

Subleading-power N -jet operators and the LBK amplitude in SCET

Martin Beneke^{*†}, Mathias Garry, Robert Szafron, Jian Wang

*Physik Department T31, James-Frank-Straße 1, Technische Universität München,
85748 Garching, Germany*

We construct a complete and minimal basis of subleading-power N -jet operators in position-space soft-collinear effective theory (SCET) and discuss how the Low-Burnett-Kroll amplitude is recovered in this framework. We begin a systematic investigation of the anomalous dimension of next-to-leading power N -jet operators in view of resummation of logarithmically enhanced terms in partonic cross sections and discuss the explicit result at the one-loop order for fermion-number two N -jet operators at the second order in the power expansion parameter of SCET.

*13th International Symposium on Radiative Corrections (Applications of Quantum Field Theory to Phenomenology)
25-29 September, 2017
St. Gilgen, Austria*

^{*}Speaker.

[†]TUM-HEP-1120/17, arXiv:1712.07462 [hep-ph]

1. Introduction

In this proceedings contribution, we summarize results from our study of subleading-power effects in high-energy scattering, in particular the anomalous dimension of subleading-power N -jet operators [1] in the framework of position-space soft-collinear effective theory (SCET). An N -jet operator describes the hard scattering of N energetic, massless particles with (outgoing) momenta p_i with $s_{ij} = 2p_i \cdot p_j + i0$, $i, j = 1 \dots N$, and all s_{ij} of the order of the square of some large scale Q . The ultraviolet (UV) divergences of such SCET operators are related to the universal infrared (IR) properties of soft and collinear radiation [2]. Up to the two-loop order the leading-power soft-collinear anomalous dimension has the very simple structure

$$\Gamma = -\gamma_{\text{cusp}}(\alpha_s) \sum_{i < j} \mathbf{T}_i \cdot \mathbf{T}_j \ln \left(\frac{-s_{ij}}{\mu^2} \right) + \sum_i \gamma_i(\alpha_s) \quad (1.1)$$

in colour-operator notation. Given the advances in the understanding of multi-loop corrections to the leading-power anomalous dimension [3, 4], it is also timely to ask about the next-to-leading power (NLP) term in the M/Q expansion, where $M \ll Q$ refers to a smaller scale generated by soft or collinear radiation. Besides shedding light on the formal structure of IR divergences at subleading power, the NLP N -jet anomalous dimension is a key ingredient for summing logarithmically enhanced loop effects to all orders in perturbation theory. NLP logarithms have recently been investigated at fixed NNLO accuracy for the threshold limit of the Drell-Yan process [5] and 0-jettiness observables [6, 7].

2. SCET essentials

We consider physical processes for which the virtuality of collinear modes in any of the N jet directions is of the same order, and parametrically larger than the one of the soft mode. The power-counting parameter λ is set by the transverse momentum $p_{\perp i} \sim \lambda$ of collinear momenta with virtuality $\mathcal{O}(\lambda^2)$. The components of soft momentum are all $\mathcal{O}(\lambda^2)$ and consequently soft virtuality scales as λ^4 . The term ‘‘NLP’’ refers to $\mathcal{O}(\lambda)$ and $\mathcal{O}(\lambda^2)$, since the first non-vanishing power correction to most physical processes of interest is $\mathcal{O}(\lambda^2)$.

After integrating out the hard interactions, the infrared physics is described by the SCET Lagrangian including all subleading-power interactions. For N widely separated collinear directions, the Lagrangian

$$\mathcal{L}_{\text{SCET}} = \sum_{i=1}^N \mathcal{L}_i(\psi_i, \psi_s) + \mathcal{L}_s(\psi_s) \quad (2.1)$$

is the sum of N copies of collinear Lagrangians with N pairs of separate light-like reference vectors $n_{i\pm}$, $i = 1, \dots, N$ satisfying $n_{i-} \cdot n_{j-} = \mathcal{O}(1)$. The collinear fields ψ_i all interact with the same soft field ψ_s but not among each other. The following properties of the position-space SCET formulation [8, 9] are essential for the following analysis:

- In products with collinear fields $\psi_i(x)$, soft fields $\psi_s(x)$ must be multipole-expanded in x around $x_{i-}^\mu = (n_{i+} \cdot x) n_{i-}^\mu / 2$.

- There are no purely collinear subleading-power interactions. That is, writing $\mathcal{L}_i(\psi_i, \psi_s) = \sum_{n=0}^{\infty} \mathcal{L}_i^{(n)}(\psi_i, \psi_s)$ any interaction vertex from the subleading power Lagrangians $n > 0$ contains at least one soft field. The NLP Lagrangians $n = 1, 2$ are known and higher-order terms can be easily constructed.
- The SCET Lagrangian is invariant under N separate collinear gauge transformations, which operate on a single collinear direction, and a soft gauge transformation under which all fields transform, see Ref. [9].

