

# PoS

# Amplitude analyses with charm decays at $e^+e^-$ machines

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> Amplitude analyses provide uniquely powerful sensitivity to the magnitudes and phases of interfering amplitudes. This makes amplitude analyses of charm decays particularly useful for studies of CP violation, *D* mixing, and other phenomena. A summary of some recent analyses of this type at  $e^+e^-$  machines is presented here.

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#### 1. Introduction

Decays of a heavy meson into three of more light mesons provide an ideal environment for CP studies. The prevalence of light meson resonances leads to a significant amount of phase motion in a non-trivial distribution over the Dalitz plot. For the special case of decays to three pseudoscalars, the phase space density is uniform across the Dalitz plot. This means that any visible structure is directly related to the dynamics of the accessible amplitudes. An amplitude analysis of such a reaction provides a complete description of the data. With this tool, it is possible to measure the decay amplitudes and phases of intermediate reactions, accurately measure branching fractions, and study the effects of final state interactions.

Amplitude analyses at  $e^+e^-$  machines have some significant benefits over similar studies at hadron machines. In particular,  $e^+e^-$  machines provide a relatively clean environment, an efficient reconstruction of neutral particles like  $\pi^0$  and  $\eta$  mesons, and a nearly 100% trigger efficiency. Of course, different  $e^+e^-$  machines provide different environments with particular benefits. The lack of phase space to produce fragmentation particles makes charm threshold production with detectors like CLEOc and BESIII an excellent source of clean charm samples, though with much lower statistics than those produced at the B-factories. Charm threshold samples are also particularly useful for studies that make use of the quantum correlation of the *D* meson pairs.

The largest samples of charm decays come from the *B*-factories, including CLEO, Belle, and BaBar. Typically, charm samples are collected by reconstructing a slow pion from a prompt  $D^*$  decay and using the pion charge to tag the flavor of the  $D^0$ . There are indications that the large charm samples from *B*-factories could also be useful to constrain charm interference parameters using charm mixing [1]. Very high statistics samples from the Belle II detector will become available in the next decade. Belle II is scheduled to start taking data in 2017, with a goal of collecting 50 ab<sup>-1</sup> by 2026.

#### 2. Amplitude analysis

The most commonly performed amplitude analyses make use of the isobar model and its extensions. In this model, the total amplitude is expressed as a coherent sum of quasi-two-body contributions. Often, a (complex) non-resonant term is also included. Fits to the data can be either binned or unbinned, but naturally come with inherent model dependence. The strong interaction dynamics, typically in the form of line shapes, barrier factors, etc., must be described by the amplitude. Alternative methods to avoid model dependence usually involve binning the data to avoid making assumptions about the dynamics (see for example Ref. [2], discussed below).

Amplitude analyses often suffer from limitations due to model dependence. Notable effects include dependence on line shapes from coupled-channel and threshold effects. The most prevalent isobar models use a sum of Brett-Wigner line shapes. This model has the distinct drawback that it violates unitarity, especially for broad, overlapping resonances. Another particular limitation to isobar models is that it is difficult to differentiate S-wave amplitudes and non-resonant terms. This can lead to unphysical phase variations in the amplitudes. More robust methods exist, but still suffer from model dependence to some degree. These methods include using scattering data to constrain phase variations or using input from theory such as dispersion relations.

Many analyses make use of the K-matrix formalism, which provides an elegant means to express the unitarity of the S-matrix for two-body scattering, under the assumption that the twobody system is isolated from the rest of the final state. For a single resonance in a single channel, the K-matrix formalism reduces to the constant width Brett-Wigner form. The K-matrix formalism is preferable in the case of multiple amplitudes, for which the Brett-Wigner masses and widths may be shifted, especially for two nearby resonances. Some relatively recent charm analyses make use of extensive studies of S-wave dynamics using the K-matrix formalism [3, 4].

Another possible source of a description for the S-wave dynamics in charm amplitude analyses include a recent study of the  $\pi^0 \pi^0$  system in radiative  $J/\psi$  decays at BESIII [5]. Since the  $\pi^0 \pi^0$ system does not interact with the final state photon, this reaction presents a very simple environment in which to study the scalar spectrum. In addition, the all-neutral channel is very clean relative to the charged channel. The amplitude analysis is further simplified by the fact that only  $J^{PC} = \text{even}^{++}$ amplitudes are accessible. By extracting the  $\pi^0 \pi^0$  amplitudes in bins of  $\pi^0 \pi^0$  invariant mass, this study reports the function describing the strong interaction dynamics in a model independent way (Fig. 1).



