-r^2Q_0^2/4\}$ [18]. The left plot refers to the LO BK equation (2.1). For a given Y, the respective solution looks like a front which interpolates between a weak scattering regime at large ρ (small r), where the amplitude is small and rapidly decreasing with ρ ('color transparency'), and a strong scattering regime at lower values of ρ (larger r), where the amplitude approaches the unitarity (or 'black disk') limit $T = 1$. The transition between these two regimes occurs at the saturation scale $\rho_s(Y) \equiv \ln[Q_s^2(Y)/Q_0^2]$, conveniently defined as the value of ρ where $T(\rho,Y) = 0.5$. With increasing Y, the saturation front progresses towards larger values of ρ, meaning that $\rho_s(Y)$ increases with Y — roughly linearly. However, the speed of this progression predicted by the LO evolution — say, as measured by the saturation exponent $\lambda_s \equiv {\rm d}\rho_s/{\rm d}Y$ — is too large to be consistent with the phenomenology: as visible in Fig. 4, the LO BK equation with fixed coupling $\bar{\alpha} = 0.25$ yields $\lambda_s \simeq 1$ for $Y \gtrsim 10$, whereas the fits to the high-energy data at either HERA or the LHC require a lower value $\lambda_s \simeq 0.2 \div 0.3$.

The curves denoted as 'DLA at NLO' in either Fig. 3 (middle) or Fig. 4 (left) are obtained by keeping just the double-logarithmic piece of the NLO corrections and illustrate the instability introduced by this piece (see [18] for details). The other curves refer to various resummations which stabilize and slow down the evolution, to be later described.

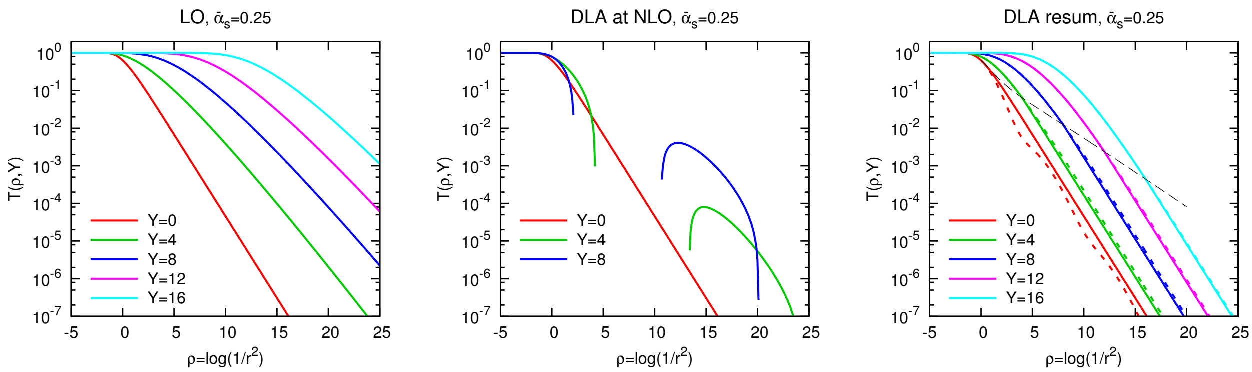

Figure 3: *Numerical solutions to various versions of the BK equation (all with fixed coupling $\bar{\alpha} = 0.25$). [18]. Left: LO. Middle: DLA at NLO (meaning that one keeps just the DCL among the NLO corrections). Right: after resumming DCL's to all orders (that is, by solving Eq. (4.1) with fixed coupling and $\mathscr{K}_{\rm SL} \to 1$).*

