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Off-shell amplitudes and 4-jet production in k_T-factorization

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We compute tree level QCD scattering amplitudes in the k_T-factorization approach, which implies that incoming partons are off the mass-shell. To do that, we use a generalisation of the Britto-Cachazo-Feng-Witten (BCFW) recursion relation, which works, in principle, for any number of legs; we then use these results to perform a study of both Single (SPS) and Double Parton Scattering contributions (DPS) to the inclusive 4-jet production in the k_T-factorization framework at leading order and center of mass energies $\sqrt{s} = 7$ TeV and 13 TeV. We discuss the importance of double parton scattering for relatively soft cuts on the jet transverse momenta and find out that symmetric cuts do not suit well the k_T-factorization predictions, because of a kinematic effect suppressing the double parton scattering contribution. We thus propose the systematic use of asymmetric cuts in future experimental analyses and propose new variables to pin down DPS via the analysis of the corresponding differential distributions.

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1. Introduction

Physical, measurable cross sections are obtained by convoluting parton-level scattering amplitudes, describing the interaction of the elementary constituents of the colliding hadrons on a smaller time scale, with universal functions (PDFs) describing the distributions of such partons inside the protons, which account for evolution phenomena taking place on time scales much longer than the parton scattering itself.

In the *High Energy Factorization* (HEF) (aka *k_T-factorisation*) approach [1, 2] to QCD, the amplitudes entering the calculation of cross sections feature particles with off-shell momenta, due to a non vanishing transverse component of the partons with respect to the hadron longitudinal momentum which is taken into account. The importance of the additional contributions thus arising w.r.t. the collinear case is expected to be increasingly relevant as the energy scale of the hadronic collision increases, opening up the possibility of having hard scatterings from partons carrying a small longitudinal component of the proton momentum.

In order for results to be physical, amplitudes need to be gauge invariant, a property whose definition is far form trivial in case there are off-shell legs. A dramatic improvement in the calculation of on-shell scattering amplitudes has been achieved ever since 2005, when the BCFW recursion procedure was first introduced, originally for pure Yang-Mills theories [3, 4] and later extended to include amplitudes with fermions [5]. The question whether this recursion can be generalised to tree level amplitudes with off-shell partons was solved in [6, 7] and a numerical implementation of the procedure is available as well [8]. Such novel QCD scattering amplitudes, endowed with proper generalised Parton Distribution Functions, which we build using the Kimber-Martin-Ryskin presctiption [9, 10], can be used to extract k_T-factorization predictions for pure QCD processes.

The estimation of multi-jet production cross sections is, at the same time, one of the key backgrounds for the searches of new physics and, in the case of 4 jets, one of the preferential channels for the prospective observation of Double Parton Scattering. The latter is defined as the occurrence of two hard (and thus well differentiated from the underlying event) parton-level scatterings in which both protons involved in the hadron collision contribute two of the four hard colliding projectiles. If the final state is a 4-parton one, which is thus observed in the detector as 4 jets, then we can use our methods to compute the relevant $2 \to 4$ and $2 \to 2$ matrix elements in order to study how much DPS is expected to contribute both to the total cross sections and to the differential distributions.

This allows us, in particular, to expand the analysis in [11] and assess the differences between the collinear approach and the k_T-factorization framework.

In the following, we give an overview of the procedure for the computation of gauge-invariant off-shell scattering amplitudes [6, 7] and subsequently present some selected results of our phenomenological analyses [12, 13].

