PROCEEDINGS OF SCIENCE

# Theory overview on amplitude analyses with charm decays

**Benoit Loiseau***
*Sorbonne Universités, Université Pierre et Marie Curie, Sorbonne Paris Cité, Université Paris Diderot, et IN2P3-CNRS, UMR 7585, Laboratoire de Physique Nucléaire et de Hautes Énergies, 4 place Jussieu, 75252 Paris, France*
*E-mail:* loiseau@lpnhe.in2p3.fr

This contribution about amplitude analyses in multibody hadronic charm decays deals with some attempts to introduce theoretical constraints. Different effective hadronic formalism approaches are mentioned. A recent work, based on a basic weak interaction process and a Chiral unitary model to account for the final state interaction, is described in details for the $f_0(980)$ production in $D_s^+ \to \pi^+\pi^+\pi^-$ and $D_s^+ \to \pi^+K^+K^-$ decays. Within the framework of the diagrammatic approach and flavor symmetry, a global analysis of two-body $D$ decays into a vector meson and a pseudoscalar meson is presented. A quasi-two-body QCD factorization model for $D$ decays into three mesons and its recent application to $D^0 \to K_S^0\pi^+\pi^-$ is outlined. For processes with final-state pions and kaons and as an alternative to the sum of Breit-Wigner amplitudes, often used in experimental Dalitz-plot analyses, amplitude parametrizations, in term of unitary $\pi\pi$, $\pi K$ and $K\bar{K}$ form factors, are proposed. These parametrizations are derived from quasi-two-body factorization models.

*VIII International Workshop On Charm Physics*
*5-9 September, 2016*
*Bologna, Italy*

*Speaker.

## 1. Introduction

There is an impressive set of hadronic multibody decay data for $D^0$, $D^+$ and $D_s^+$ decays [1, 2, 3]. The Dalitz plots are characterized by an accumulation of events displaying the presence of meson resonances and their interferences [4, 5, 6, 7, 8]. The Standard model (SM) predicts null $CP$ asymmetries and some deviation could be a signal of physics beyond SM [9, 10]. Furthermore the study of $D^0$-$\bar{D}^0$ mixing might indicate the presence of new physics contributions [11, 12, 13, 14]. Multibody hadronic decays of $D_{(s)}$ mesons consist of a weak process, microscopic quark flavor changing process like $c \to d$ or $c \to s$ via the $W$ meson interaction, followed by hadronization and final state meson-meson strong interaction processes. Basic amplitude analyses are usually performed via the isobar model or sum of relativistic Breit-Wigner terms representing the different possible implied resonances plus a non-resonant background: can one go beyond?

Section 2 is devoted to some final state interaction (FSI) studies, section 3 to a diagrammatic approach and flavor symmetry for $D \to VP$ decays ($V, P \equiv$ vector, pseudoscalar mesons), section 4 to a quasi-two-body QCD factorization model for $D$ decays into three mesons and section 5 to amplitude parametrizations based on quasi-two-body factorization and to some conclusions.

## 2. Final-state interaction constraints

### 2.1 Different effective hadronic formalism approaches

The $K^-\pi^+$ FSI in the $D^+ \to K^-\pi^+\pi^+$ data of the E791 Collaboration has been the subject of many studies using different effective hadronic formalism approaches [15, 16, 17, 18, 19]. In Ref. [15] it is shown that final state interactions are important in shaping the Dalitz plot and that several weak and hadronic processes are required. In Ref. [16] the theoretical treatment of this decay includes a rich dynamic behavior that mix weak and strong interactions in a non trivial way. The authors of Ref. [17] assume the dominance of the weak vector current together with a Chiral effective Lagrangian and phenomenological form factors. In Ref. [18] a full dispersive Khuri-Treiman formalism is applied, resumming rescattering contributions to all orders using $\pi\pi$ and $\pi K$ phase shifts as input, and fitting subtraction constants to the experimental Dalitz plots from CLEO and FOCUS. S. X. Nakamura [19] has performed a coupled-channel analysis of pseudo-data generated from the isobar model of the E791 Collaboration. The authors of Ref. [20] have studied the $D^+ \to K^+K^-K^+$ process with a multi-meson model as an alternative to isobar model, with free parameters predicted by the theory to be fine-tuned by a fit to data.

### 2.2 Basic weak interaction plus Chiral unitary approach

The study of $f_0(980)$ production in $D_s^+ \to \pi^+\pi^+\pi^-$ and $D_s^+ \to \pi^+K^+K^-$ decays performed by the authors of Ref. [21] is described below. For the sake of completeness the main steps of their derivation is reproduced here. They start from the Cabibbo favored $c \to s\bar{d}u$ flavor changing process. Then the $c\bar{s}$ pair of the $D_s^+$ decays into a $\bar{d}u$ (which hadronizes into a $\pi^+$) and a $s\bar{s}$ pair. Insertion, in the $s\bar{s}$ pair, of a $q\bar{q}$ with the quantum numbers of the vacuum, $\bar{u}u + \bar{d}d + \bar{s}s$, leads then, via hadronization, to the production of two pseudoscalar mesons. This dominant process is depicted in Fig. 1 of Ref. [21]. To find out the meson-meson components in the $s\bar{s}$ pair one can define the following $q\bar{q}$ $M$ matrix:

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}, \tag{2.1}$$

which satisfies

$$M \cdot M = M \times (\bar{u}u + \bar{d}d + \bar{s}s) \tag{2.2}$$

In the standard $\eta - \eta'$ mixing [22] the matrix $M$ is related to [23]