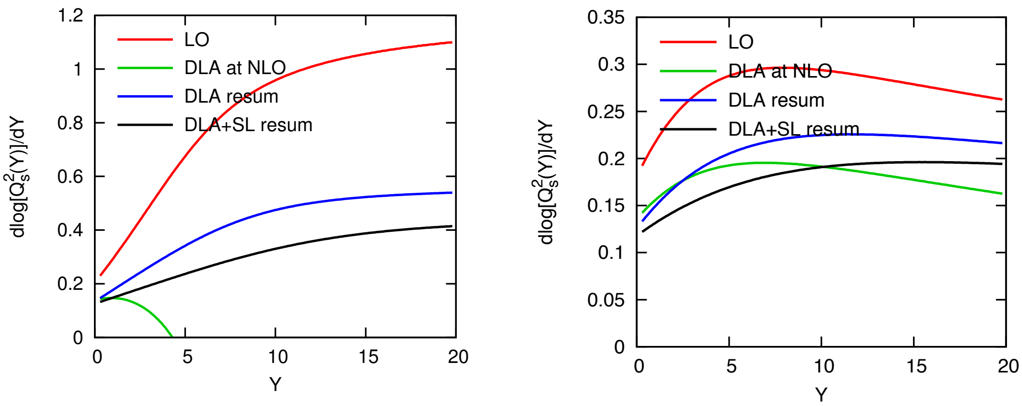

Figure 4: *The saturation exponent $\lambda_s \equiv {\rm d}\ln Q_s^2/{\rm d}Y$ predicted by various versions of the BK equation with either fixed coupling $\bar{\alpha} = 0.25$ (left), or one-loop running coupling (right). The 'DLA at NLO' curves show the instability of the strict NLO approximation, whereas the curves denoted as 'DLA resum' and 'DLA+SL resum' demonstrate the effect of successive resummations in stabilizing and slowing down the evolution.*

3. Time ordering and double-collinear logarithms

The NLO correction $\sim \bar{\alpha}\rho^2$ to the kernel arises from a particular 2-loop contribution of order $\bar{\alpha}^2 Y \rho^3$, which looks anomalously large: it involves a total of 4 (energy or transverse) logarithms, like the respective LLA contribution $\sim (\bar{\alpha} Y \rho)^2$. As a matter of facts, this particular NLO contribution is generated by the same 2-loop diagrams (in terms of topology and kinematics) that are responsible for 2 successive steps in the LLA evolution described by Eq. (2.2): namely, Feynman graphs involving 2 gluon emissions which are strongly ordered in both longitudinal momentum and transverse momentum (or transverse size). The physical interpretation of the enhanced contributions becomes most transparent when the 2-loop diagrams are computed within light-cone, or time-ordered, perturbation theory [18].

For example, let us examine the diagram in Fig. 5 where the longitudinal momenta of the

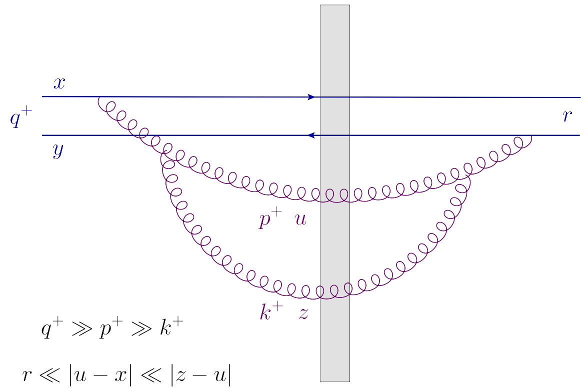

Figure 5: A typical diagram yielding a NLO correction enhanced by a DCL.

emitted gluons obey $q^+ \gg p^+ \gg k^+$, whereas their transverse sizes are ordered according to

$$r = |\boldsymbol{x}-\boldsymbol{y}| \ll |\boldsymbol{u}-\boldsymbol{x}| \simeq |\boldsymbol{u}-\boldsymbol{y}| \ll |\boldsymbol{z}-\boldsymbol{x}| \simeq |\boldsymbol{z}-\boldsymbol{y}| \simeq |\boldsymbol{z}-\boldsymbol{u}| \ll 1/Q_0\,. \tag{3.1}$$

We implicitly assumed here that the dipole projectile is a right mover with large longitudinal momentum q^+, while the hadronic target is a left mover with large momentum P^-. As visible in Fig. 5, the softer gluon k is emitted after and absorbed before the harder one p. This particular time-ordering introduces the energy denominator $1/(k^- + p^-)$ which in turn implies that the largest logarithmic contributions occur when the *lifetimes* of the two gluons are also strongly ordered: $\tau_k \equiv 2k^+/\boldsymbol{k}^2 \ll \tau_p \equiv 2p^+/\boldsymbol{p}^2$. Here $\boldsymbol{p}$ is the transverse momentum of the gluon p, related to its transverse size via the uncertainty principle, $\boldsymbol{p}^2 \sim 1/(\boldsymbol{u}-\boldsymbol{x})^2$, and similarly for the gluon k. Indeed, when this condition $\tau_k \ll \tau_p$ is satisfied, then the four integrations over p^+, u, k^+, and z are all logarithmic[1]. Different hookings of the two gluons lead to 32 diagrams like the one in Fig. 5. Adding all of these contributions, we find in the regime defined in Eq. (3.1) (with simplified notations $|\boldsymbol{u}-\boldsymbol{x}| \to u$ and $|\boldsymbol{z}-\boldsymbol{u}| \to z$)