3. Subleading-power N -jet operator basis

We construct a complete and minimal basis of subleading N -jet operators in SCET. The general structure of an N -jet operator

$$J = \int dt C(\{t_{i_k}\}) J_s(0) \prod_{i=1}^N J_i(t_{i_1}, t_{i_2}, \dots) \quad (3.1)$$

can be described by products of operators J_i associated with collinear directions n_{i-} , each of which is itself composed of a product of n_i gauge-invariant collinear “building blocks” ψ_{i_k} ,

$$J_i(t_{i_1}, t_{i_2}, \dots) = \prod_{k=1}^{n_i} \psi_{i_k}(t_{i_k} n_{i+}), \quad (3.2)$$

and a purely soft-field operator J_s . In general, each of the collinear building blocks is integrated over the corresponding light-like direction in position space, $C(\{t_{i_k}\})$ is a hard matching coefficient, and $dt = \prod_{ik} dt_{i_k}$. Apart from the light-like displacements, the operators are evaluated at position $X = 0$, corresponding to the location of the hard interaction.

The guiding principle for constructing building blocks is the requirement of collinear and soft gauge covariance. Because each collinear sector transforms under its own collinear gauge transformation, each collinear building block must be a collinear gauge singlet. However, the soft field may interact with different collinear sectors so we only need to assume that collinear building blocks transform covariantly under the soft gauge transformation, hence

$$J_i(x) \xrightarrow{\text{coll.}} J_i(x), \quad J_i(x) \xrightarrow{\text{soft}} U_s(x_{i-}) J_i(x), \quad (3.3)$$

where U_s must be taken in the (not necessarily irreducible) colour representation of J_i . For the matrix adjoint representation we would have $J_i(x) \xrightarrow{\text{soft}} U_s(x_{i-}) J_i(x) U_s^\dagger(x_{i-})$ with U_s in the fundamental. The elementary collinear-gauge-invariant collinear building blocks are given by

$$\psi_i(t_i n_{i+}) \in \begin{cases} \chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i & \text{collinear quark} \\ \mathcal{A}_{\perp i}^\mu(t_i n_{i+}) \equiv W_i^\dagger [iD_{\perp i}^\mu W_i] & \text{collinear gluon} \end{cases} \quad (3.4)$$

for the collinear quark and gluon field in the i -th direction, respectively. W_i is the path-ordered exponential of $n_{i+} A_i$ (“ i -collinear Wilson line”) and the covariant derivative includes only the collinear gluon field. Both, the quark and gluon building blocks scale as $\mathcal{O}(\lambda)$. Objects containing $in_{i+} D_i$ or

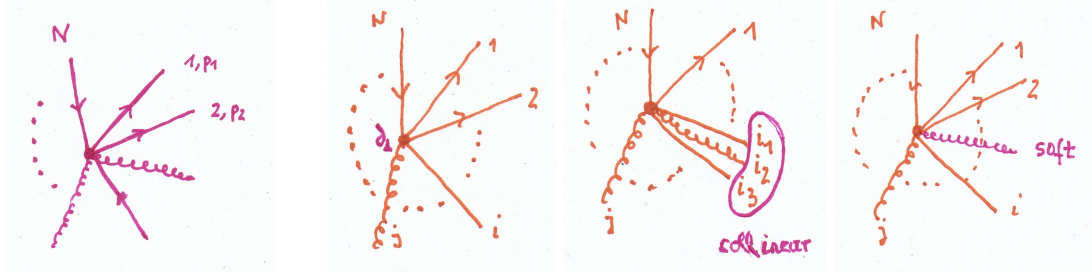


Figure 1: Three ways (second to fourth panel) to generate subleading-power N -jet operators from the leading-power operator $\prod_{i=1}^N J_i^{A0}(t_i)$ (first panel).

$in_{i+}\partial$ are redundant. At leading power, only a single building block contributes to each direction, i.e. $n_i = 1$ for all $i = 1, \dots, N$, and the $J_i(t_{i_1}, t_{i_2}, \dots)$ are given by

$$J_i^{A0}(t_i) = \psi_i(t_i n_{i+}). \quad (3.5)$$

We are interested in N -jet operators that are suppressed by one or two powers of λ relative to the leading power. In general, this suppression can arise in three ways, shown in Fig. 1:

- (i) via higher-derivative operators, i.e. acting with either $i\partial_{\perp i}^{\mu} \sim \mathcal{O}(\lambda)$ or $in_{i-}D_s \equiv in_{i-}\partial + g_s n_{i-}A_s(x_{i-}) \sim \mathcal{O}(\lambda^2)$ on the elementary building blocks ψ_{i_k} . Here it is important to note that since the elementary building blocks transform under the soft gauge transformation with $U_s(x_{i-})$, the covariant soft derivative is the ordinary derivative for the transverse direction and $in_{i-}D_s$ for the n_{i-} projection, which guarantees a homogeneous scaling in λ ;
- (ii) by adding more elementary building blocks $\chi_i, \mathcal{A}_{\perp i}^{\mu}$ in a given direction;
- (iii) via the new elementary collinear building block $n_{i-}\mathcal{A}_i \equiv W_i^{\dagger} in_{i-}D_i W_i - in_{i-}D_s$ that appears at subleading power, and via purely soft building blocks in J_s .