**Figure 1:** The 0<sup>++</sup> amplitude from a mass independent amplitude analysis of the  $\pi^0 \pi^0$  system produced in radiative  $J/\psi$  decays at BESIII [5].

#### 3. Three-body charm decays

Weak three-body decays of open heavy flavor mesons are a useful environment to study the interference between intermediate resonances. Three-body decays have the advantage of additional kinematic freedom relative to two-body final states. Intermediate resonances dominate three-body reactions, causing a non-uniform distribution of events over the Dalitz plot. Since all events have the same final state, multiple resonances in close proximity interfere with each other, yielding an opportunity to measure phases.

Final state interactions (FSI) pose a challenge to measurements of decay rates and phases of decay amplitudes, which can be significantly modified due to the effects of FSI. The rich substructure apparent in, for example,  $D^+$  decays to  $K_S \pi^+ \pi^0$  [2] indicates the complexity of the final state interactions involved in the reaction (Fig. 2). A better understanding of FSI in charm decays is important to reduce the uncertainties related to  $D^0 - \overline{D^0}$  mixing parameters and of the CKM angle  $\gamma$ . Experimental measurements can also serve to help improve theoretical models of related phenomena.



**Figure 2:** The Dalitz plot (a) and projections (b-d) for  $D^+$  decays to  $K_S \pi^+ \pi^0$  [2].

The recent analysis of  $D^+$  decays to  $K_S \pi^+ \pi^0$  by BES III, using a 2.91 fb<sup>-1</sup> sample collected at the  $\psi(3770)$ , includes an isobar model fit with six quasi-two-body Cabibbo-favored (CF) amplitudes plus a non-resonant term. This reaction is a golden mode to study the  $K\pi$  S-wave in Ddecays. A series of models, including a  $\kappa$  pole and Breit-Wigner functions, are fit to the data. The results are cross-checked with a model independent analysis that gives consistent results (Fig. 3).

#### 4. Time integrated amplitude analyses

Standard model predictions for CP asymmetries in the charm sector are expected to be quite small. A larger value for experimentally measured CP asymmetries could therefore be an indication of new physics or an enhancement due to FSI. Direct CP asymmetries due to interference between tree and penguin-level diagrams could be exhibited in singly Cabibbo-suppressed (SCS) decays, making them of interest for CP violation (CPV) studies. A thorough search of this type was recently



Figure 3: The magnitude and phase of the  $K\pi$  S-wave for model dependent and model independent fits [2].

performed by the BaBar collaboration in  $D^+$  decays to  $K^+K^-\pi^{\pm}$  [6], with the goal of probing the Dalitz-plot substructure for asymmetries in both the magnitudes and phases of intermediate states. This study includes a time integrated (TI) model dependent amplitude analysis with an isobar model of coherent Breit-Wigner line shapes (Fig. 4). The  $K\pi$  S-wave is parametrized with a  $\kappa(800)$ , a  $K^*(1430)$ , and a non-resonant amplitude. The case of no CPV is initially assumed to determine the relative fit fraction of intermediate resonances. Next, this assumption is relaxed for resonances with a fit fraction of at least 1%. The resulting charge asymmetries are consistent with zero.

Similar analyses of four-body decays like  $D^0 \to K^+ K^- \pi^+ \pi^-$  may be useful in measurements of the CKM angle  $\gamma$  in *B* decays, but there is insufficient knowledge of the substructure of the *D* decays to make a reliable assessment of the potential sensitivity. An amplitude analysis of  $D^0$  decays to  $K^+ K^- \pi^+ \pi^-$  can expose effects that may be diluted or concealed by the more inclusive T-odd correlation approach, but is complicated due to the non-uniform phase space of four-body decays and the possibility to have two separate intermediate resonances contributing. The time integrated amplitude analysis of this reaction by the CLEO collaboration (see Fig. 5 includes amplitudes for single intermediate resonances as in  $D^0 \to K^* K \pi$ ;  $K^* \to K \pi$ , quasi-two-body amplitudes like  $D^0 \to \phi \rho^0$ ;  $\phi \to KK$ ;  $\rho^0 \to \pi \pi$ , and cascade amplitudes such as  $D^0 \to K_1 K$ ;  $K_1 \to K^* \pi$ ;  $K^* \to K \pi$ 