$$\Delta T(r) = \bar{\alpha}^2 \int \frac{\mathrm{d}k^+}{k^+}\frac{\mathrm{d}z^2}{z^2} \int \frac{\mathrm{d}p^+}{p^+}\frac{\mathrm{d}u^2}{u^2}\,\Theta\left(p^+u^2 - k^+z^2\right)\frac{r^2}{z^2}\,T(z)\,, \tag{3.2}$$

where the step-function implements the lifetime constraint. If there were not for this constraint, Eq. (3.2) would look identical as two iterations of the LO equation (2.2). By integrating out the intermediate gluon p^+,

$$\bar{\alpha}\int_{r^2}^{z^2}\frac{\mathrm{d}u^2}{u^2}\int_{k^+}^{q^+}\frac{\mathrm{d}p^+}{p^+}\,\Theta\left(p^+u^2 - k^+z^2\right) = \bar{\alpha}\left(\ln\frac{z^2}{u^2}\ln\frac{q^+}{k^+} - \frac{1}{2}\ln^2\frac{z^2}{u^2}\right) = \bar{\alpha}Y\rho - \frac{\bar{\alpha}\rho^2}{2}\,, \tag{3.3}$$

one finds the expected LLA contribution $\bar{\alpha}Y\rho$ plus a term independent of Y, namely $-\bar{\alpha}\rho^2/2$, to be interpreted as a NLO correction to the kernel for emitting the softer gluon k^+. This correction reproduces the DCL part of the NLO correction in Eq. (2.3), thus clarifying the physical interpretation of the latter: it expresses the reduction in the rapidity interval ΔY available to the intermediate

[1] Other diagrams which are not time-ordered may contain double logarithms individually, but they cancel in the final answer [18].

gluon due to the time-ordering constraint. This argument extends to all orders [18]: the perturbative corrections enhanced by DCLs can be resummed to all orders by enforcing time-ordering within the 'naive' LLA. However this procedure has the drawback to produce an evolution equation which is *non-local* in rapidity [18, 20], as already manifest in Eq. (3.2). This non-locality reflects the fact that the natural evolution variable at high energy is not the longitudinal momentum k^+ of a gluon from the projectile, or the associated rapidity $Y = \ln(q^+/k^+)$, but rather its lifetime $\tau_k = 2k^+/k_\perp^2$, or equivalently $\eta \equiv Y - \rho$ with $\rho = \ln(Q^2/k_\perp^2)$: the evolution is local in η, but not in Y.

4. The collinearly-improved BK equation

At this stage, something remarkable happens: the non-local equation with time-ordering can be *equivalently* rewritten as an equation *local* in Y, where however both the kernel and the initial condition at $Y = 0$ resum corrections to all orders in $\bar{\alpha}\rho^2$ [18]. This equation can furthermore be extended to resum the *single* transverse logarithms that appear at NLO, cf. Eq. (2.3), namely the single collinear logarithms (SCL) which represent the beginning of the DGLAP evolution and the one-loop running coupling corrections [19].

The SCL too arises from successive emissions in which the second gluon is much softer, both in transverse and longitudinal momenta, than the first one, but now it is the region $\tau_k \sim \tau_p$ which gives the relevant contribution. Its coefficient $A_1 \equiv 11/12 + N_f/6N_c^3$ can be recognized as the first non-singular term in the small-ω expansion of the DGLAP anomalous dimension [19, 15]. This implies that in order to resum such SCLs, it suffices to include A_1 as an 'anomalous dimension', i.e. as a power-law suppression in the evolution kernel. The running coupling corrections can be resummed by choosing the renormalization scale μ as the hardest scale in the problem: $\bar{\alpha} \to \bar{\alpha}(r_{\min})$, where $r_{\min}$ is the size of the smallest dipole, $r_{\min} \equiv \min\{|\boldsymbol{x}-\boldsymbol{y}|, |\boldsymbol{x}-\boldsymbol{z}|, |\boldsymbol{y}-\boldsymbol{z}|\}$.