In the following, we label operators that consist of a single building block by J_i^{An} , where $n = 1, 2$ indicates the relative power suppression due to additional derivatives. Operators with two, three, ... elementary building blocks are denoted as $J_i^{Bn}, J_i^{Cn}, \dots$, and here n indicates the power suppression relative to J_i^{A0} . The $\mathcal{O}(\lambda)$ operator basis consists of

$$J_i^{A1}(t_i) = i\partial_{\perp i}^{\nu} J_i^{A0}, \quad (3.6)$$

$$J_i^{B1}(t_{i_1}, t_{i_2}) = \psi_{i_1}(t_{i_1} n_{i+}) \psi_{i_2}(t_{i_2} n_{i+}) \in \begin{cases} \mathcal{A}_{\perp i}^{\mu}(t_{i_1} n_{i+}) \chi_i(t_{i_2} n_{i+}) \\ \chi_i(t_{i_1} n_{i+}) \chi_i(t_{i_2} n_{i+}) \\ \mathcal{A}_{\perp i}^{\mu}(t_{i_1} n_{i+}) \mathcal{A}_{\perp i}^{\nu}(t_{i_2} n_{i+}) \\ \chi_i(t_{i_1} n_{i+}) \tilde{\chi}_i(t_{i_2} n_{i+}), \end{cases} \quad (3.7)$$

and at $\mathcal{O}(\lambda^2)$

$$J_i^{A2}(t_i) = i\partial_{\perp i}^{\nu} i\partial_{\perp i}^{\rho} J_i^{A0}, \quad (3.8)$$

$$J_i^{B2}(t_{i_1}, t_{i_2}) \in \begin{cases} \psi_{i_1}(t_{i_1} n_{i+}) i\partial_{\perp i}^{\mu} \psi_{i_2}(t_{i_2} n_{i+}) \\ i\partial_{\perp i}^{\mu} [\psi_{i_1}(t_{i_1} n_{i+}) \psi_{i_2}(t_{i_2} n_{i+})], \end{cases} \quad (3.9)$$

$$J_i^{C2}(t_{i_1}, t_{i_2}, t_{i_3}) = \Psi_{i_1}(t_{i_1} n_{i_+}) \Psi_{i_2}(t_{i_2} n_{i_+}) \Psi_{i_3}(t_{i_3} n_{i_+}), \quad (3.10)$$

where $\Psi_{i_1} \Psi_{i_2}$ in J_i^{B2} can be any combination from J_i^{B1} , and Ψ_{i_3} can be any of the elementary building blocks from Eq. (3.4). This exhausts the options (i), (ii) from above at $\mathcal{O}(\lambda^2)$. Concerning (iii), it can be shown (App. B of Ref. [1]) that $n_{i_-} \mathcal{A}_i$ can be eliminated by the collinear field equation, and the same is true for the soft covariant derivatives, which operate on collinear building blocks in the form

$$in_{i_-} D_s \chi_i, \quad [in_{i_-} D_s, \mathcal{A}_{\perp i}^\mu]. \quad (3.11)$$

It follows that one can use a basis of collinear building blocks that does not involve soft fields through covariant derivatives and is constructed entirely from ordinary transverse derivatives and the elementary building block for the quark field and the transverse gluon field.

In addition to the collinear building blocks, the N -jet operator may also contain a pure soft building block $J_s(x)$. In the pure soft sector there is no need to perform the SCET multipole expansion of the soft fields and therefore the soft gauge transformation $U_s(x)$ in this case depends on x rather than on x_- . The covariant pure soft building blocks start at $\mathcal{O}(\lambda^3)$, for example

$$q(x) \sim \lambda^3, \quad F_s^{\mu\nu} \sim \lambda^4, \quad iD_s^\mu q(x) \sim \lambda^5, \quad (3.12)$$

where *on soft building blocks* $iD_s^\mu(x) = i\partial^\mu + g_s A_s^\mu(x)$ and the soft field-strength tensor is defined as $ig_s F_s^{\mu\nu} = [iD_s^\mu, iD_s^\nu]$. When a pure soft building block appears in the product (3.1) with collinear fields in several directions, the multipole expansion with respect to each of them requires that one sets $x = 0$. However, due to the $\mathcal{O}(\lambda^3)$ suppression, $J_s(0)$ in Eq. (3.1) can be dropped at $\mathcal{O}(\lambda^2)$. Therefore, soft fields enter neither via the soft nor via the collinear building blocks for our basis choice, up to $\mathcal{O}(\lambda^2)$, and option (iii) (fourth panel in Fig. 1) is not relevant. This implies that the emission of a soft gluon from the hard process, which generates the N -jet operator, is entirely accounted for by Lagrangian interactions. We discuss this somewhat surprising result further in the next section.