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}. \tag{2.3}$$

Neglecting $\eta'$ contribution, its mass being too large, one has:

$$s\bar{s}(\bar{u}u + \bar{d}d + \bar{s}s) \equiv (\Phi \cdot \Phi)_{33} = K^-K^+ + \bar{K}^0K^0 + \frac{1}{3}\eta\eta, \tag{2.4}$$

which are the produced states before FSI. After rescattering the $K^+K^-$ pair can produce $\pi^+\pi^-$ and/or $K^+K^-$ pairs. The $D_s^+$ decay width into a $\pi^+$ and two mesons, labelled, $\Gamma_{P^+P^-}$, where $P^+P^- \equiv K^+K^-$ or $\pi^+\pi^-$, satisfies

$$\frac{d\Gamma_{P^+P^-}}{dM_{inv}} = \frac{1}{(2\pi)^3}\frac{p_\pi \tilde{p}_P}{4M_{D_s}^2}|T_{P^+P^-}|^2, \tag{2.5}$$

with,

$$T_{K^+K^-} = V_0 \left(1 + G_{K^+K^-}\, t_{K^+K^- \to K^+K^-} + G_{K^0\bar{K}^0}\, t_{K^0\bar{K}^0 \to K^+K^-} + \frac{2}{3}\frac{1}{\sqrt{2}} G_{\eta\eta}\, t_{\eta\eta \to K^+K^-}\right), \tag{2.6}$$

$$T_{\pi^+\pi^-} = V_0 \left(G_{K^+K^-}\, t_{K^+K^- \to \pi^+\pi^-} + G_{K^0\bar{K}^0}\, t_{K^0\bar{K}^0 \to \pi^+\pi^-} + \frac{2}{3}\frac{1}{\sqrt{2}} G_{\eta\eta}\, t_{\eta\eta \to \pi^+\pi^-}\right). \tag{2.7}$$

The function $G_l$ is the loop function,

$$G_l(s) = i\int \frac{d^4q}{(2\pi)^4} \frac{1}{(p-q)^2 - m_1^2 + i\varepsilon} \frac{1}{q^2 - m_2^2 + i\varepsilon}, \tag{2.8}$$

$m_1$, $m_2$ being the meson masses in loop $l$. The integral on $q^0$ is analytical and a cut-off, $|\mathbf{q}_{max}| =$ 600 MeV/c is needed in the integral on $\mathbf{q}$ in order to reproduce the experimental amplitudes. The $t_{i\to j}$ matrices are obtained by solving the coupled-channel Bethe-Salpeter equation

$$t_{i\to j}(s) = V_{ij}(s) + \sum_{l=1}^{5} V_{il}(s)G_l(s)t_{l\to j}(s), \tag{2.9}$$

where the indices $i, j, l$ running from 1 to 5 denote the different channels: 1 for $\pi^+\pi^-$, 2 for $\pi^0\pi^0$, 3 for $K^+K^-$, 4 for $K^0\bar{K}^0$, and 5 for $\eta\eta$. The kernel $V_{ij}$ are the tree-level transition amplitudes built from phenomenological Lagrangians in Ref. [24].

Adjusting $V_0$ and comparing theoretical amplitudes $T_{K^+K^-}$ and $T_{\pi^+\pi^-}$ with those available from the experimental data [25, 26] leads to a fair agreement as can be seen in Figs. 2, 3 (and 4) of Ref. [24]. Invariant mass distributions for $D_s^+ \to \pi^+\pi^-\pi^+$ and $D_s^+ \to \pi^+K^-K^+$ are shown in Fig. 5 of Ref. [24] The $f_0(980)$ signals in the spectra are (up to a global common normalization factor) predictions of the Chiral Unitary approach with no free parameters. The mechanism displayed above was used in Ref. [27] to obtain branching ratios for $a_0(980)$ and $f_0(980)$ production in good agreement with experiment. An interesting issue will be the study of the $\pi^+\pi^0\eta$ decay mode which generates the $a_0(980)$ and can lead to informations on possible $f_0(980)$ and $a_0(980)$ mixing.

## 3. Diagrammatic approach for $D$ decays into a vector and a pseudoscalar meson

### 3.1 $D \to VP$ decays within SU(3) flavor symmetry

The $c$ quark mass, $m_c$, being too high to apply Chiral perturbation theory and too light to use heavy quark expansion approaches, one can use a diagrammatic approach with flavor-flow diagrams classified according to the topologies of weak interactions with all strong interaction effects included. This model-independent analysis, based on flavor SU(3) symmetry, determines topological amplitudes allowing to specify the relative importance of different underlying decay mechanisms. In such an approach, introduced by L. L. Chau [28], the conceivable topologies of weak interactions are shown in Fig. 1 of Ref. [29].

We describe here some results of the recent work of Ref. [30] in which all two-body charmed meson decays $D \to VP$ are studied in this diagrammatic framework. There, within SU(3) flavor symmetry, only four types of amplitudes exist: color-allowed $T$, color-suppressed $C$, $W$-exchange $E$, and $W$-annihilation $A$. Subscript $P$ or $V$ to each amplitude, *e.g.*, $T_{P(V)}$, denote the amplitude in which the spectator quark goes to the pseudoscalar or vector meson in the final state. These two kinds of amplitudes do not have *a priori* any obvious relationship. All the flavor amplitude magnitudes and their associated strong phases are then extracted through a fit on existing experimental branching fractions.

