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Time lag in transient cosmic accreting sources

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Models are developed for time lag between maxima of the source brightness in different wavelengths during a transient flash of luminosity connected with a short period of increase of the mass flux onto the central compact object. A simple formula is derived for finding the time delay among events in different wavelengths, valid in general for all disk accreting cosmic sources. In close binaries with accretion disks the time lag is connected with effects of viscosity defining a radial motion of matter in the accretion disk. In AGN flashes, the falling matter has a low angular momentum, and the time lag is defined by the free fall time to the gravitating center. The validity of these models is shown by means of several examples of galactic and extragalactic accreting sources.

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1. Introduction

The optical behavior of the Be star in the high-mass X-ray transient A0535+26/ HDE245770, as discussed in [11], shows that at periastron the luminosity is typically enhanced by 0.02 to a few tenths mag, and the X-ray outburst occurs eight days after the periastron. Indeed, at the periastron an increase of the mass flux occurs. This sort of flush reaches the external part of the temporary accretion disk around the neutron star and moves to the hot central parts of the accretion disk and the neutron star's surface. The time necessary for this way is dependent on the turbulent viscosity in the accretion disk, as discussed in [9]. Behaviour of A0535+26/HDE245770 during the year 2014 was discussed in [8] and relationship ΔV_{mag} vs I_x was derived , predicting not only the arrival time of X-ray outbursts, but also its intensity I_x.

A quantitative model have been constructed in [9] for explaining the time delay between optical and X-ray outbursts. The mechanism for explaining the X-ray – optical delay in A 0535+26/HDE 245770 is based on an enhanced mass flux propagation through the viscous accretion disk. The observed time delay is related to the motion of a high-mass flux region from the outer boundary of the neutron star Roche lobe to the Alfven surface due to the action of the α-viscosity. This mechanism, known as UV-optical delay (the delay of the EUV flash with respect to the optical flash) was observed also for cataclysmic variables [26],[27].

Lags between the optical and X-ray maxima in flashes had been observed also in some AGNs. In [20] delay was observed of ~ 4 days between UV and X-ray emissions in NGC 7469; in [17] it was found a delay of ~ 100 days between optical and X-ray emissions in the Seyfert galaxy NGC 3516; in [18] it was found a delay of ~ 15 days between optical and X-ray emissions in Mkr 509, and in [6] it was found a delay of ≈ 10 days between R, I and X-ray luminosities in the Seyfert galaxy 3C 120; in [25] it was found a delay of 2.4 ± 1.0 days between optical and X-ray emissions in NGC 4051. These tame lags in AGNs are much shorter than we would expect from the simple scaling. In the disk accretion model we expect time lags orders of magnitude larger than in galactic X-ray transients, and from observations they are of the same order. We suggest that it is connected with much faster contraction of accreting matter, what happens in the case of quasispherical accretion of the matter with low angular momentum. A simple formula is derived for the time delay among events in different wavelengths, valid in general for all disk accreting cosmic sources. A model for time lag formation in AGNs is discussed quantitatively in [4].

2. Equations of the accretion disk structure

When the characteristic tine of variability of the mass flux along the accretion disk is larger than the relaxation time of the local disk equilibrium, it is possible to use the approximation of local equilibrium [24], see also [1], for calculating the transient disk structure. The artistic vision of the accretion into magnetized neutron star is given in Figure 1. The equilibrium along a radius of the accretion disk around a star with a mass M is determined by keplerian rotational velocity Ω_K

$$\Omega = \Omega_K = \left(\frac{GM}{r^3}\right)^{1/2}. \tag{2.1}$$

Writing the equation of the vertical equilibrium in approximate algebraic form we get

$$h = \sqrt{2}\frac{v_s}{\Omega}, \tag{2.2}$$

where $v_s = \sqrt{P/\rho}$ is a speed, proportional to the sound velocity, P and ρ are the (gas + radiation) pressure and density at the symmetry plane of the accretion disk, h is a semi-thickness of the accretion disk. The specific angular momentum l of the matter in the accretion disk is connected with the rotation velocity as

$$l = r v_\phi = r^2\Omega, \tag{2.3}$$

The mass flux through the disk at radius r is connected with the radial velocity v_r as