2. BCFW recursion

2.1 Basic formalism

We always consider scattering amplitudes with all particles outgoing. The momentum k^μ can be decomposed in terms of its light-like direction p^μ, satisfying $p \cdot k = 0$ and, if the particle is off-shell, of a transversal part, according to

$$k^\mu = x(q)p^\mu - \frac{\kappa}{2}\frac{\langle p|\gamma^\mu|q]}{[pq]} - \frac{\kappa^*}{2}\frac{\langle q|\gamma^\mu|p]}{\langle qp\rangle}\,, \tag{2.1}$$

with q^μ an auxiliary light-like 4-momentum

$$x(q) = \frac{q\cdot k}{q\cdot p} \quad , \quad \kappa = \frac{\langle q|\not{k}|p]}{\langle qp\rangle} \quad , \quad \kappa^* = \frac{\langle p|\not{k}|q]}{[pq]}\,, \qquad q\cdot p \neq 0\,. \tag{2.2}$$

The coefficients κ and κ^* can be shown to be independent of the auxiliary momentum q^μ, in the sense that any other light-like vector q' can be used in its place, provided $k\cdot q' \neq 0$ and

$$k^2 = -\kappa\kappa^*\,. \tag{2.3}$$

We consider *color-ordered* amplitudes, which contain only planar Feynman graphs and are built with color-stripped Feynman rules. Every scattering amplitude, including the basic 3-point functions with off-shell particles, can be found via the recursion itself, provided one knows 3-point on-shell amplitudes, which can be built from first symmetry principles. No use of Feynman rules is necessary at any step. However, the knowledge of the Feynman rules is necessary to identify the poles in the scattering amplitudes (see below) when applying the recursion. The derivation of such rules for gluons and fermions can be found in [14, 15].

We will be assuming that the reader is sufficiently familiar with the spinor-helicity formalism, which is otherwise sketched thoroughly enough in the aforementioned theoretical papers. For every generally off-shell particle whose momentum is k_i^μ, an orthogonal direction p_i^μ can be always constructed by

$$\begin{aligned} &k_1^\mu + k_2^\mu + \cdots + k_n^\mu = 0 \qquad \text{momentum conservation} \\ &p_1^2 = p_2^2 = \cdots = p_n^2 = 0 \qquad \text{light-likeness} \\ &p_1\cdot k_1 = p_2\cdot k_2 = \cdots = p_n\cdot k_n = 0\,. \qquad \text{eikonal condition} \end{aligned}$$

In the case of an on-shell particle, direction and momentum are simply the same vector.

The polarisation vectors for gluons can be expressed as

$$\varepsilon_+^\mu = \frac{\langle q|\gamma^\mu|g]}{\sqrt{2}\langle qg\rangle}\,, \quad \varepsilon_-^\mu = \frac{\langle g|\gamma^\mu|q]}{\sqrt{2}[gq]}\,, \tag{2.4}$$

where q is the auxiliary light-like vector and g is a short-hand notation for the gluon momentum. We denote gluon spinors with the numbers of the corresponding particles, whereas quarks and antiquarks are always indicated by q and $\bar{q}$ respectively.

2.2 The idea behind BCFW: artificial complex poles

The starting point of the on-shell BCFW recursion relation is Cauchy's theorem

$$\lim_{z\to\infty} f(z) = 0 \Rightarrow \oint \frac{dz}{2\pi \mathrm{i}}\frac{f(z)}{z} = 0\,, \tag{2.5}$$

where the integration contour encloses all the poles of a given rational function $f(z)$ and extends to infinity, implying that the function at the origin $f(0)$ is given by the sum over the residues at the single poles in the complex plane,

$$f(0) = -\sum_i \frac{\lim_{z\to z_i} f(z)\,(z-z_i)}{z_i}\,. \tag{2.6}$$

Now, if $f(z) = \mathscr{A}(z)$, where $\mathscr{A}(z)$ is a scattering amplitude which has somehow been turned into a function of a complex variable without spoiling momentum conservation and on-shellness, it is enough to identify the single poles in z appearing in some of the propagators in order to reconstruct the amplitude in terms of simpler building blocks. On the ground of general unitarity requirements, these are found to be products of lower-point on-shell amplitudes times an intermediate propagator [3, 4].