The case of N -jet operators differs from that of heavy-to-light currents, which consist of one collinear direction and a soft heavy-quark field, whose decay is the source of large energy for the collinear final state. The basis of subleading SCET operators listed in Ref. [10] does contain soft covariant derivatives at $\mathcal{O}(\lambda^2)$ due to the presence of the soft heavy-quark building block at leading power. The absence of soft building blocks in N -jet operators at $\mathcal{O}(\lambda^2)$ is also an important difference and simplification of the position-space vs. the label-field SCET formalism [11, 12], where soft fields must be included in the basis operators at $\mathcal{O}(\lambda^2)$ [13, 14]. The difference arises from a different split into collinear and soft, since in the label formalism only the large and transverse component of collinear momentum are treated as labels, while the residual spatial dependence of all fields, collinear and soft, is soft. The difference in the operator basis due to this is compensated by a corresponding difference in the soft-collinear interactions in the Lagrangian in the two formulations of SCET.

For the derivation of the anomalous dimension and renormalization group equation it is convenient to adopt the interaction picture and treat the subleading SCET Lagrangians $\mathcal{L}_i^{(n)}$ as perturbations, such that all operator matrix elements are understood to be evaluated with the leading-power SCET Lagrangian. The basis of subleading power N -jet operators at a given order in λ then includes further ‘‘non-local’’ operators from the time-ordered products of the current operators J at

lower orders in λ with the subleading terms in the SCET Lagrangian. The “local” (in reality, light-cone) currents do not mix into the non-local time-ordered product operators, but the latter can, in principle, mix into the former. At $\mathcal{O}(\lambda)$ the time-ordered product operators are of the form

$$J_i^{T1}(t_i) = i \int d^4x T \left\{ J_i^{A0}(t_i), \mathcal{L}_i^{(1)}(x) \right\}. \quad (3.13)$$

The generalization to higher orders in λ should be evident and includes subleading currents J^{A1} , J^{B1}, \dots , subleading Lagrangians $\mathcal{L}_i^{(n)}$ ($n > 1$), and time-ordered products with multiple subleading Lagrangian interactions. In contrast to the local current operators, the time-ordered products always contain soft fields.

For physical applications as well as the presentation of the anomalous dimension it is useful to work in terms of collinear momentum fractions by defining the Fourier transforms of the operators with respect to the positions t_{i_k} in the collinear direction,

$$J_i(P_i, \{x_{i_k}\}) \equiv P_i^{n_i} \int \prod_{k=1}^{n_i} dt_{i_k} e^{-iP_i \sum_{k=1}^{n_i} t_{i_k} x_{i_k}} J_i(\{t_{i_k}\}), \quad (3.14)$$

where x_{i_k} are fractions of the collinear momentum in direction i , carried by the k -th building block such that $\sum_{k=1}^{n_i} x_{i_k} = 1$. P_i is the total (outgoing) collinear momentum in direction i and $n_{i+} p_{i_k} = x_{i_k} P_i > 0$ for an outgoing momentum in direction i , such that from Eq. (3.14) also $P_i > 0$ and $x_{i_k} \in (0, 1)$ for all momenta outgoing, which we shall assume in the following. In general, the basis of N -jet operators can then be written in the form

$$J(\{P_i\}, \{x_{i_k}\}) = \prod_{i=1}^N J_i(P_i, \{x_{i_k}\}) \quad (3.15)$$

The operators are given by $J_i \in \{J_i^{An}, J_i^{Bn}, J_i^{Cn}, \dots\}$, depending on the number of collinear building blocks and the order in λ . The total power suppression of the N -jet operator is then obtained from adding up the suppression factors in λ from each direction. For example, at $\mathcal{O}(\lambda^2)$, it is possible to either have a J_i^{X2} operator (with $X = A, B, C$) in one direction and J_i^{A0} operators in the remaining $N - 1$ directions, or two operators $J_i^{X1} J_j^{Y1}$, with $X, Y = A, B$, and J_i^{A0} operators in the remaining $N - 2$ directions.

4. LBK amplitude from SCET

The Low-Burnett-Kroll [15, 16] formula

$$-g_s \sum_{i=1}^N \mathbf{T}_i \left(\frac{p_i \cdot \varepsilon(k)}{p_i \cdot k} + \frac{\varepsilon_\mu(k) k_\nu J_i^{\mu\nu}}{p_i \cdot k} \right) A_0(\{p_i\}) \quad (4.1)$$

expresses the radiative amplitude for the emission of a soft gluon with momentum k in terms of the corresponding non-radiative amplitude A_0 (for example, the left panel in Fig. 1) including the NLP correction to the eikonal amplitude. The next-to-soft term contains the angular momentum operator

$$J_i^{\mu\nu} = p_i^\mu \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\mu}} + \Sigma_i^{\mu\nu}, \quad (4.2)$$

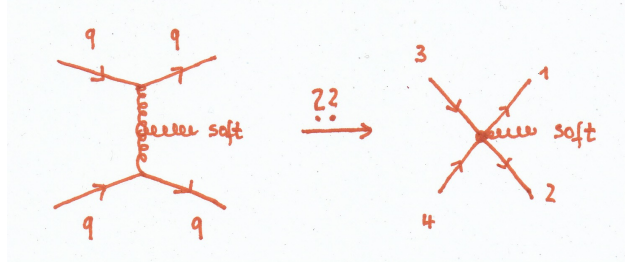


Figure 2: Soft emission in quark-quark scattering.