**Figure 4:** The Dalitz plot (top-left) and projections for  $D^+$  decays to  $K^+K^-\pi^{\pm}$  assuming no CPV [6]. The regions in the Dalitz plot are used in a model-independent analysis. The horizontal lines in the pulls for the fit projections indicate  $3\sigma$  deviations.

and makes use of both flavor-tagged and CP-tagged samples [7]. Most resonances are parametrized by a relativistic Breit-Wigner line shape. The exception is the coupled-channel Flatte parametrization for the  $f_0(980)$ . Non- resonant states with orbital angular momentum between daughters are modeled as broad resonances, wherein the spin factor alone alters the distribution over phase space. Measures of direct CP asymmetry are mostly consistent with zero.

## 5. Time dependent amplitude analyses

The first evidence for  $D^0 - \overline{D^0}$  mixing came almost a decade ago from studies by the BaBar [8], Belle [9], and CDF [10] collaborations. None of these directly measured the normalized mass and



Figure 5: The invariant-mass squared projections for the CLEO analysis of  $D^0$  decays to  $K^+K^-\pi^+\pi^-$  [7].

width differences. Time dependent (TD) amplitude analyses in decays to CP conjugate final states have a unique sensitivity to charm mixing parameters, which are defined for example in Ref. [11]. Any variation of the decay rate from exponential behavior depends on the magnitudes and phases of direct and mixing amplitudes as q/p, as well as the mixing parameters x and y. The most recent measurements of these quantities are using in global fits by the Heavy Flavor Averaging Group [11] to determine the world average mixing and CP violation parameters.

A recent analysis of  $D^0$  decays to  $K_S \pi^+ \pi^-$  was conducted by the Belle collaboration [12] to study  $D^0 - \overline{D^0}$  mixing and search for CPV. A time dependent amplitude analysis is carried out with Dalitz amplitudes expressed as a sum of quasi-two-body amplitudes. Relativistic Breit-Wigner line shapes are used to describe P- and D-wave decays while a K-matrix formalism is used to describe the  $\pi\pi$  S-wave dynamics. A mixture of Breit-Wigner line shapes and a scattering-like parametrization is used for the  $K\pi$  S-wave. First, a time integrated fit is used to extract the

magnitudes and phases of intermediate states (Fig. 6). Subsequent fits allow the mixing parameters,  $D^0$  lifetime, parameters of the proper decay-time resolution function, and amplitude model parameters to be free parameters. The fit is also performed with and without the assumption of no CPV by adding parameters for |q/p| and  $\arg(q/p)$ . The charm mixing parameters are determined to be  $x = (0.56 \pm 0.19^{+0.03+0.06}_{-0.09-0.09})\%$  and  $y = (0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.06})\%$ , assuming no CPV, where the uncertainties are statistical, experimental systematic, and systematic due to the amplitude model, respectively. These results are consistent, but more precise than previous measurements and give an estimated significance of  $D^0 - \overline{D^0}$  mixing of  $2.5\sigma$ . The results of a search for CPV give the most accurate values for |q/p| and  $\arg(q/p)$  for a single experiment at  $0.9^{+0.16+0.05+0.06}_{-0.15-0.04-0.05}$  and  $(-6 \pm 11 \pm 3^{+3}_{-4})^\circ$ , respectively, consistent with no CPV.



**Figure 6:** The Dalitz plot (top-left) and projections for  $D^0$  decays to  $K_S \pi^+ \pi^-$  from a recent amplitude analysis by the Belle collaboration [12].