Now, in order to make a scattering amplitude a rational function of a complex variable z in a way that suits the off-shel case as well, two particles are picked up, say i and j, and each particle's direction is chosen to be the reference vector for the other, so that their momenta with transverse component are

$$\begin{aligned} k_i^\mu &= x_i(p_j)\,p_i^\mu - \frac{\kappa_i}{2}\frac{\langle i|\gamma^\mu|j]}{[ij]} - \frac{\kappa_i^*}{2}\frac{\langle j|\gamma^\mu|i]}{\langle ji\rangle} \\ k_j^\mu &= x_j(p_i)\,p_j^\mu - \frac{\kappa_j}{2}\frac{\langle j|\gamma^\mu|i]}{[ji]} - \frac{\kappa_j^*}{2}\frac{\langle i|\gamma^\mu|j]}{\langle ij\rangle}\,. \end{aligned} \tag{2.7}$$

Let the shift vector be

$$e^\mu = \frac{1}{2}\langle i|\gamma^\mu|j]\,, \quad p_i\cdot e = p_j\cdot e = e\cdot e = 0\,. \tag{2.8}$$

The shifted momenta are thus

$$\begin{aligned} \hat{k}_i^\mu &= k_i + z e^\mu \;=\; x_i(p_j)\,p_i^\mu - \frac{\kappa_i - z[ij]}{2}\frac{\langle i|\gamma^\mu|j]}{[ij]} - \frac{\kappa_i^*}{2}\frac{\langle j|\gamma^\mu|i]}{\langle ji\rangle} \\ \hat{k}_j^\mu &= k_j - z e^\mu \;=\; x_j(p_i)\,p_j^\mu - \frac{\kappa_j}{2}\frac{\langle j|\gamma^\mu|i]}{[ji]} - \frac{\kappa_j^* + z\langle ij\rangle}{2}\frac{\langle i|\gamma^\mu|j]}{\langle ij\rangle} \end{aligned} \tag{2.9}$$

Momentum conservation and either on-shellness or the eikonal conditions $p_i\cdot\hat{k}_i = 0$ and $p_j\cdot\hat{k}_j = 0$ are preserved by the shift (2.9). The changes induced in the momenta or in the directions by this shift are:

$$e^\mu \;=\; \frac{1}{2}\langle i|\gamma^\mu|j] \Leftrightarrow \begin{cases} i \text{ off-shell:} & \hat{\kappa}_i = \kappa_i - z[ij] \\ i \text{ on-shell:} & |\hat{i}] = |i] + z|j] \\ j \text{ off-shell:} & \hat{\kappa}_i^* = \kappa_j^* + z\langle ij\rangle \\ j \text{ on-shell:} & |\hat{j}\rangle = |j\rangle - z|i\rangle \end{cases} \tag{2.10}$$

It is basic to the BCFW argument that (2.10) implies that the large z behaviours of the polarisation vectors of shifted gluons are

$$e^\mu = \frac{1}{2}\langle i|\gamma^\mu|j] \Rightarrow \varepsilon_{i-}^\mu \sim \frac{1}{z} \quad \text{and} \quad \varepsilon_{j+}^\mu \sim \frac{1}{z}\,, \tag{2.11}$$

whereas the opposite helicity polarisation vectors of shifted gluons stay constant. It is important for us, in order for our argument to work in general, to include in our amplitudes the propagators of the external off-shell particles, who play the same role as the gluon polarisation vectors in the on-shell case.

Not all of the shift vector choices are suitable to apply the BCFW recursion, because some of them lead to a violation of the basic hypothesis of the residue theorem

$$\lim_{z\to\infty} \mathscr{A}(z) = 0\,. \tag{2.12}$$

However, one legitimate choice always exists, for amplitudes with at least one gluon on the external lines, as discussed in the original literature on on-shell amplitudes [4, 5] and later generalised to the off-shell case as well [6, 7]. So, we can conclude that we have a method to recursively compute scattering amplitudes with at least one gluon, completely bypassing the rapidly unmanageable Feynman diagram computations. As for amplitudes with no gluon, they can be numerically evaluated by exploiting the Dyson-Schwinger recursion.