We are thus led to the following, *collinearly-improved*, version of the BK equation, which faithfully includes the NLO effects enhanced by large (double or single) transverse logarithms, but improves over the strict NLO approximation by resumming similar corrections to all orders:

$$\frac{\partial T_{\boldsymbol{xy}}}{\partial Y} = \int \frac{\mathrm{d}^2\boldsymbol{z}}{2\pi}\, \bar{\alpha}(r_{\min})\, \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\, \mathscr{K}_{\rm DLA}\, \mathscr{K}_{\rm SL}\left[-T_{\boldsymbol{xy}} + T_{\boldsymbol{xz}} + T_{\boldsymbol{zy}} - T_{\boldsymbol{xz}}T_{\boldsymbol{zy}}\right]. \tag{4.1}$$

As compared to the LO BK equation (2.1), the above equation involves the running coupling $\bar{\alpha}(r_{\min})$ with the smallest dipole prescription, together with two multiplicative corrections to the kernel, $\mathscr{K}_{\rm DLA}$ and $\mathscr{K}_{\rm SL}$, which encode the resummations of double and respectively single collinear logarithms. Physicswise, $\mathscr{K}_{\rm DLA}$ implements the condition of time-ordering for the successive soft gluon emissions by the projectile, whereas $\mathscr{K}_{\rm SL}$ resums a subset of the DGLAP logarithms (see Refs. [18, 19] for details). Specifically, $\mathscr{K}_{\rm DLA}$ is defined as the function

$$\mathscr{K}_{\rm DLA}(\rho) = \frac{\mathrm{J}_1\big(2\sqrt{\bar{\alpha}\rho^2}\big)}{\sqrt{\bar{\alpha}\rho^2}} = 1 - \frac{\bar{\alpha}\rho^2}{2} + \frac{(\bar{\alpha}\rho^2)^2}{12} + \cdots, \tag{4.2}$$

evaluated at $\rho^2 = L_{\boldsymbol{xz}r}L_{\boldsymbol{yz}r}$, with $L_{\boldsymbol{xz}r} \equiv \ln[(\boldsymbol{x}-\boldsymbol{z})^2/r^2]$. (This kernel $\mathscr{K}_{\rm DLA}$ has previously been identified in transverse momentum space [17], as an approximation to resummations performed in relation with the NLO BFKL equation [14, 15].) If the double logarithm $L_{\boldsymbol{xz}r}L_{\boldsymbol{yz}r}$ is negative, then one uses its absolute value and the Bessel function J_1 gets replaced by the modified Bessel function I_1. Note however that if, e.g., $(\boldsymbol{x}-\boldsymbol{z})^2 \ll r^2$, so that $L_{\boldsymbol{xzr}} < 0$, then $(\boldsymbol{y}-\boldsymbol{z})^2 \simeq r^2$ and hence $L_{\boldsymbol{yzr}} \simeq 0$. Accordingly, the relatively small daughter dipoles bring no significant contributions to the difference $\mathscr{K}_{\rm DLA} - 1$. Furthermore,

$$\mathscr{K}_{\rm SL} = \exp\left\{-\bar{\alpha}A_1 \left|\ln \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{\min\{(\boldsymbol{x}-\boldsymbol{z})^2, (\boldsymbol{y}-\boldsymbol{z})^2\}}\right|\right\}. \tag{4.3}$$

If one keeps only the terms of order $\bar{\alpha}$ within the product $\mathscr{K}_{\rm DLA}\mathscr{K}_{\rm SL}$, one recovers the NLO collinear logarithms exhibited in Eq. (2.3), as expected. However, any finite-order expansion of $\mathscr{K}_{\rm DLA}\mathscr{K}_{\rm SL}$ would artificially enhance the importance of the 'collinear' regions in phase-space — the regions where the successive gluon emissions (or dipole splittings) are strongly ordered in transverse sizes, or momenta. This is the origin of the instability of the strict NLO approximation to the high-energy evolution, as previously mentioned.