where the spin operator $\Sigma_i^{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu]$ for a Dirac fermion. The term $p_i^\nu \frac{\partial}{\partial p_{i\mu}}$ produces a local contribution to (4.1), which appears to correspond to an N -jet operator with a soft building block. In view of the above result that no such operators exist at NLP, it is instructive to reproduce the LBK formula in position-space SCET.¹

As an example, we consider the scattering of two quarks of different flavour. The tree-level non-radiative amplitude is given by

$$A_0 = -g_s^2 \bar{u}(p_1) \gamma^\rho T^a u(-p_3) \frac{-i}{(p_1 + p_3)^2} \bar{u}(p_2) \gamma_\rho T^a u(-p_4), \quad (4.3)$$

and carries a non-trivial dependence on the external momenta. The tree-level radiative amplitude is given by five diagrams. When the soft emitted gluon is attached to the internal propagator as shown on the left in Fig. 2, the duplicated internal propagator appears to give rise to the hard four-jet operator with a soft gluon building block shown on the right. However, contrary to this expectation, we find that the full-theory diagram on the left is reproduced to NLP by time-ordered products with the sub-leading lagrangian interactions alone, just as the other four diagrams with a soft gluon attached to one of the external legs, in the following way:

- We choose a reference frame in which every p_i is aligned with the corresponding collinear reference vector n_{i-} , that is, $p_{\perp i} = 0$. In this frame the $\mathcal{O}(\lambda)$ correction to the LP eikonal term vanishes, as does the contribution from $J_i^{A2}(t_i)$ at NLP $\mathcal{O}(\lambda^2)$.
- The time-ordered product $\int d^4x T \{ J_i^{A0}(t_i), \mathcal{L}_i^{(2)}(x) \}$ with the $\mathcal{O}(\lambda^2)$ collinear quark-soft gluon interaction produces the spin contribution and an additional term, which is cancelled by a contribution from $\int d^4x T \{ J_i^{A1}(t_i), \mathcal{L}_i^{(1)}(x) \}$.
- The orbital angular momentum term also originates fully from the above two time-ordered products. The required derivatives on the non-radiative amplitude arise from the multipole-expanded soft-gluon interaction vertices. For example, $n_{i-} \cdot x$ in $\mathcal{L}_i^{(2)}(x)$ yields a factor of it_i in the convolution of the hard coefficient function with the N -jet operator along the light-like direction, which turns into a momentum-derivative on the momentum-space coefficient function/non-radiative amplitude.

¹The LBK amplitude was analyzed in the framework of label-SCET in Ref. [13]. It turns out that the two formulations of SCET recover the LBK formula in rather different ways.

- For this to work the hard coefficient of $J_i^{A1}(t_i)$ must be related in a specific way to the one of $J_i^{A0}(t_i)$. The required relation follows from invariance of SCET under reparameterizations of the light-like direction reference vectors [17].

This result appears to be generic, and we checked that it holds as well for the heavy-to-light decay considered in Ref. [10]. Note that, unlike in the original derivation of the LBK amplitude, one does not have to invoke gauge invariance and the Ward identity to fix the local contribution to the LBK amplitude, since gauge invariance is manifest in every SCET current and Lagrangian term.

5. Anomalous dimension of NLP N -jet operators

Operator renormalization in renormalized perturbation theory is given by

$$\langle \mathcal{O}_P(\{\phi_{\text{ren}}\}, \{g_{\text{ren}}\}) \rangle_{\text{ren}} = \sum_Q Z_{PQ} \prod_{\phi \in Q} Z_\phi^{1/2} \prod_{g \in Q} Z_g \langle \mathcal{O}_{Q,\text{bare}}(\{\phi_{\text{ren}}\}, \{g_{\text{ren}}\}) \rangle, \quad (5.1)$$

where P, Q label the N -jet operators as well as time-ordered products of N -jet operators with insertions of power-suppressed interactions $\mathcal{L}_{\text{SCET}}$. At one-loop, writing $Z_{PQ} = \delta_{PQ} + \delta Z_{PQ}$, demanding that the left-hand side is finite and accounting for the fact that soft interactions do not change the large collinear momentum fractions implies the $\overline{\text{MS}}$ scheme renormalization conditions

$$0 = \langle J_P(x) \rangle_{1\text{-loop, div.}}^{\text{soft, ij}} + \sum_Q \delta Z_{PQ}^{s, ij}(x) \langle J_Q(x) \rangle_{\text{tree}}, \quad (5.2)$$

$$0 = \langle J_P(x) \rangle_{1\text{-loop, div.}}^{\text{coll, i}} + \sum_Q \int \prod_{k>1} dy_{ik} \left[\delta Z_{PQ}^{c, i}(x, y) + \delta_{PQ} \prod_{k>1} \delta(x_{ik} - y_{ik}) \left(\frac{1}{2} \sum_{\phi \in J_{P_i}} \delta Z_\phi + \sum_{g \in J_{P_i}} \delta Z_g \right) \right] \langle J_Q(y) \rangle_{\text{tree}}, \quad (5.3)$$

from which the collinear-loop and soft-loop contributions to the renormalization factor can be determined. The anomalous dimension matrix is defined by

$$\Gamma = -\mathbf{Z}^{-1} \frac{d}{d \ln \mu} \mathbf{Z}, \quad (5.4)$$

where we use matrix notation involving both discrete indices (P, Q) labelling the set of N -jet operators including open Lorentz, spinor and colour indices as well as continuous indices (x, y) for the collinear momentum fractions associated with each building block. To extract the UV divergences, we regularize the IR divergences by assuming that the external states have small off-shellness $p_{i_k}^2 \neq 0$. At the end of the computation, the soft and collinear part are combined and only then the limit $p_{i_k}^2 \rightarrow 0$ can be taken. The cancellation of the off-shell regulator dependence serves as an additional check of the computation.