$$\dot{M} = -4\pi h\rho r v_r, \quad \dot{M} > 0, \quad v_r < 0. \tag{2.4}$$

We use α approximation for the turbulent viscosity [24], when $(r\phi)$ component of a stress tensor $t_{r\phi}$ is written as

$$t_{r\phi} = \alpha P, \tag{2.5}$$

where the phenomenological non-dimensional parameter $\alpha \leq 1$. The condition of stationarity of the angular momentum, in which the outward viscous radial flux of the angular momentum is balanced by the angular momentum of the inward flux of the mass is written as [1]

$$r^2 h\alpha P = \frac{\dot{M}}{4\pi}(l - l_{in}) \tag{2.6}$$

The main input into the time lag comes from the outer regions of the disk with $l \gg l_{in}$. Then we have from (2.4),(2.6) the expression for the radial velocity in the form

$$v_r = -\alpha\frac{v_s^2}{v_\phi}. \tag{2.7}$$

Write also a definition of the surface density Σ, and equation (2.6) with account of (2.3), and the condition $l \gg l_{in}$, in the form

$$\Sigma = 2\rho h, \quad \dot{M}\Omega = 4\pi\alpha P h. \tag{2.8}$$

The equation of the local thermal balance in the accretion disk, when the heat produced by viscosity Q_+ is all emitted through the sites of the optically thick accretion disk with a total flux Q_-, at $l \gg l_{in}$ is written as [1]

$$\frac{3}{2}\dot{M}\Omega^2 = \frac{16\pi acT^4}{3\varkappa\Sigma}. \tag{2.9}$$

Here T is a temperature in the symmetry plane of the accretion disk, a is the constant of the radiation energy density, c is the speed of light, $\varkappa$ is the Thompson (scattering) opacity of the matter.

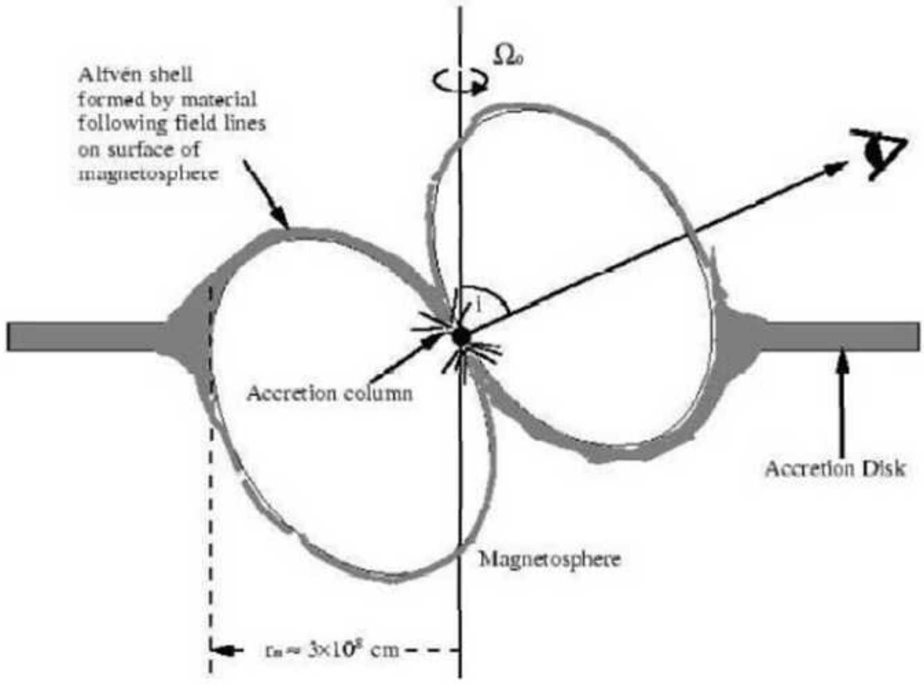

Figure 1: Sketch of the accretion flow in a disk being picked up by a strong neutron star magnetic field [9].