2.3 Reconstructing the amplitude from the residues

Our single poles in z always appear due to the vanishing denominators of the gluon or fermion propagators. Our scattering amplitude $\mathscr{A}(0)$ is given by a sum over 4 possible kinds of residues

$$\mathscr{A}(0) = \sum_{s=g,f}\left(\sum_{p}\sum_{h=+,-}\mathrm{A}^s_{p,h} + \sum_i \mathrm{B}^s_i + \mathrm{C}^s + \mathrm{D}^s\right)\,. \tag{2.13}$$

The index s refers to the particle species, namely gluons or fermions; h is the helicity. K^μ will denote the momentum flowing through the propagators exhibiting poles. The residues for poles in both gluons and fermions poles are of the following general kinds:

$\mathrm{A}^s_{p,h}$ are due to the poles which appear also in the on-shell BCFW recursion. The index p stands for the cyclically ordered partitions of the particles into two subsets; the shifted particles are never on the same sub-amplitude. The pole is due to an intermediate virtual gluon, whose shifted momentum squared, $K^2(z)$, is on-shell for

$$z = -\frac{K^2}{2\,e\cdot K}\,.$$

B^s_i residues are due to poles appearing in specific auxiliary eikonal quark propagators, which are needed to account for off-shell particles in a gauge-invariant way. Vanishing of the denominators of the propagators for this eikonal particles means $p_i\cdot\hat{K}(z) = 0$, where $\hat{K}$ is the momentum flowing through the propagator. The location of these poles is

$$z = -\frac{2\,p_i\cdot K}{2\,p_i\cdot e}\,.$$

If the i-th particle is on-shell, these terms are not present.

Finally, C^g and D^g denote the same kind of residues: they appear respectively when the shifted i-th or j-th gluons are off-shell. They are due to the vanishing of the shifted momentum in of this particles: $k_i^2(z) = 0$ or $k_j^2(z) = 0$.

3. Phenomenological analysis of 4-jet production

3.1 Single-parton scattering production of four jets

The collinear factorization formula for the calculation of the inclusive partonic 4-jet cross section reads

$$\sigma^B_{4-jets} = \sum_{i,j} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} x_1 f_i(x_1,\mu_F)\, x_2 f_j(x_2,\mu_F) \times \frac{1}{2\hat{s}} \prod_{l=1}^{4} \frac{d^3 k_l}{(2\pi)^3 2E_l} \Theta_{4-jet}\, (2\pi)^4\, \delta\left(x_1 P_1 + x_2 P_2 - \sum_{l=1}^{4} k_l\right) \overline{|\mathcal{M}(i,j \to 4\,\text{part.})|^2}\,. \tag{3.1}$$

Here $f_i(x_{1,2},\mu_F)$ are the collinear PDFs for the $i-th$ parton, carrying $x_{1,2}$ momentum fractions of the proton and evaluated at the factorization scale μ_F; the index l runs over the four partons in the final state, P is the total initial state partonic momentum, associated to the center of mass energy $\hat{s} = P^2 = (P_i + P_j)^2 = 2\,P_i \cdot P_j$; the Θ function takes into account the kinematic cuts applied and $\mathcal{M}$ is the partonic on-shell matrix element, which includes symmetrization effects due to the possible identity of the final state particles.