5.1 $F = 2$ operators

In the following, we will focus on the case in which one of the collinear directions carries fermion number $F = 2$. The simplification of this choice results from the absence of a leading-power operator J_i^{A0} (and consequently all J_i^{An}), since one needs two fermion fields in the same

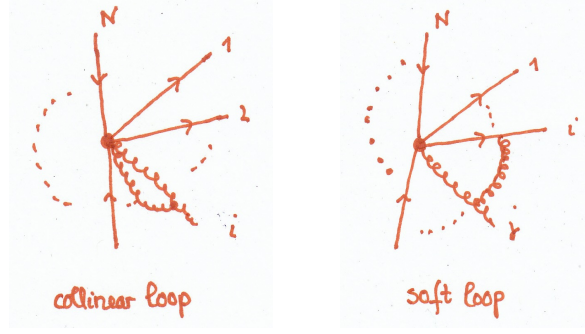


Figure 3: Collinear (left) and soft (right) loop diagram.

direction to begin with. Nevertheless, this simpler case allows us to display most of the features of the anomalous dimension at $\mathcal{O}(\lambda^2)$. The $F = 2$ operator basis at $\mathcal{O}(\lambda)$ consists of the single collinear operator

$$J_{\chi_\alpha \chi_\beta}^{B1}(t_1, t_2) = \chi_{i\alpha}(t_1, n_{i+}) \chi_{i\beta}(t_2, n_{i+}). \quad (5.5)$$

We keep open the Dirac spinor indices α, β , because they will in general be contracted with components of the N -jet operator from the other collinear directions $j \neq i$. The same rule applies to Lorentz and colour indices, and we only assume that the total N -jet operator transforms as a colour singlet. The $\mathcal{O}(\lambda^2)$ basis operators are constructed as described in Sec. 3 and include the three-body operator

$$J_{\mathcal{A}^\mu \chi_\alpha \chi_\beta}^{C2}(t_1, t_2, t_3) = \mathcal{A}_{\perp i}^\mu(t_1, n_{i+}) \chi_{i\alpha}(t_2, n_{i+}) \chi_{i\beta}(t_3, n_{i+}). \quad (5.6)$$

We refer to Ref. [1] for the details of this computation and restrict ourselves here to the following remarks:

- Collinear loops always connect lines within the same collinear sector, while soft loops always connect lines of two different collinear directions, see Fig. 3.
- Collinear loops within a single building block (as in the left panel of Fig. 3) are trivial. However, collinear loops connecting different building blocks in the same collinear direction change the distribution of momentum fraction and result in momentum-dependent renormalization factors. For example, the collinear contribution to the anomalous dimension of the B1 operator (5.5) reads

$$\delta Z_{\chi_\alpha \chi_\beta, \chi_\gamma \chi_\delta}^{c,i}(x, y) = -\delta(x-y) \delta_{\alpha\gamma} \delta_{\beta\delta} X_{i_1 i_2} + \frac{1}{\epsilon} \gamma_{\chi_\alpha \chi_\beta, \chi_\gamma \chi_\delta}^i(x, y), \quad (5.7)$$

with

$$X_{i_1 i_2} \equiv \frac{\alpha_s}{4\pi} \left\{ \frac{2}{\epsilon^2} (\mathbf{T}_{i_1} + \mathbf{T}_{i_2})^2 + \frac{2}{\epsilon} (\mathbf{T}_{i_1} + \mathbf{T}_{i_2}) \cdot \left[\mathbf{T}_{i_1} \ln \left(\frac{\mu^2}{-p_1^2} \right) + \mathbf{T}_{i_2} \ln \left(\frac{\mu^2}{-p_2^2} \right) \right] + \frac{3}{2\epsilon} (\mathbf{T}_{i_1}^2 + \mathbf{T}_{i_2}^2) \right\}, \quad (5.8)$$

and

$$\begin{aligned} \gamma_{\chi_\alpha \chi_\beta \chi_\gamma \chi_\delta}^j(x, y) = & \frac{\alpha_s \mathbf{T}_{i_1} \cdot \mathbf{T}_{i_2}}{2\pi} \left\{ \delta_{\alpha\gamma} \delta_{\beta\delta} \left(\theta(x-y) \left[\frac{1}{x-y} \right]_+ + \theta(y-x) \left[\frac{1}{y-x} \right]_+ \right. \right. \\ & \left. \left. - \theta(x-y) \frac{1-\frac{\bar{x}}{y}}{y} - \theta(y-x) \frac{1-\frac{x}{\bar{y}}}{y} \right) \right. \\ & \left. - \frac{1}{4} (\sigma_\perp^{\nu\mu})_{\alpha\gamma} (\sigma_{\perp\nu\mu})_{\beta\delta} \left(\theta(x-y) \frac{\bar{x}}{y} + \theta(y-x) \frac{x}{y} \right) \right\}. \end{aligned} \quad (5.9)$$