3. Calculation of time lag in the model of disk accretion for galactic X-ray sources

With account of (2.8) and (2.2), this relation may be written as

$$\frac{T^4}{\rho v_s} = \frac{9}{8\sqrt{2}} \frac{\dot{M}\varkappa\Omega}{\pi ac}. \tag{3.1}$$

From the second relation in (2.8), with account of (2.2) we obtain the relation

$$\rho v_s^3 = \frac{\dot{M}}{4\pi\sqrt{2}} \frac{v_\phi^2}{\alpha r^2}. \tag{3.2}$$

The time lag in transient X ray sources is usually formed in the regions of the accretion disk where there is a gas pressure with P_g, and scattering opacity $\varkappa = 0.2(1+X)$, X is a hydrogen mass fraction. The gas constant $\mathscr{R}$ for the hydrogen-helium fully ionized plasma, and the gas pressure, are written as [16]

$$\mathbf{P_g} = \rho\mathscr{R}\mathbf{T}, \quad \mathscr{R} = \frac{k}{\mu m_p}, \quad \mu = \frac{4}{5X+3}. \tag{3.3}$$

Here k is the Boltzmann constant, m_p is the proton mass. Multiplying (3.1) and (3.2) we obtain, with account of (3.3) the relation

$$T^5 = \frac{9}{64\pi^2}\frac{\dot{M}^2 \varkappa \Omega^3}{\alpha ac\mathscr{R}}. \tag{3.4}$$

Our main goal is to express the local parameters of the accretion disk via the observed parameters: mass of the compact star (or a black hole) M, mass flux into the star $\dot{M}$ measured through its bolometric luminosity, effective temperature T_{eff} of the object, evaluated from its spectra. The radius where radiation is the most effective during the transient accretion may be found from the thermal equilibrium condition, accepting that the radiation is emitted from the sides of the disk with effective temperature T_{eff}

$$Q_+ = \frac{3}{8\pi}\dot{M}\frac{GM}{r^3} = \sigma T_{eff}^4, \tag{3.5}$$

giving

$$r = \left(\frac{3}{8\pi}\frac{GM\dot{M}}{\sigma T_{eff}^4}\right)^{1/3}. \tag{3.6}$$

with account of (2.1) and (3.6), the relation (3.4) is written in the form

$$T^5 = \left(\frac{3\dot{M}}{8\pi}\right)^{1/2} \frac{\varkappa}{ac\alpha\mathscr{R}}(\sigma T_{eff}^4)^{3/2}. \tag{3.7}$$

The radial velocity from (2.7) may be written as

$$v_r = -\alpha\mathscr{R}T\sqrt{\frac{r}{GM}}. \tag{3.8}$$

With account of (3.6),(3.7) we obtain

$$v_r = -\left(\frac{3}{8\pi}\right)^{4/15}\left(\frac{\varkappa}{ac}\right)^{1/5}\frac{(\alpha\mathscr{R})^{4/5}}{(GM)^{1/3}}\dot{M}^{4/15}(\sigma T_{eff}^4)^{2/15}. \tag{3.9}$$

For a given $\dot{M}$ in the flash, the value of T_{eff} is changing in time, while the wave of a large $\dot{M}$ is moving to the star along the accretion disk [9], [8], [3]. The equation for the radius of the hot layer flash wave moving to the star is written as

$$\frac{dr}{dt} = v_r. \tag{3.10}$$

We can reduce it to the variable T_{eff} instead of r, using equations (3.6),(3.9), when it is written in the form

$$\frac{dT_{eff}}{dt} = -\frac{3}{4}\frac{v_r}{r}T_{eff}$$
$$= \frac{3}{4}\left(\frac{8\pi}{3}\right)^{1/15}\left(\frac{\varkappa}{ac}\right)^{1/5}\frac{(\alpha\mathscr{R})^{4/5}}{(GM)^{2/3}}\frac{T_{eff}}{\dot{M}^{1/15}}(\sigma T_{eff}^4)^{7/15}. \tag{3.11}$$

Writing Eq. (3.11) as

$$\frac{dT_{eff}}{dt} = A\,T_{eff}^{43/15}, \quad A = \frac{3}{4}\left(\frac{8\pi}{3}\right)^{1/15}\left(\frac{\varkappa}{ac}\right)^{1/5}\frac{(\alpha\mathscr{R})^{4/5}}{(GM)^{2/3}}\frac{\sigma^{7/15}}{\dot{M}^{1/15}}, \tag{3.12}$$

we obtain its solution in the form

$$-\frac{15}{28A}T_{eff}^{-28/15} = t + const. \tag{3.13}$$

For the initial condition $T_{eff}(0) = T_0$, we find the time τ, when the effective temperature $T_{eff} = T_1$, by the relation