The analogous formula to (3.1) for HEF is

$$\sigma^B_{4-jets} = \sum_{i,j} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} d^2k_{T1} d^2k_{T2}\, \mathscr{F}_i(x_1,k_{T1},\mu_F)\, \mathscr{F}_j(x_2,k_{T2},\mu_F) \times \frac{1}{2\hat{s}} \prod_{l=1}^{4} \frac{d^3 k_l}{(2\pi)^3 2E_l} \Theta_{4-jet}\, (2\pi)^4\, \delta\left(x_1 P_1 + x_2 P_2 + \vec{k}_{T1} + \vec{k}_{T2} - \sum_{l=1}^{4} k_l\right) \overline{|\mathcal{M}(i^*,j^* \to 4\,\text{part.})|^2}\,. \tag{3.2}$$

Here $\mathscr{F}_i(x_k,k_{Tk},\mu_F)$ is a transverse momentum dependent (TMD) parton density for a given type of parton. Similarly as in the collinear factorization case, x_k is the longitudinal momentum fraction and μ_F is a factorization scale. The new degree of freedom is introduced via the transverse k_{Tk}, which is perpendicular to the collision axis. The formula is valid when the x's are not too large and not too small and, in order to construct a full set of TMD parton densities, we apply the Kimber-Martin-Ryskin (KMR) prescription [9, 10], which, at the end of the day, amounts to applying the Sudakov form factor onto the PDFs.

$\mathcal{M}(i^*,j^* \to 4\,\text{part.})$ is the gauge invariant matrix element for $2 \to 4$ particle scattering with two initial off-shell legs. We rely on the numerical Dyson-Schwinger recursion in the AVHLIB[1] for its computation. If a complete calculation of 4-jet production in k_T factorization was still missing in the literature, it was mainly because computing gauge-invariant amplitudes with off-shell legs is definitely non trivial. Techniques to compute such amplitudes in gauge invariant ways are by now analytically and numerically well established [6–8].

We use a running α_s provided with the MSTW2008 PDF sets and set both the renormalization and factorization scales equal to half the transverse energy, which is the sum of the final state transverse momenta, $\mu_F = \mu_R = \frac{\hat{H}_T}{2} = \frac{1}{2}\sum_{l=1}^{4} k_T^l$, working in the $n_F = 5$ flavour scheme.

[1] available for download at https://bitbucket.org/hameren/avhlib

3.2 Double-parton scattering production of four jets

The SPS contribution is expected to dominate for high momentum transfer because, as it is intuitively clear, it is highly unlikely that two partons from one proton and two from the other one are energetic enough for two very hard scatterings to take place, as the well-known behaviour of the PDFs for large momentum fractions suggests. However, if the cuts on the transverse momenta of the final state are lowered, a window opens to observe significant double parton scattering effects, as often stated in the literature on the subject and recently analysed for 4-jet production in the framework of collinear factorization [11]. Here we perform the same analysis in HEF, with the goal to assess the differences in the predictions.

First of all, let us present the standard, phenomenologically motivated formula for the computation of differential DPS cross sections, tailored to a four-parton final state,

$$\frac{d\sigma^B_{4-jet,DPS}}{d\xi_1 d\xi_2} = \frac{m}{\sigma_{eff}} \sum_{i_1,j_1,k_1,l_1;i_2,j_2,k_2,l_2} \frac{d\sigma^B(i_1 j_1 \to k_1 l_1)}{d\xi_1} \frac{d\sigma^B(i_2 j_2 \to k_2 l_2)}{d\xi_2}\,, \tag{3.3}$$

where the $\sigma(ab \to cd)$ cross sections are obtained by restricting formulas (3.1) and (3.2) to a single channel and the symmetry factor m is $1/2$ if the two hard scatterings are identical, in order to avoid double counting, and is otherwise 1, whereas ξ_1 and ξ_2 denote generic kinematical variables for the first and second scattering, respectively.

The effective cross section σ_{eff} can be loosely interpreted as a measure of the transverse correlation of the two partons inside the hadrons. In this paper we stick to the widely used value σ_{eff} = 15 mb.