### 3.2 Fit on branching fractions of $D \to VP$ decays

The 8 complex amplitudes, $T_{P(V)}, C_{P(V)}, E_{P(V)}, A_{P(V)}$ (15 real parameters, $T_V$ chosen to be real) are determined by performing a $\chi^2$ fit of 16 experimental branching fractions for Cabibbo-favored $D^0$ and $D^+_{(s)}$ decays proportional to the Cabibbo-Kobayashi-Maskawa (CKM) factors $V^*_{cs}V_{ud} \sim \mathcal{O}(1)$. All the data are extracted from the Particle Data Group [31] but the branching fraction $\mathcal{B}_{\rho^+\eta'}$ is taken from Ref. [32]. The determination of $D_s^+ \to \pi^+\rho^0$ using results from Refs. [26, 31] (see Ref. [30]) allows the extraction of the $A_{P,(V)}$ amplitudes. Among the several solutions found by the authors there is one favored, named (A1). The results of the fits are shown in the Table II of Ref. [30]. Comparison is made in particular with the pole model of Ref. [33] which is built using generalized factorization and addition of poles in annihilation diagrams.

The Cabibbo-favored amplitudes, resulting from the branching fraction fit results (solution A1) are shown in Table 1.

| $\|T_P\|$ | $\delta_{T_P}$ | $\|C_V\|$ | $\delta_{C_V}$ | $\|C_P\|$ | $\delta_{C_P}$ | $\|E_V\|$ |
|---|---|---|---|---|---|---|
| $8.46^{+0.22}_{-0.25}$ | $57^{+35}_{-41}$ | $4.09^{+0.16}_{-0.25}$ | $-145^{+29}_{-39}$ | $4.08^{+0.37}_{-0.36}$ | $-157\pm2$ | $1.19^{+0.64}_{-0.46}$ |
| $\delta_{E_V}$ | $\|E_P\|$ | $\delta_{E_P}$ | $\|A_P\|$ | $\delta_{A_P}$ | $\|A_V\|$ | $\delta_{A_V}$ |
| $-85^{+42}_{-39}$ | $3.06\pm0.09$ | $98\pm5$ | $0.64^{+0.14}_{-0.27}$ | $152^{+48}_{-50}$ | $0.52^{+0.24}_{-0.19}$ | $122^{+70}_{-42}$ |

**Table 1:** Cabibbo-favored amplitudes resulting from the branching fraction fit results [30] (solution A1) for a $\eta$-$\eta'$ mixing angle of 43.5°. Units: $10^{-6}$, strong phases in degrees. $|T_V| = 4.21^{+0.18}_{-0.19}$.

The modulus of color-allowed tree $T_P$ amplitude is the largest. The moduli of color-allowed tree $T_V$, color-suppressed tree $C_{V(P)}$ and $W$-exchange $E_P$ are of the same magnitude. The moduli of the $W$-annihilation $A_{P(V)}$ amplitudes are the smallest.

Branching fraction predictions, with no flavor SU(3) breaking, for singly Cabibbo-suppressed decays proportional to $V^*_{cd}V_{ud} \sim \mathscr{O}(\lambda)$ and to $V^*_{cs}V_{us} \sim \mathscr{O}(\lambda)$ with $\lambda = 0.22543$ (CKMfitter, see Ref. [14]) can be seen in Table III of Ref. [30] and predictions for doubly Cabibbo-suppressed decays (no SU(3) breaking) proportional to $V^*_{cd}V_{us} \sim \mathscr{O}(\lambda^2)$ in Table IV. The predictions for the doubly Cabibbo-suppressed channels are in good agreement with data but the singly Cabibbo-suppressed ones have some flavor SU(3) symmetry breaking effects.

### 3.3 Concluding remarks on this $D \to VP$ study

Exact flavor SU(3) describes reasonably well the available data. If $T$ and $C$ amplitudes are factorizable, the effective Wilson coefficients $a_{1,2}$, $|a_2/a_1|$ and $\arg(a_2/a_1)$ (see next section) can be extracted from Cabibbo-favored $D^+ \to \overline{K}^{*0}\pi^+$ and $\overline{K}^0\rho^+$ [solution (A1)]. The results are shown in Table VII of Ref. [30]. SU(3) symmetry breaking in color-allowed $T$ and color-suppressed $C$ tree amplitudes is needed in general to have a better agreement with experiment. Nevertheless, the exact flavor SU(3)-symmetric approach alone is adequate to provide an overall explanation for the current data. The impact of this symmetry on $D \to PP$ decays has been presented in this workshop by P. Santorelli [34]. Within the diagrammatic approach one should quote the validity study of flavor SU(3) [35].

## 4. Factorization approach for hadronic three-body $D$ decays

### 4.1 Quasi-two-body factorization for $D^0 \to K^0_S\pi^+\pi^-$

QCD factorization beyond naïve factorization, expansion in $\alpha_s$ (strong coupling constant) and $1/m_b$ ($m_b$ $b$-quark mass), applies with success to charmless nonleptonic two-body $B$ decays [36]. In $D$ decays, $m_c \sim m_b/3$ leads a priori to significant corrections to the factorized results and factorization is more a phenomenological approach, based on the seminal work by Bauer, Stech and Wirbel [37]. It is then applied successfully to $D$ decays, treating Wilson coefficients as phenomenological parameters to account for non-factorizable corrections. A part from a recent attempt to extend the framework of QCD factorization to non-leptonic $B$ decays into three light mesons [38], so far there exists no factorization theorem for three-body decays. However there are important contributions from intermediate resonances as $\rho(770)$, $K^*(892)$ and $\phi(1020)$ and three-body decays may