Vice-versa, the all-order resummation of such corrections within the factor $\mathscr{K}_{\rm DLA}\mathscr{K}_{\rm SL}$ suppresses the contributions from the 'collinear' regions and thus restores the convergence of perturbation theory. This is rather obvious for the second factor $\mathscr{K}_{\rm SL}$, which exponentially cuts off the configurations where the daughter dipoles are either much smaller, or much larger, than the parent dipole. But this is also true for the other factor $\mathscr{K}_{\rm DLA}$, which, as already mentioned, becomes important only when the daughter dipoles are sufficiently large, such that $\bar{\alpha}\rho^2 \gg 1$. In that case, the Bessel function $\mathrm{J}_1\big(2\sqrt{\bar{\alpha}\rho^2}\big)$ is rapidly oscillating when varying the position $\boldsymbol{z}$ of the emitted gluon, hence the integral over the regions in space where '$\boldsymbol{z}$ is large' (in the sense that $|\boldsymbol{z}-\boldsymbol{x}| \sim |\boldsymbol{z}-\boldsymbol{y}| \gg r$) averages to zero.

As previously mentioned, the resummation of the double collinear logarithms refers not only to the kernel, where it introduces the factor $\mathscr{K}_{\rm DLA}$, but also to the initial condition to Eq. (4.1) at $Y = 0$. The corresponding resummation is discussed in Ref. [18].

In contrast to the strict NLO BK equation, the evolution described by the resummed equation (4.1) is totally stable. This is illustrated by the right plot in Fig. 3, which refers to the resummation of the DCL's alone. That is, the amplitude $T(\rho,Y)$ shown in that plot is obtained by numerically solving Eq. (4.1) with a fixed coupling $\bar{\alpha} = 0.25$ and $\mathscr{K}_{\rm SL} \to 1$ [18].

The right plot in Fig. 2, taken from [22], convincingly demonstrates that the terms enhanced by collinear logarithms represent indeed the largest among all the NLO corrections. That plot exhibits the evolution rate $(\partial_Y T)/T$ (plotted as a function of r at $Y = 0$) obtained by numerically solving the 'collinearly-improved' version of the fully NLO BK equation. That is, on top of Eq. (4.1) one has added all the "pure-α_s" corrections, i.e. the NLO terms which are not enhanced by transverse logarithms. The evolution rate is seen to be positive (in contrast to the strict NLO result without resummation, Fig. 2 left), which confirms the stabilizing role of the resummation. It is furthermore significantly lower than the respective LO result (also shown in Fig. 2 right), thus demonstrating the role of the higher-order corrections in slowing down the evolution (see also below). But the most striking feature in that plot is the fact that the "pure-α_s" NLO corrections seem to play no role: their overall effect (as singled out in that plot) is consistent with zero. This strongly suggests that after the collinear and running coupling resummations, the perturbation theory becomes rapidly convergent. Accordingly, one may expect the predictions of the collinearly improved BK equation (4.1) to be quite close to the actual physical results.

Another effect of the various resummations, of utmost importance for the phenomenology, is to considerably slow down the evolution: the corresponding "evolution speed" — the saturation exponent $\lambda_s \equiv \mathrm{d}\ln Q_s^2/\mathrm{d}Y$, with $Q_s(Y)$ the saturation momentum — is substantially smaller than that predicted by the LO evolution. This is illustrated by the curves denoted as 'resum' in Fig. 4. Also notice the important role played by the running of the coupling in slowing down the evolution (cf. Fig. 4 right). Already in the absence of any collinear resummation, the running coupling version of the LO BK equation (i.e. Eq. (2.1) with $\bar{\alpha} \to \bar{\alpha}(r_{\min})$) predicts a value $\lambda_s \simeq 0.3$, which is about 3 times smaller than the respective prediction of the fixed-coupling scenario with $\bar{\alpha} = 0.25$. After also including the collinear resummations, one finds $\lambda_s \simeq 0.2$, which is about the right value to be consistent with the phenomenology.

Indeed, using Eq. (4.1) together with appropriate forms for the initial condition, we have been able to obtain good quality fits to the HERA data [21] for the *ep* reduced cross section at small Bjorken $x \leq 10^{-2}$, with only 4 free parameters (for similar fits, without inclusion of the SCLs, see [23]). The evolution speed extracted from the fits is $\lambda_s = 0.20 \div 0.24$. A remarkable feature about these fits is that they are rather discriminatory: they exclude several models for the initial conditions previously used in the literature and also some previous choices for the running coupling. Conversely, they favor the running-coupling version of the McLerran-Venugopalan model for the initial condition and the 'smallest dipole' prescription $\bar{\alpha}(r_{\min})$ for the running of the coupling.

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