We also have to use an ansatz for DPDFs; for collinear-factorization this is

$$D_{1,2}(x_1,x_2,\mu) = f_1(x_1,\mu)\, f_2(x_2,\mu)\, \theta(1-x_1-x_2)\,, \tag{3.4}$$

where $D_{1,2}(x_1,x_2,\mu)$ is the DPDF and $f_i(x_i,\mu)$ are the ordinary PDFs. The subscripts 1 and 2 distinguish the two generic partons in the same proton. Of course this ansatz can be automatically generalised to the case when parton transverse momenta are included by simply including the dependence on the transverse momentum.

3.3 Comparison to the collinear approach and to ATLAS data with hard central kinematic cuts

Our HEF calculation was first tested against the 8 TeV data recently reported by the ATLAS collaboration [16]. The kinematic cuts are $p_T > 100$ GeV for the leading jet and $p_T > 64$ GeV for the first three subleading jets; in addition $|\eta| < 2.8$ is the pseudorapidity cut and $\Delta R > 0.65$ is the constraint on the jet cone radius parameter.

We employ the running NLO α_s coming with the MSTW2008 sets. For such cuts, not much difference is expected between the two approaches and indeed we find none; DPS effects are irrelevant with this kinematics.

3.4 Comparison to CMS data with softer cuts

As discussed in Ref. [11], so far the only experimental analysis of four-jet production relevant for the DPS studies was realized by the CMS collaboration [17]. The cuts used in this analysis are

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$p_T > 50$ GeV for the first and second jets, $p_T > 20$ GeV for the third and fourth jets, $|\eta| < 4.7$ and the jet cone radius parameter $\Delta R > 0.5$. In the rest of this section, we present our results for such cuts.

As for the total cross section for the four jet production, the experimental and theoretical LO results are:

$$\begin{aligned} \text{CMS collaboration}: &\quad \sigma_{tot} = 330 \pm 5\,(\text{stat.}) \pm 45\,(\text{syst.})\,nb \\ \text{LO collinear factorization}: &\quad \sigma_{SPS} = 697\,nb\,, \quad \sigma_{DPS} = 125\,nb\,, \quad \sigma_{tot} = 822\,nb \\ \text{LO HEF } k_T\text{-factorization}: &\quad \sigma_{SPS} = 548\,nb\,, \quad \sigma_{DPS} = 33\,nb\,, \quad \sigma_{tot} = 581\,nb \end{aligned} \tag{3.5}$$

The LO results need refinements from NLO contributions, much more than it does in the case of the ATLAS hard cuts, as we are of course not that deep into the perturbative region. For this reason, in the following we will always perform comparisons only to data normalised to the total (SPS+DPS) cross sections. We find that this is better than introducing fixed K-factors, whose phase-space dependence is never really under control. What is immediately apparent in the HEF total cross section is the dramatic damping of the DPS contribution with respect to the collinear case. This damping effect is of kinematical nature. The point is that the emission of gluon radiation, which is taken into account via the TMDs in our approach and via the real contribution in a collinear NLO calculation, alters the exact momentum balance of the final state two-jet system, so that a lot of events are not taken into account for the higher transverse momentum just above the cut. In Fig. 1