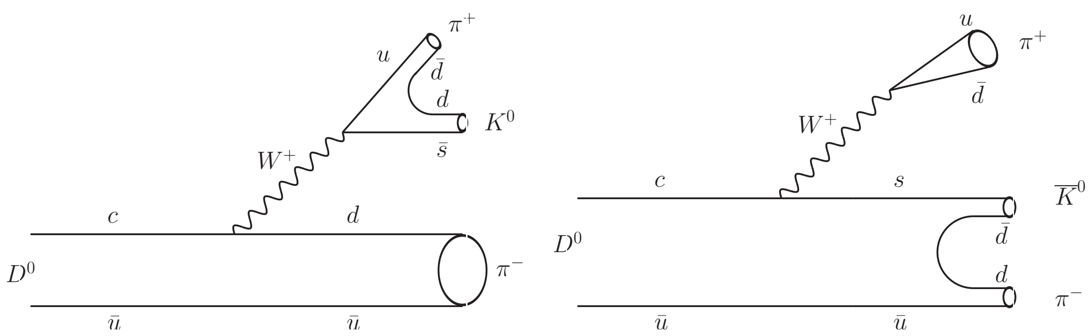


**Figure 1:** Microscopic quark tree diagrams for doubly Cabibbo-suppressed (left) and Cabibbo-favored (right) amplitudes

be considered as quasi-two-body decays. One makes the hypothesis that two of the three final-state mesons form a single state originating from a quark-antiquark pair. This leads to a quasi-two-body final state to which the factorization procedure is applied.

Within this framework we report on the Dalitz plot studies of $D^0 \to K_S^0\pi^+\pi^-$ decays performed in Ref. [39]. There is no penguin ($W$-loop diagram) in this decay and the weak effective Hamiltonian reads

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_{i=1,2} C_i(\mu) O_i(\mu) + h.c., \tag{4.1}$$

where $V_{CKM}$ represents the quark mixing couplings, $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$ the Fermi coupling, $C_i(\mu)$ the QCD Wilson coefficients arising from $W$ exchange and $\mu$ the renormalization scale with $\mu \sim m_c = 1.3$ GeV. The left-handed quark current-current operators $O_1$ reads

$$O_1 = j_1 \otimes j_2, j_1 = \bar{s}_\alpha \gamma^\nu (1-\gamma^5) c_\alpha \equiv (\bar{s}c)_{V-A}, j_2 = \bar{u}_\beta \gamma_\nu (1-\gamma_5) d_\beta \equiv (\bar{u}d)_{V-A}, \tag{4.2}$$

$\alpha$ and $\beta$ being color indices. The operator $O_2$ has a similar expression.

In the amplitude and at leading order in $\alpha_s$, the following real effective QCD coefficients $a_1(m_c)$ and $a_2(m_c)$ will appear,

$$a_1(m_c) = C_1(m_c) + \frac{C_2(m_c)}{N_C}, \qquad a_2(m_c) = C_2(m_c) + \frac{C_1(m_c)}{N_C}, \tag{4.3}$$

$N_C = 3$ being the number of colors (from now on $a_i(m_c) \equiv a_i, i = 1,2$). The use of the Operator Product Expansion and of the fact that the $W$ mass is large leads to the two-body factorization approximation,

$$\langle M_1 M_2 | O_i(\mu) | D^0 \rangle = \langle M_1 | j_1 | 0 \rangle \langle M_2 | j_2 | D^0 \rangle + \text{higher order corrections.} \tag{4.4}$$

In the quasi-two-body approximation, the final state $\bar{K}^0\pi^+\pi^-$ is assimilated to the two-body states $[\bar{K}^0\pi^\pm]_L\, \pi^\mp$ and $\bar{K}^0\, [\pi^+\pi^-]_L$ where the meson-meson pair can be in $L = S, P$ and $D$ states. The mesons $M_{1,2}$ of the factorization equation (4.4) are then $M_1 = [\pi^+ K^0]_L, M_2 = \pi^-$ or $M_1 = \bar{K}^0, M_2 = [\pi^+\pi^-]_L$. In the $D^0 \to K_S^0\pi^+\pi^-$ decays the possible $c$ quark flavor changing are $c \to$



PoS(CHARM2016)033

$su\bar{d}$ leading to Cabibbo-favored (CF) amplitudes $\propto V_{CKM} = V_{cs}^* V_{ud} \equiv \Lambda_1$ and $c \to du\bar{s}$ for doubly Cabibbo-suppressed (DCS) amplitudes $\propto V_{CKM} = V_{cd}^* V_{us} \equiv \Lambda_2$. Here, besides tree amplitudes, there are annihilation ones arising from $W$ exchange between the quarks $c$ and $\bar{u}$ of the $D^0$. The interested reader will find the detailed expressions of the different amplitudes obtained applying the above quasi-two-body factorization approximation to the $\langle \bar{K}^0 \pi^- \pi^+ | H_{eff} | D^0 \rangle$ matrix element. Let us exemplify some terms entering these amplitudes. In the DCS tree amplitude (see Eq. (10) of Ref. [39]) for $L = S$ one encounters the contribution of a term like (see left diagram of Fig. 1)

$$\frac{G_F}{2} \Lambda_2 a_1 \langle \pi^- | (\bar{d}\, c)_{V-A} | D^0 \rangle \cdot \langle [K^0 \pi^+]_S | (\bar{u}\, s)_{V-A} | 0 \rangle, \tag{4.5}$$

where the first matrix element is related to the $D^0 \to \pi$ transition form factor and the second one to the scalar $K\pi$ form factor. In the CF tree amplitude (see Eq. (6) of Ref. [39]) for $L = S$ one has the contribution of a term as (see right diagram of Fig. 1)

$$\frac{G_F}{2} \Lambda_1 a_1 \langle \pi^+ | (\bar{u}\, d)_{V-A} | 0 \rangle \cdot \langle [\bar{K}^0(p_0) \pi^-]_S | (\bar{s}\, c)_{V-A} | D^0(p_{D^0}) \rangle, \tag{4.6}$$