- At $\mathcal{O}(\lambda^2)$ there is operator mixing among the B2 and C2 operators, but C2 operators do not mix into B2 operators.
- The renormalization factor of the three-body operator (5.6) follows from the ones of two-body quark-quark and quark-gluon operators, since in the one-loop order only two collinear building blocks can be connected through a loop.
- For the $F = 2$ case considered, the soft renormalization is trivial. It is diagonal in operator space and momentum fraction, spin-independent, and has the same dipole form as the first term in Eq. (1.1). The possible mixing of the time-ordered product operators into the current operators also vanishes.

5.2 Structure of the anomalous dimension

Summing over all N collinear directions, combining the soft and collinear loop contributions, and further using colour conservation for the total (colour-singlet) N -jet operator, we can cast the one-loop anomalous dimension into the form

$$\begin{aligned} \Gamma_{PQ}(x, y) = & \delta_{PQ} \delta(x-y) \left[-\gamma_{\text{cusp}}(\alpha_s) \sum_{i < j} \sum_{l, k} \mathbf{T}_{i_l} \cdot \mathbf{T}_{j_k} \ln \left(\frac{-s_{ij} x_{i_l} x_{j_k}}{\mu^2} \right) + \sum_i \sum_l \gamma_i(\alpha_s) \right] \\ & + 2 \sum_i \delta^{[i]}(x-y) \gamma_{PQ}^i(x, y), \end{aligned} \quad (5.10)$$

where $\gamma_{\text{cusp}}(\alpha_s) = \frac{\alpha_s}{\pi}$, $\gamma_i(\alpha_s) \equiv -\frac{\alpha_s}{2\pi} \mathbf{T}_{i_l}^2 c_{i_l} = -\frac{3\alpha_s}{4\pi} C_F(0)$ for collinear quarks (gluons). The short-hand delta-functions are defined as $\delta(x-y) \equiv \prod_i \prod_{k=2}^n \delta(x_{i_k} - y_{i_k})$, $\delta^{[i]}(x-y) \equiv \prod_{j \neq i} \prod_{k > 1} \delta(x_{j_k} - y_{j_k})$. The second line captures the off-diagonal contributions such as Eq. (5.9) and arises only from single $1/\varepsilon$ poles. The first line is diagonal and contains a logarithm whose coefficient is related to the familiar cusp anomalous dimension.

We note the similarity of the first line with the leading-power anomalous dimension (1.1). There is an additional sum over the number of building blocks in every collinear direction. Indeed, Eq. (5.10) reduces to Eq. (1.1) when there is only a single building block in each collinear direction (i.e. $l, k = 1, x_{i_l}, x_{j_k} \rightarrow 1$), such that in the notation used above $\delta(x-y) \equiv \prod_i \prod_{k > 1} \delta(x_{i_k} - y_{i_k}) \rightarrow 1$ is an empty product equal to unity. Furthermore, possibly non-diagonal contributions encapsulated in γ_{PQ}^i vanish at the leading power.

For the case in which one of the collinear directions contains two fermionic building blocks (direction i , say), there is only a single N -jet operator of this kind at $\mathcal{O}(\lambda)$, given by the product of

$J_i = J_{\chi\chi}^{B1}(t_{i_1}, t_{i_2})$ defined in Eq. (5.5) for the direction labelled by i and leading-power building blocks for all other $N - 1$ directions $J_{j \neq i} = J_j^{A0}$. In this case, the anomalous dimension is off-diagonal in the collinear momentum fractions in direction i ,

$$\sum_{j=1}^N \delta^{[j]}(x-y) \frac{\gamma_{PQ}^j(x,y)}{\varepsilon} \rightarrow \frac{1}{\varepsilon} \gamma_{\chi\chi, \chi\chi}^j(x_{i_1}, y_{i_1}), \quad (5.11)$$

where the right-hand side is given by Eq. (5.9), and we have used $\gamma_{PQ}^j(x,y) = 0$ for all leading-power building blocks $j \neq i$. Furthermore the product of delta functions for the $N - 1$ other directions $\delta^{[i]}(x-y) \equiv \prod_{j \neq i} \prod_{k > 1} \delta(x_{j_k} - y_{j_k}) \rightarrow 1$ also collapses to unity.