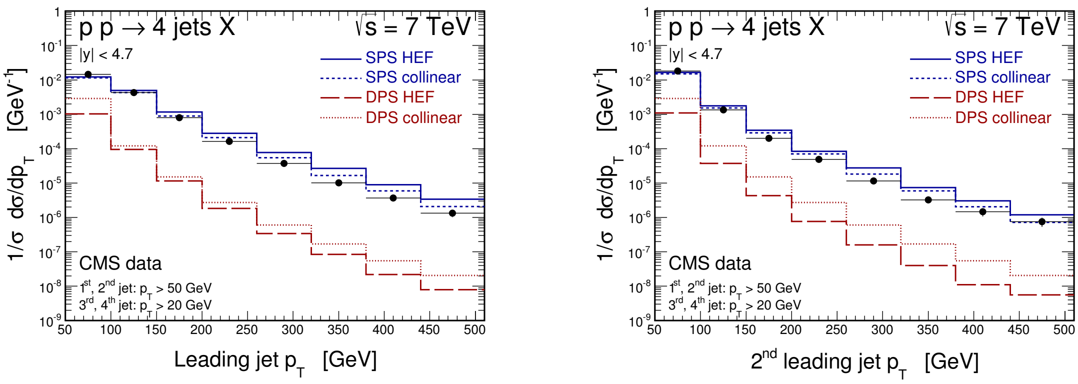

Figure 1: Comparison of the LO collinear and HEF predictions to the CMS data for the 1st and 2nd leading jets.

we compare the predictions in HEF to the CMS data for the 1st and 2nd leading jets transverse momenta spectra. Here both the SPS and DPS contributions are normalized to the total cross section, i.e. the sum of the SPS and DPS contributions. In all cases the renormalized transverse momentum distributions agree quite well with the CMS data.

3.5 HEF predictions for a possible set of asymmetric cuts

Now we propose a set of asymmetric cuts. Specifically, we require $p_T > 35$ GeV for the leading jet, $p_T > 20$ GeV for all the other jets and we stick to $|\eta| < 4.7$, $\Delta R > 0.65$ for rapidity and jet size parameter. An experimental analysis with such cuts is not available at the moment. Of course it would be desirable to have such an analysis in the future.

The theoretical total cross sections for these cuts for four-jet production are:

$$\begin{aligned} \text{LO collinear factorization}: \quad & \sigma_{SPS} = 1969\,nb\,, \quad \sigma_{DPS} = 514\,nb \\ \text{LO HEF } k_T\text{-factorization}: \quad & \sigma_{SPS} = 1506\,nb\,, \quad \sigma_{DPS} = 297\,nb \end{aligned} \tag{3.6}$$

When comparing to (3.5), it is apparent that now the drop in the total cross section for DPS when moving from LO collinear to HEF approach is considerably smaller, as argued.

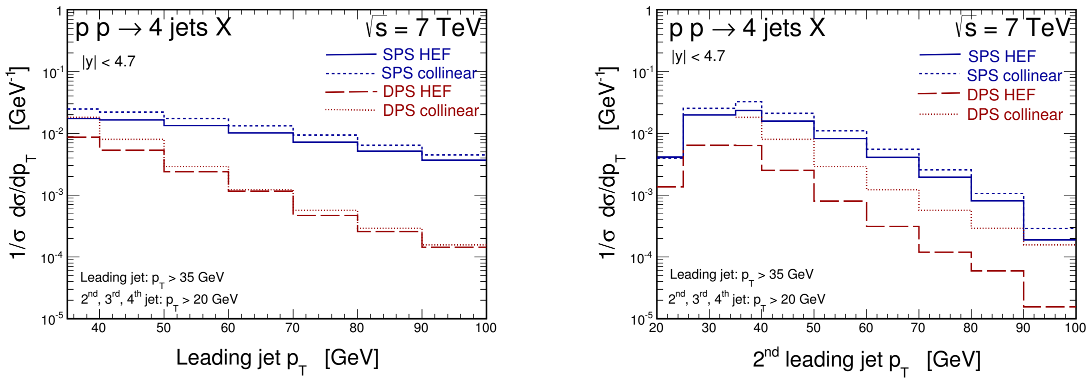

Figure 2: LO collinear and HEF predictions for the 1st and 2nd leading jets with the asymmetric cuts.

In Fig. 2 we show our predictions for the normalized transverse momentum distributions with the new set of cuts.

4. Looking for new variables to enhance DPS

We proposed in [13] that the analysis of cross sections which are differential with respect to three other variables might be interesting in view of the goal of clearly identifying DPS.