where the first matrix element is related to the $\pi$ decay constant and the second one to the scalar $D^0 \to K\pi$ transition form factor. Its evaluation is less straightforward, it could be experimentally evaluated from semi-leptonic processes such as, $D^0 \to K^- \pi^+ \mu^+ \mu^-$ [40, 41]. Here assuming this transition to proceed through the dominant intermediate resonance $K_0^*(1430)$, it can be written in terms of the $K\pi$ scalar form factor. The corresponding amplitude given by Eq. (13) of Ref. [39] is

$$T^{CF}_{[\bar{K}^0 \pi^-]_S\, \pi^+}(s_0, s_-, s_+) = -\frac{G_F}{2}\, a_1 \Lambda_1\, \chi_1\, \left(m_{D^0}^2 - s_-\right)\, f_\pi\, F_0^{D^0 K_0^*(1430)^-}(m_\pi^2)\, F_0^{\bar{K}^0 \pi^-}(s_-), \tag{4.7}$$

where $s_\pm = (p_{\pi^\pm} + p_{K^0})^2 = m_\pm^2, s_0 = (p_{\pi^+} + p_{\pi^-})^2 = m_0^2$. The scalar $K\pi$ form factor $F_0^{\bar{K}^0 \pi^-}(s_-)$ includes the contributions of the $K_0^*(800)$ and $K_0^*(1430)$ resonances. The factor $\chi_1$, related to the strength of this scalar form factor is taken as a complex constant to be fitted. It can be estimated from the $K_0(1430)$ properties [39]. Here $f_\pi$ is the pion decay constant. Following Ref. [42] the value of 0.48 is used for the $D^0$ to $K_0^*(1430)$ transition form factor $F_0^{D^0 K_0^*(1430)^-}(m_\pi^2)$.

What we have shown just above for amplitudes with $K\pi$ final state pair in $S$ wave, works also for the $P$-wave case and for amplitudes with the $\pi^+\pi^-$ final state pair in $S$ and $P$ wave. So, amplitudes with the $\pi^+\pi^-$ ($K\pi$) final state pair in $S$ and $P$ wave are described in terms of scalar and vector $\pi\pi$ ($K\pi$) form factors. Form factors with final meson pair in $D$ wave are represented by relativistic Breit-Wigner formulae. The 13 tree-amplitudes and 14 $W$-exchange ones derived in Ref. [39] can be recast into 10 amplitudes. Summary of these CF and DCS, amplitudes associated to the different quasi two-body channel together with the contributing dominant resonances are listed in Table 1 of Ref. [39].

### 4.2 Form factors

It can be shown from field theory and using dispersion relations [43] that meson-meson form factors can be calculated exactly, using Muskhelishvili-Omnès equations, if one knows the meson-meson strong interactions at all energies. The details and corresponding references of the unitary scalar and vector $K\pi$ and $\pi\pi$ form factors used in the best fits of the $D^0 \to K_S^0 \pi^+ \pi^-$ Dalitz plot



PoS(CHARM2016)033

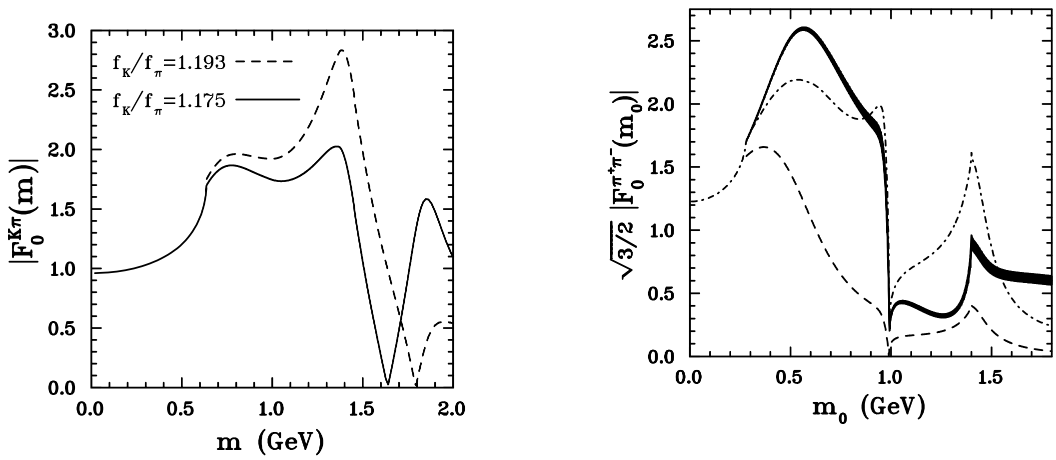


**Figure 2:** Left panel: modulus of the scalar $K\pi$, $F_0^{K\pi}$, form factor as a function of the effective $K\pi$ mass for two values of the $f_K/f_\pi$ ratio ($f_K$ being the kaon decay constant). Right panel: modulus of the scalar $\pi\pi$, $F_0^{\pi\pi}$, form factor as a function of the effective $\pi\pi$ mass, dark band variation within the parameter errors of the fit to Belle data [44], dashed line same form factor but different parameters (Ref. [45]), dot-dash line is the result of Ref. [46] using Muskhelishvili-Omnès equations.

performed in Ref. [39] can be found in this reference. We just plot here, in Fig. 2, the modulus of the scalar $K\pi$ ($F_0^{K\pi}$, left panel) and $\pi\pi$ ($F_0^{\pi\pi}$, right panel) form factors as function of the $K\pi$ ($m_\pm$) and $\pi\pi$ ($m_0$) effective masses. The scalar $K\pi$ form factor is characterized by two bumps arising from the $K_0^*(800)$ and $K_0^*(1430)$ contributions and the $\pi\pi$ form factor by a dip coming from the $f_0(980)$) and two bumps from the $f_0(500)$ and $f_0(1400)$.