$$\tau = \frac{15}{28A}(T_0^{-28/15} - T_1^{-28/15}). \tag{3.14}$$

If T_0 corresponds to maximum of optics, and T_1 corresponds to X-ray maximum, the value of τ represents the time lag between these two maxima, which for the transient X-ray source A0535+26/HDE245770 is close to 8 days [11]. For $T_1 \gg T_0$, with account of (3.12) we have

$$\tau = \frac{15}{28A}T_0^{-28/15} = \frac{5}{7}\left(\frac{3}{8\pi}\right)^{1/15}\left(\frac{ac}{\varkappa}\right)^{1/5}\frac{(GM)^{2/3}}{(\alpha\mathscr{R})^{4/5}}\frac{\dot{M}^{1/15}}{(\sigma T_0^4)^{7/15}} \tag{3.15}$$

Substituting numbers into (3.15), and introducing dimensionless values

$$m = \frac{M}{M_\odot}, \; \dot{m} = \frac{\dot{M}}{10^{-8}M_\odot/\text{year}}, \; T_4 = \frac{T_0}{10^4\text{K}}, \tag{3.16}$$

we obtain the value of a time lag between optical and X-ray maxima in the transient Be/X-ray source, resulting from a rapid increase of the mass flux of matter with $X = 0.7$ through the accretion disk, with account of (3.3), in the form

$$\tau = 6.9\frac{m^{2/3}\dot{m}^{1/15}}{\alpha^{4/5}T_4^{28/15}}\ \text{days}. \tag{3.17}$$

This formula is useful for having an approximate idea about the time lag between optical and X-ray maximum emissions in a cosmic disk accreting source, independent of its nature: white dwarf, neutron star or black hole. What we can remark is that the time lag depends on the mass almost linearly ($m^{2/3}$), while on the mass flux very weakly, $\dot{m}^{1/15}$. The dependence on viscosity α and on T_4 is more serious.

In the case of **A0535+26/HDE245770**, using the neutron star mass M = 1.5 $M_\odot$, with the mass accretion rate $\dot{M} \simeq 7.7\times10^{-7}$ $M_\odot$ yr^{-1}, $\dot{m} = 77$ [7], the viscosity coefficient α = 0.15, and $T_4 \simeq 2.8$ [5], the time-delay, computed with the formula (3.17), $\tau = 8.06$ days, coincides with the experimental time-delay (X-ray–optical) of ~ 8 days.

In the cataclysmic variable **SS Cygni**, with the mass of white dwarf $M = 0.97$ $M_\odot$ [10], mass accretion rate $\dot{M} = 4\times10^{17}$ g s^{-1} $\approx 6.3\times10^{-9}$ $M_\odot yr^{-1}$, $\dot{m} = 0.63$ [12], viscosity α = 0.2 [27], and $T_4 = 4$ [13], the time-delay, computed with the formula (3.17), is $\tau \simeq 1.8$ days. The experimental time-delay (UV–optical) of 0.9–1.4 days [29], and for $\alpha = 0.3$ we obtain $\tau = 1.35$ days, well inside the error box.

In the X-ray source **Aql X-1**, with the mass of the neutron star $M = 1.4$ $M_\odot$ [28], mass accretion rate $\dot{M} = 4\times10^{17}$ g s^{-1} $\approx 6.3\times10^{-9}$ $M_\odot yr^{-1}$, $\dot{m} = 0.63$ [30],[19], viscosity α = 0.2, and

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$T_4 \simeq 2.8$, the time-delay computed with the formula (3.17) is $\tau \simeq 4.4$ days. The observed time-delay (X-ray–optical) of ~ 3 days [23] is better reproduced for $\alpha = 0.3$, when $\tau = 3.2$ days follows from formula (3.17).