The first of such variables was found to be the maximum rapidity distance

$$\Delta Y \equiv max_{\substack{i,j\in\{1,2,3,4\} \\ i\neq j}} |\eta_i - \eta_j|\,. \tag{4.1}$$

A second candidate is the azimuthal correlations between the jets which are most remote in rapidity

$$\varphi_{jj} \equiv |\varphi_i - \varphi_j|\,, \quad \text{for} \quad |\eta_i - \eta_j| = \Delta Y\,. \tag{4.2}$$

Finally, we propose that the minimal sum of two azimuthal distances, defined as

$$\Delta\varphi_{3j}^{min} \equiv min_{\substack{i,j,k\in\{1,2,3,4\} \\ i\neq j\neq k}} \left(|\varphi_i - \varphi_j| + |\varphi_j - \varphi_k|\right)\,, \tag{4.3}$$

can be specially interesting. Lack of space prevents us from repeating here the analysis of [13], but it is at least worth illustrating the key concept in Fig. 3, where it can be seen that, especially for 13 TeV data, the DPS contribution to the differential cross section grows up to $1/2$ for high values of $\Delta\varphi_{3j}^{min}$.

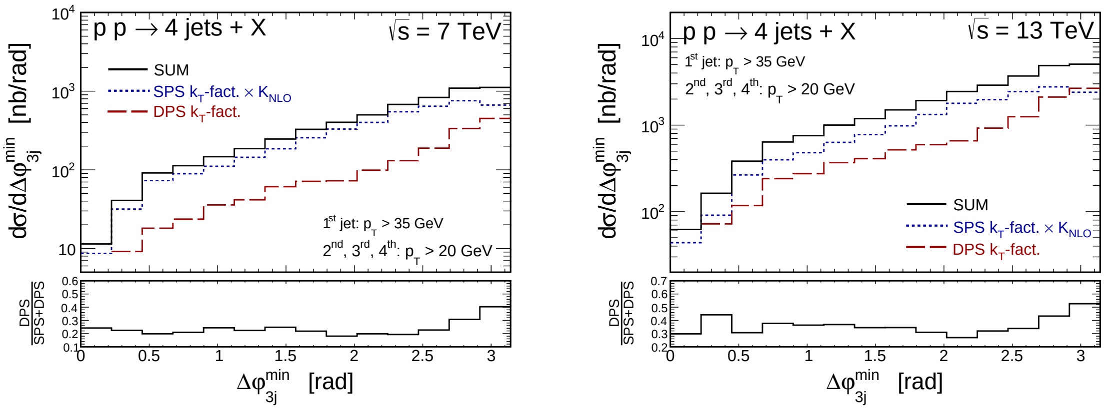

Figure 3: Distribution in $\Delta\varphi_{3j}^{min}$ angle for the asymmetric cut for $\sqrt{s} = 7$ TeV (left) and $\sqrt{s} = 13$ TeV (right). The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.

5. Conclusions

We have compared the perturbative predictions for four-jet production at the LHC in leading-order collinear and high-energy (k_T-)factorization. We find that there is no significant difference between the collinear and HEF approach for hard central cuts, but significant differences show up, especially for DPS, when the cuts on the transverse momenta are lowered. Our approach is able to describe existing CMS data on jet rapidity distributions and we have presented our predictions for differential distributions with respect to other variables as well. We observed that HEF severely tames DPS for symmetric cuts, due to gluon-emission effects encoded in the PDFs which alter the transverse-momentum balance between final state partons. We have found that the damping is sensibly reduces when cuts are not identical.

We find that, for sufficiently small cuts on the transverse momenta, DPS effects are enhanced relative to the SPS contribution when rapidities of jets are large, for large rapidity distances between the most remote jets, for small azimuthal angles between the two jets most remote in rapidity and, finally, for large values of $\Delta\varphi_{3j}^{min}$. In general, the relative effects of DPS in the k_T-factorization approach are somewhat smaller than those found previously in the LO collinear approach. A complete treatment of the subject of this proceeding is extensively given in [12, 13].

Acknowledgments

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