### 4.3 Dalitz-plot fit

The Dalitz plot distribution of the best fit to the Belle data [44] is shown in Figs. 9 of Ref. [39]. This distribution reproduces well that of Belle. The corresponding input parameters and the obtained 33 free parameters are given in section IV and in Table II of Ref. [39], respectively. The Dalitz plot shows a rich interference pattern with the dominance of the $K^*(892)$.

Branching fraction (Br), listed in Table V of Ref. [39], compare well with those of Belle's analysis. Their sum, equal to 133 %, shows the importance of interferences. The largest Br come from the amplitudes, $\mathcal{M}_1$ [contributions of $K_0^*(800)^-$, $K_0^*(1430)^-$] with a Br of 25. %) , $\mathcal{M}_2$ [contribution of $f_0(500)$, $f_0(980)$, $f_0(1400)$] with a Br of 16.9 %, $\mathcal{M}_3$ [contributions of $K^*(892)^-$] with a Br of 62.7 %) and $\mathcal{M}_4$ [contributions of $\rho(770)^0$] with a Br of 21.9 %. The sum of the Belle Br



PoS(CHARM2016)033

with the final $\pi^+\pi^-$ pair state in $S$ wave, equal to 18.6%, is close the Br of $\mathscr{M}_2$, equal to 16.9 %. The annihilation ($W$ exchange) contributions can be important.

## 5. Concluding remarks

### 5.1 Amplitude parametrizations

As a sound alternative to the simplistic and widely used isobar model, explicit amplitude parametrizations, that can be readily implemented in experimental analysis, are suggested in Refs. [47, 48] for the study of the decays $D^+ \to \pi^-\pi^+\pi^+$, $D^+ \to K^-\pi^+\ \pi^+$, $D^0 \to K^0_S\ \pi^+\ \pi^-$ and $D^0 \to K^0_S\ K^+K^-$. These parametrizations, where two-body hadronic final state interactions are taken into account in terms of unitary $S$- and $P$-wave $\pi\pi$, $\pi K$ and $K\bar{K}$ form factors, are derived from different phenomenologically successful works (except for the $D^0 \to K^0_S\ K^+K^-$ not tested yet) based on quasi two-body factorization approach.

### 5.2 Outlook

#### 5.2.1 Some other studies on hadronic $D$ decays

Within the factorization approximation and taking into account final state interaction, two-body hadronic $D^0$, $D^+$ and $D_s^+$ decays have been studied in Ref. [49]. A reasonable agreement with the data, which show some large flavor SU(3) symmetry violation, is obtained. CP violating asymmetries are also discussed.

The authors of Ref. [50] explore consequences of constraints from CPT symmetry on three-body $D$ decays. They simulate the $D^\pm \to \pi^\mp K^+K^-$ decays and discuss correlations with measured $D^\pm \to \pi^\mp\pi^+\pi^-$.

The impact of New Dynamics is studied in Ref. [51].

The authors of Ref. [52] perform an analysis of pure annihilation type diagrams for two-body $D \to PP(V)$ decays based on the $k_T$ factorization. Their results agree with the existing experimental data for most channels.

#### 5.2.2 Concluding summary

There is and there will be an impressive amount of high-quality hadronic-multibody decay data of $D^0, D^+, D_s^+$. Improved models are and will be needed for extracting accurate informations from these data. In this brief review I described some available potentialities for constraining amplitude analyses in some of these charm decays. I have listed different effective hadronic formalism approaches. Detailed outcomes of a model, with combination of basic elements of weak interaction together with final state constraints in the framework of a Chiral unitary approach in coupled channel, have been reported.

I have described the diagrammatic-approach framework. It consists of a model-independent analysis, based on flavor SU(3) symmetry topological amplitudes, allowing to understand the relative importance of different underlying decay mechanisms. In this framework a global analysis of two-body $D$ decays into a vector meson and a pseudoscalar meson has been presented.

A quasi-two-body QCD factorization model for $D$ decays into three mesons and its recent application to $D^0 \to K^0_S\pi^+\pi^-$ was shown to describe successfully the data. It suggests to go beyond the superposition of Breit-Wigner amplitudes in the analysis of three-body decays with pions and kaons in the final state. It is advocated to use a phenomenological quasi-two-body factorization approach in which two-body hadronic final state interactions are fully taken into account in terms of unitary $S$- and $P$-wave $\pi\pi$, $\pi K$ and $K\bar{K}$ form factors.

It could be interesting to see if the above phenomenological quasi-two-body factorization approach could be used in amplitude analysis of four-body hadronic $D$ decays such as $D^0 \to K^+K^-\pi^+\pi^-$ recently analyzed by the CLEO Collaboration [53], $D^0 \to \pi^+\pi^-\pi^+\pi^-$ [54] and $D^0 \to K^-\pi^+\pi^+\pi^-$ presently studied by the BES III Collaboration [8].

Acknowledgments
I thank Jean-Pierre Dedonder, Robert Kamiński and Leonard Leśniak for their fruitful collaboration on the quasi-two-body factorization work on the $D^0 \to K_S^0\pi^+\pi^-$ decays (IN2P3-COPIN collaboration agreement, project N° 08-127). I appreciate helpful discussions and fruitful comments of Jean-Pierre Dedonder during the preparation of my talk and the writing of this contribution. I am obliged to Diogo Boito and Bruno El-Bennich for their help on the amplitude parametrizations proposed in section 5.1.

## References

[1] J. H. Alvarenga Nogueira *et al.*, *Summary of the 2015 LHCb workshop on multi-body decays of D and B mesons*, hep-ex/1605.03889.