In the black hole X-ray transient **GRO J1655-40**, with the mass of the black hole $M \simeq 7\ M_\odot$ [21], and the approximate relationship $M/\dot{M} \sim 10^7$ yr [15] we have $\dot{M} \sim 7 \times 10^{-7}\ M_\odot yr^{-1}$. Accepting $T_4 = 3$, we obtain from the formula (3.17) the time-delay $\tau \simeq 15.6$ days at $\alpha = 0.2$, and $\tau \simeq 11.3$ days at $\alpha = 0.3$. The optical precursor was observed ~ 6 days before the X-ray flash [22], so the formula (3.17) overestimates the observed value. This time delay was observed in the X-ray flash of the LMXB X-ray nova with a black hole, which is usually connected with instability, developed in the accretion disk after accumulation of sufficiently large mass in the quiescent state. Development of the instability leads to non-stationary behaviour with larger viscosity. This explains why formula (3.17), obtained for a stationary accretion disk, overestimates the value of the time lag. However, if we use a larger value of viscosity, like $\alpha = 0.6$, the time lag computed with the formula (3.17) is $\tau \simeq 6.5$ days, in agreement with the observed time delay.

4. Time lags in observations of AGNs

The accretion disks around SMBH have much wider regions, with $P = P_{rad}$, so the formula for the time lag is written as [4]

$$\tau_r = 4.9 \times 10^7 \frac{m^{2/3}}{\alpha \dot{m}^{5/6} T_4^{14/3}} \text{ days.} \tag{4.1}$$

This formula gives the values of a time lag orders of magnitude longer than observational ones. Therefore another model of the formation of time lags in AGN is considered.

The accretion on SMBH in AGN takes place from surrounding gas, with low angular momentum, so formation of an accretion disk may not happen, and accretion could take place in the form of a spherical flow. The flashes in AGNs, connected presumably with tidal disruptions of surrounding stars in close encounter with SMBH, are accompanied by rapid ejection of matter with formation of a jet flowing outside, and another rapid jet directed toward the SMBH. Large part of the inner jet moves to the SMBH with a velocity, of the order of free-fall speed. The tidal disruption of star (more than 600 000 sites in GOOGLE) leads to the optical flash, and the X-ray flash starts when the matter of the inner jet heats sufficiently to radiate in the X-ray region. We suggest, that in the observed events this part is sufficiently large for creation of a strong initial flash. The sources of radiation during tidal disruption event are given in in Figure 2. The time delay between the optical and X-ray flashes follows from observations, the radius, at which the optical flash happens, is calculated for the motion with free fall velocity v_{ff} as

$$v_{ff} = \sqrt{\frac{2GM}{r}}, \quad \frac{dr}{dt} = v_{ff}, \quad \tau_{ff} = \frac{2}{3}\frac{r^{3/2}}{\sqrt{GM}}. \tag{4.2}$$

Taking τ_{ff} equal to observational time delay τ_{obs}, we obtain a radius of the optical flash r_{opt} as

$$r_{opt} = 1.65 \times 10^{12} \tau_{obs} m^{1/3} \text{ cm}, \tag{4.3}$$

where τ_{obs} is expressed in days, and SMBH mass m in solar masses.

A tidal disruption of a star happens when the tidal force from SMBH F_t becomes comparable with gravitational force of the star F_s at the radius of a star R_s with a mass M_s. The radius of tidal disruption r_t when these two forces become equal is written as, see e.g. [2]

$$F_t = 2\frac{GM}{r^3}R_s, \quad F_s = \frac{GM_s}{R_s^2},$$
$$r_t = R_s\left(2\frac{M_{BH}}{M_s}\right)^{1/3} = R_s(2\frac{m}{m_s})^{1/3}. \tag{4.4}$$

Table 1: The values of masses of the disrupted stars $m_s = \frac{M_s}{M_\odot}$, the distance from the central black hole at which the optical flash happens r_{opt}, and radii of stars R_s, which are disrupted at the tidal radius r_t, identified with $r_{opt} = r_t$, are given for SMBH in AGNs, listed above.

Source	$\mathrm{r_{opt} = r_t}$	$\mathrm{R_S}$
Mrk 509	5.2×10^{15}cm	$114\,m_s^{1/3}R_\odot$
NGC 7469	8.95×10^{14}cm	$47\,m_s^{1/3}R_\odot$
3C 120	3.3×10^{15}cm	$100\,m_s^{1/3}R_\odot$
NGC 3516	1.1×10^{16}cm	$409\,m_s^{1/3}R_\odot$
NGC 4051	2.5×10^{14}cm	$36\,m_s^{1/3}R_\odot$

We see, that the flashes in AGN originating from tidal disruption may happen, when a giant star with a radius between few tens and few hundreds solar radius enters the tidal radius. The observational time lag between the optical and X-ray flashes is quite consistent with the model, considered here, in which the optical flash happens at the radius of tidal disruption, and the X-ray flash happens when the matter accreting with the free fall speed becomes hot enough due to adiabatic heating.