A very recent analysis of  $D^0$  decays to  $\pi^+\pi^-\pi^0$  by the BaBar collaboration [13] uses an

amplitude analysis similar to the method proposed by CLEO in  $D^0$  decays to  $K_S \pi^+ \pi^-$  [14]. Unlike the original measurements of D mixing, such a TD amplitude analysis of self-conjugate decays allows for a direct measurement of the mixing parameters. The  $\pi^+\pi^-\pi^0$  channel is dominated by  $\rho(770)^{\pm,0}$  intermediate states with small additional contributions from isoscalar and excited  $\rho$  states plus a small non-resonant amplitude (Fig. 7). This is the first measurement with this reaction of the charm mixing parameters, which are determined to be  $x = (1.5 \pm 1.2 \pm 0.6)\%$  and  $y = (0.2 \pm 0.9 \pm 0.5)\%$ . The authors suggest that statistical uncertainties will dominate even with data samples 10 to 100 times larger than the 468.1 fb<sup>-1</sup> sample, recorded at the  $\Upsilon(4S)$ , used in this analysis. This is due to the ability to reduce several systematic uncertainties with the larger data samples.



**Figure 7:** The (a) Dalitz plot and (b) difference between the Dalitz plot and fit model, as well as the fit projections (c-e) for  $D^0$  decays to  $\pi^+\pi^-\pi^0$  from a recent amplitude analysis by the BaBar collaboration [13].

# 6. Where are the baryons?

It is worth noting that there has been little recent activity on amplitude analyses of decays of charmed baryons. While the BESIII [15] and Belle [16] collaborations have recently performed measurements of branching fractions of  $\Lambda_c$  decays, neither study included an amplitude analysis.

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The most recent amplitude analysis of  $\Lambda_c$  decays (to  $pK^+\pi^-$ ) was performed by E791 in the year 2000 using data produced with  $\pi$ -N interactions [17]. This amplitude analysis was performed with about 1000 events, similar to the statistics available at Belle and BESIII.

One caveat to the claim about recent activity on amplitude analyses of charmed baryon decays is that BESIII has recently measured SCS decays of the  $\Lambda_c$  to  $p\pi^+\pi^-$  and  $pK^+K^-$ , using an amplitude analysis to determine the detection efficiency [18]. The results of the amplitude analysis are not reported.

#### 7. Summary

Amplitude analysis is an extremely useful tool for heavy flavor decays, where FSI have significant effects on decay rates, phases of amplitudes, and analytic structures. This is of particular interest for charm decays, where deviations from the small expected CP asymmetries may provide indications for new physics. Amplitude analyses of charm decays are also of interest for measurements of charm mixing and can provide inputs for measurements of the CKM angle  $\gamma$  in the B system. With additional data expected at BESIII and Belle II, new high precision amplitude analyses of charm decays should bring significant progress on these fronts.

### References

- [1] Harnew, S and J. Rademacker, Phys. Lett. B 728, 296-302 (2014).
- [2] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. D 89, 052001 (2014).
- [3] B. Aubert et al. (BaBar Collaboration), Phys. Rev. D 78, 034023 (2008).
- [4] Anisovich, V.V. and A.V. Sarantsev, Eur. Phys. J. A. 16, 229-258 (2003).
- [5] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. D 92, 052003 (2015).
- [6] B. Aubert et al. (BaBar Collaboration), Phys. Rev. D 87, 052010 (2013).
- [7] M. Artuso et al. (CLEO Collaboration), Phys. Rev. D 85, 122002 (2012).
- [8] B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 98, 211802 (2007).
- [9] M. Staric et al. (Belle Collaboration), Phys. Rev. Lett. 98, 211803 (2007).
- [10] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 100, 121802 (2008).
- [11] Y. Amhis et al. (Heavy Flavor Averaging Group), arXiv:1412.7515 (2014).
- [12] T. Peng et al. (Belle Collaboration), Phys. Rev. D. 89, 091103(R) (2014).
- [13] B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. 93, 112014 (2016).
- [14] D. M. Asner et al. (CLEO Collaboration), Phys. Rev. D 72, 012001 (2005).
- [15] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 116, 052001 (2016).
- [16] A. Zupanc et al. (Belle Collaboration), Phys. Rev. Lett. 113, 042002 (2014).
- [17] E. M. Aitala et al. (E791 Collaboration), Phys. Lett. B 471, 449-459 (2000).
- [18] M. Ablikim et al. (BESIII Collaboration), arXiv:1608.00407 (2016).