[2] M. Gersabeck, *Experimental Overview*, Contribution to this workshop.

[3] R. A. Briere, *Quantum Coherence and Charm*, Contribution to this workshop.

[4] J. Rademacker, *Multibody Charm Decays*, Contribution to this workshop.

[5] Alberto C. dos Reis, for the LHCb collaboration, *Amplitude analysis of charm decays at LHCb*, Contribution to this workshop.

[6] J. Bennett, *Amplitude analyses with charm decays at $e^+e^-$ machines*, Contribution to this workshop.

[7] T. Evans, *Recent results on Dalitz-plot analyses of D mesons decays at LHCb*, Contribution to this workshop.

[8] Yu Lu for BESIII Collaboration, *Amplitude analysis of $D^0 \to K^-\pi^+\pi^+\pi^-$ and measurements of D two-body decays*, Contribution to this workshop.

[9] A. Paul, *Theoretical overview on NP search in rare and (semi-) leptonic decays*, Contribution to this workshop.

[10] F. Buccella, M. Lusignoli, A. Pugliese, P Santorelli, *CP violation in D meson decays: Would it be a sign of new physics?*, Phys. Rev. D **88** (2013) 074011, [hep-ph/1305.7343].

[11] M. R. J. Williams, *CP Violation and mixing in charm decays at LHCb*, Contribution to this workshop.

[12] A. Lenz, *Theory Overview*, Contribution to this workshop.

[13] M. Ciuchini, *$D^0 - \bar{D}^0$ mixing theory*, Contribution to this workshop.

[14] D. Derkach, *Impact of Charm physics to CKM fit*, Contribution to this workshop.

[15] P.C. Magalhães, M.R. Robilotta, K.S.F.F. Guimarães, T. Frederico, W.S. de Paula, I. Bediaga, A.C. dos Reis, C.M. Maekawa, *$D^+ \to K^-\pi^+\pi^+$ : heavy meson decays and final state interactions*, hep-ph/1307.8352, FPCP-2013, SLAC eConf C130520.

[16] P. Magalhães, M. Robilotta, *A complete model for $D^+ \to K^-\pi^+\pi^+$ S wave*, hep-ph/1312.2033, proceedings CHARM 2013, SLAC eConf C130831.

[17] P. C. Magalhães, M. R. Robilotta , *$D^+ \to K^-\pi^+\pi^+$ - the weak vector current*, Phys. Rev. D **92** (2015) 094005 [hep-ph/1504.06346].

[18] F. Niecknig, B. Kubis, *Dispersion-theoretical anlysis of the $D^+ \to K^-\pi^+\pi^+$ Dalitz plot*, JHEP **1510** (2015) 142, [hep-ph/1509.03188].

[19] S. X. Nakamura, *A coupled-channel analysis of $D^+ \to K^-\pi^+\pi^+$ decay*, Phys. Rev. D **93** (2016) 014005 [hep-ph/1504.02557] and contribution to this workshop.

[20] R. T. Aoude, P. C. Magalhães, A. C. dos Reis, M. R. Robilotta, *Multi-meson model applied to $D^+ \to K^+K^-K^+$*, hep-ph/1604.02904 and contribution to this workshop.

[21] J. M. Dias, F. S.Navarra, M. Nielsen, E. Oset, *$f_0(980)$ production in $D_s^+ \to \pi^+\pi^+\pi^-$ and $D_s^+ \to \pi^+K^+K^-$ decays*, Phys. Rev. D **94** (2016) 096002 [hep-ph/1601.04635].

[22] A. Bramon, A. Grau and G. Pancheri, *Intermediate vector meson contributions to $V^0 \to P^0P^0\gamma$ decays*, Phys. Lett. B **283** (1992) 416.

[23] D. Gamermann, E. Oset, B.S. Zou, *The radiative decay of $\psi(3770)$ into the predicted scalar state $X(3700)$*, Eur. Phys. J. A **41** (2009) 85, [hep/ph0805.0499].

[24] J. A. Oller and E. Oset, *Chiral symmetry amplitudes in the S wave isoscalar and isovector channels and the sigma, $f_0(980)$, $a_0(980)$ scalar mesons*, Nucl. Phys. A **620** (1997) 438; Erratum, Nucl. Phys. A **652** (1999) 407, [hep-ph/9702314].

[25] P. del Amo Sanchez *et al.* (BABAR Collaboration), *Dalitz plot analysis of $D_s^+ \to K^+K^-\pi^+$*, Phys. Rev. D **83** (2011) 05200 [hep-ex/1011.4190].

[26] B. Aubert *et al.* (BABAR Collaboration), *Dalitz Plot Analysis of $D_s^+ \to \pi^+\pi^-\pi^+$*, Phys. Rev. D **79** 032003 (2009) [hep-ex/0808.0971].

[27] Ju-Jun Xie, L.R. Dai, E. Oset, *The low lying scalar resonances in the $D^0$ decays into $K_s^0$ and $f_0(500)$, $f_0(980)$, $a_0(980)$*, Physics Letters B **742**, 363 (2015) [arXiv:1409.0401]; work developed in *Weak decays of heavy hadrons into dynamically generated resonances*, E. Oset *et al.*, hep-ph/1601.03972.

[28] L. L. Chau, *Quark mixing in weak interactions*, Phys. Rep. **95** (1983) 1.

[29] Hai-Yang Cheng, Cheng-Wei Chiang, *Direct CP violation in two-body hadronic charmed meson decays*, Phys. Rev. D **85** (2012) 034036 [hep-ph/1201.0785].