5. Conclusions

1. The time lag in disk accreting galactic close binary sources is based on a sudden increase of the accretion flow, started from disk periphery, related to the optical maximum. The matter in the accretion disk moves inside with the speed, determined by a turbulent viscosity. The derived. formula gives results in a good accordance with observational values.

2. The optical flash in AGN happens at a disruption of a giant star entering the radius of strong tidal forces. The matter with low angular momentum falls into SMBH in the form of a quasi-spherical flow. X-ray flash happens when the falling matter reaches the hot inner regions. Knowing the SMBH masses from observations we obtain the radii of the disrupted star between few tens and few hundreds of $R_\odot$.

3. The matter with larger angular momentum, appeared in the disruption of the star, should form an accretion disk through which the matter will move to the center due to turbulent viscosity, similarly to flashes in close galactic binaries. After such a flash in AGNs we expect a long duration variability in the whole electromagnetic spectrum.

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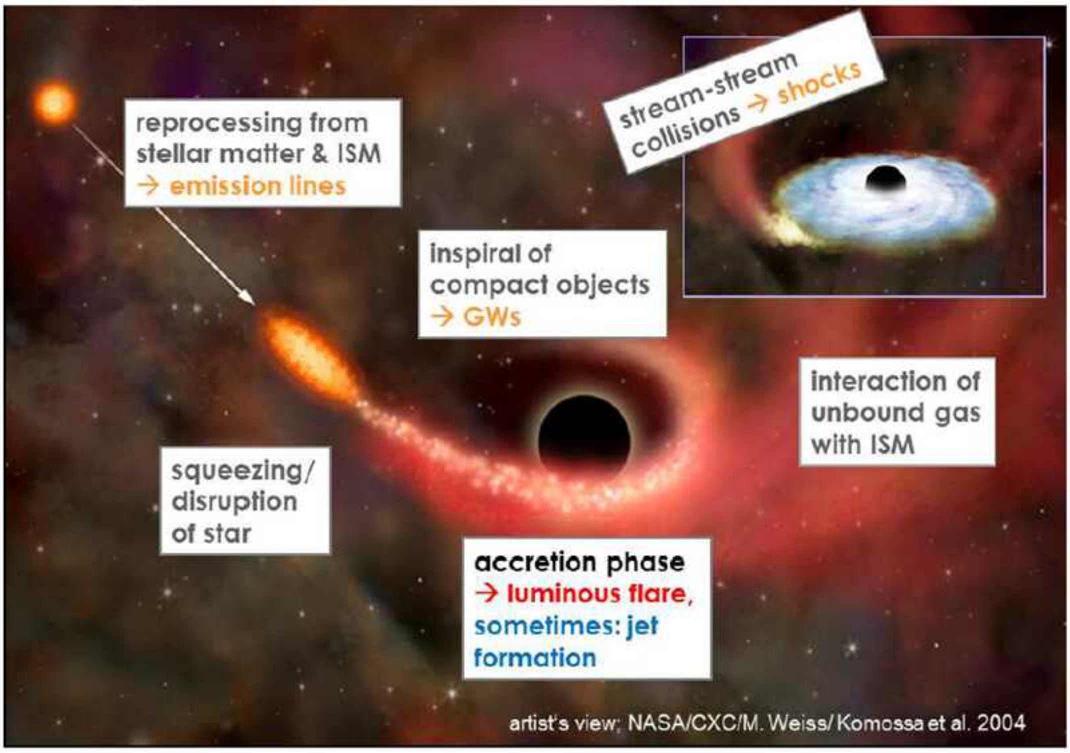

Figure 2: Sites and sources of radiation during the evolution of Tidal Disruption Event (TDE) [14]

6. Acknowledgments

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DISCUSSION

MICHELE MONTGOMERY 1. Why choose X=0.7 as I did not see Y value or Z value in μ equation $\left(\mu = \frac{4}{5X+3}\right)$?

2. What is γ in ideal gas EoS?

G. BISNOVATYI-KOGAN 1. For $Z \ll X, Y$ input of heavy elements in EoS was neglected. The value X=0.7 is related to solar composition, small deviations don't change the results.

2. $\gamma = 5/3$.