[30] Hai-Yang Cheng, Cheng-Wei Chiang, An-Li Kuo, *A global analysis of two-body D to VP decays within the framework of flavor symmetry*, Phys. Rev. D **93** (2016) 114010 [hep-ph/1604.03761].

[31] K. A. Olive *et al.* (Particle Data Group), *Review of Particle Physics*, Chin. Phys. C **38** (2014) 090001.

[32] M. Ablikim *et al.* (BESIII Collaboration), *Measurement of the branching fractions of $D_s^+ \to \eta' X$ and $D_s^+ \to \eta'\rho^+$ in $e^+e^- \to D_s^+D_s^-$*, Phys. Lett. B **750** (2015) 466 [hep-ex/1506.08952].

[33] Fu-Sheng Yu, Xiao-Xia Wang, Cai-Dian Lü, *Nonleptonic two body decays of charmed mesons*, Phys. Rev. D **84** (2011) 074019, [hep-ph/1101.4714]; see also: Qin Qin, Hsiang-nan Li, Cai-Dian Lü, Fu-Sheng Yu, *Branching ratios and direct CP asymmetries in $D \to PV$ decays*, Phys. Rev. D **89** (2014) 054006 [hep-ph/1305.7021].

[34] P. Santorelli, *CP Violation in nonleptonic two-body decays of D mesons*, Contribution to this workshop.

[35] B. Bhattacharya and J. L. Rosner, *Decays of charmed mesons to PV final states*, Phys. Rev. D **79** (2009) 034016; Erratum Phys. Rev. D **81** (2010) 099903 [hep-ph/0812.3167].

[36] M. Beneke, M. Neubert, *QCD factorization for $B \to PP$ and $B \to PV$ decays* Nuc. Phys. B **675** (2003) 333, [hep-ph/0308039].

[37] M. Bauer, B. Stech and M. Wirbel, *Exclusive non-leptonic decays of D-, $D_s$- and B-mesons*, Z. Phys. C **34** (1987)103.

[38] S. Kränkl, T. Mannel, J. Virto, *Three-body non-leptonic B decays and QCD factorization*, Nucl. Phys. B **899** (2015) 247 [hep-ph/1505.04111].

[39] J.-P. Dedonder, R. Kamiński, L. Leśniak and B. Loiseau, *Dalitz plot studies of $D^0 \to K^0_S\pi^+\pi^-$ decays in a factorization approach*, Phys. Rev. D **89** (2014) 094018 [hep-ph/1403.2971.

[40] R. Aaij *et al.*, LHCb collaboration, *First observation of the decay $D^0 \to K^-\pi^+\mu^+\mu^-$ in the $\rho^0$- $\omega$ region of the dimuon mass spectrum*, Phys. Lett. B **757** (2016) 558 [hep-ex/1510.08367].

[41] M. Fontana on behalf of the LHCb, *Rare charm decays at LHCb*, Contribution to this workshop.

[42] H.-Y. Cheng, and C.-W. Chiang, *Hadronic D decays involving even parity light mesons*, Phys. Rev. D **81** (2010) 074031 [hep-ph/1002.2466].

[43] Barton, G.: Dispersion Techniques in Field Theory, W. A. Benjamin, INC., New York (1965).

[44] A. Poluektov *et al.* (Belle Collaboration), *Evidence for direct CP violation in the decay $B^\pm \to D^{(*)}K^\pm, D \to K^0_S\pi^+\pi^-$ and measurement of the CKM phase $\phi_3$*, Phys. Rev. D **81** (2010) 112002 [hep-ex/1003.3360].

[45] J.-P. Dedonder, A. Furman, R Kamiński, L. Leśniak and B. Loiseau, *Final state interactions and CP violation in $B^\pm \to \pi^+\pi^-\pi^\pm$ decays*, Acta Phys. Pol. B **42** (2011) 2013 [hep-ph/1011.0960v2.

[46] B. Moussallam, *$N_f$ dependence of the quark condensate from a Chiral sum rule*. Eur. Phys. J. C **14**, 111 (2000) [hep-ph/9909292], B. Moussallam provided us with tables of his pion scalar form factor.

[47] D. Boito and B. Loiseau, *Parametrization of three-body hadronic D decay amplitudes*, on page 27 of Ref. [1].

[48] D. Boito, J.-P. Dedonder, B. El-Bennich, R. Escribano, R Kamiński, L. Leśniak and B. Loiseau, *Parametrizations of three-body hadronic B and D decay amplitudes in terms of unitary meson-meson form factors*, in preparation.

[49] F. Buccella, M. Lusignoli, G. Miele, A. Pugliese, P. Santorelli, *Nonleptonic weak decays of charmed mesons*, Phys. Rev. D **51** (1995) 3478 [hep-ph/9411286].

[50] I. Bediaga, I.I. Bigi, J. Miranda, A. Reis, *CP asymmetries in three-Body final states in charged D decays & CPT invariance*, Phys. Rev. D **89** (2014) 074024 [hep-ph/1401.3310].

[51] Ikaros I. Bigi, *IIB's view of theoretical landscape for LHCb experimentalists*, on page 25 of Ref. [1].

[52] Z. T. Zou, C. Li and C. D. Liu, *Pure annihilation type $D \to PP(V)$ decays in the perturbative QCD approach*, Chin. Phys. C **37** (2013) 093101 [hep-ph/1301.3444].

[53] M. Artuso *et al.* (CLEO Collaboration), *Amplitude analysis of $D^0 \to K^+K^-\pi^+\pi^-$* , Phys. Rev. D **85** (2012) 122002 [hep-ex/1201.5716].

[54] P. D'Argent, *$D \to 4\pi$ Dalitz analysis*, Contribution to this workshop.