At $\mathcal{O}(\lambda^2)$, there are two possibilities. First, that the direction i which we choose to carry fermion-number two encompasses itself the $\mathcal{O}(\lambda^2)$ suppression, i.e. it is represented by one of the three operators $J_i \in \{J_{\chi\partial\chi}^{B2}, J_{\partial(\chi\chi)}^{B2}, J_{\mathcal{A}\chi\chi}^{C2}\}$. Then the other $N - 1$ directions have to contain leading-power building blocks, as before. There is operator mixing among the three $\mathcal{O}(\lambda^2)$ suppressed operators and the structure of the anomalous dimension matrix is

$$\sum_j \delta^{[j]}(x-y) \frac{\gamma_{PQ}^j(x,y)}{\varepsilon} \rightarrow \frac{1}{\varepsilon} \begin{pmatrix} \gamma_{\chi\partial\chi, \chi\partial\chi}^j & \gamma_{\chi\partial\chi, \partial(\chi\chi)}^j & \gamma_{\chi\partial\chi, \mathcal{A}\chi\chi}^j \\ 0 & \gamma_{\partial(\chi\chi), \partial(\chi\chi)}^j & 0 \\ 0 & 0 & \gamma_{\mathcal{A}\chi\chi, \mathcal{A}\chi\chi}^j \end{pmatrix}, \quad (5.12)$$

where the non-zero contributions are given in Ref. [1]. The anomalous dimension is diagonal with respect to the other $N - 1$ directions.

The second possibility that can occur at $\mathcal{O}(\lambda^2)$ is that direction i with $F = 2$ is described by the $\mathcal{O}(\lambda)$ contribution $J_i = J_{\chi\chi}^{B1}(t_{i_1}, t_{i_2})$, and one of the other $N - 1$ directions, say direction i' , contributes an additional $\mathcal{O}(\lambda)$ suppression. The remaining $N - 2$ directions must then be represented by leading-power building blocks. Since we do not require direction i' to have a definite fermion number, there are more possibilities, in particular $J_{i'} \in \{J_{\partial\chi}^{A1}, J_{\partial\mathcal{A}}^{A1}, J_{\mathcal{A}\chi}^{B1}, J_{\mathcal{A}\mathcal{A}}^{B1}, J_{\chi\chi}^{B1}, J_{\bar{\chi}\chi}^{B1}\}$ (plus hermitian conjugated operators). In this case, we need in addition the corresponding anomalous dimension matrices $\gamma_{PQ}^{i'}$ for these operators.

6. Summary

The anomalous dimension of subleading power N -jet operators is one of the key ingredients for the resummation of logarithmically enhanced terms in partonic cross sections beyond the leading power. Schematically, an observable expanded in powers of a small scale-ratio M/Q will be represented in the form

$$d\sigma = \sum_{a,b} C_a C_b^* \otimes J_{ab} \otimes S_{ab} \quad (6.1)$$

of a convolution of the product $C_a C_b^*$ of hard matching coefficients, jet and soft functions. The N -jet operator anomalous dimension discussed here governs the renormalization-group evolution of hard functions of all possible operators up to NLP, which are convoluted with their respective jet and soft functions. For a given process the colour- and spin-space anomalous dimensions have to be projected on a basis of scalar operators. The renormalization-group equations are integro-differential equations in every collinear sector, which will most likely have to be solved approximately or numerically.

Acknowledgements

This work has been supported by the BMBF grant no. 05H15WOCAA.

References

- [1] M. Beneke, M. Garny, R. Szafron and J. Wang, *Anomalous dimension of subleading-power N -jet operators*, 1712.04416.
- [2] T. Becher and M. Neubert, *Infrared singularities of scattering amplitudes in perturbative QCD*, *Phys. Rev. Lett.* **102** (2009) 162001, [0901.0722].
- [3] Ø. Almeliid, C. Duhr and E. Gardi, *Three-loop corrections to the soft anomalous dimension in multileg scattering*, *Phys. Rev. Lett.* **117** (2016) 172002, [1507.00047].
- [4] S. Moch, J. A. M. Vermaseren and A. Vogt, *The Quark form-factor at higher orders*, *JHEP* **08** (2005) 049, [hep-ph/0507039].
- [5] D. Bonocore, E. Laenen, L. Magnea, L. Vernazza and C. D. White, *Non-abelian factorisation for next-to-leading-power threshold logarithms*, *JHEP* **12** (2016) 121, [1610.06842].
- [6] I. Moulton, L. Rothen, I. W. Stewart, F. J. Tackmann and H. X. Zhu, *Subleading Power Corrections for N -Jettiness Subtractions*, *Phys. Rev.* **D95** (2017) 074023, [1612.00450].
- [7] R. Boughezal, X. Liu and F. Petriello, *Power Corrections in the N -jettiness Subtraction Scheme*, *JHEP* **03** (2017) 160, [1612.02911].
- [8] M. Beneke, A. P. Chapovsky, M. Diehl and T. Feldmann, *Soft collinear effective theory and heavy to light currents beyond leading power*, *Nucl. Phys.* **B643** (2002) 431–476, [hep-ph/0206152].
- [9] M. Beneke and T. Feldmann, *Multipole expanded soft collinear effective theory with non-abelian gauge symmetry*, *Phys. Lett.* **B553** (2003) 267–276, [hep-ph/0211358].
- [10] M. Beneke, F. Campanario, T. Mannel and B. D. Pecjak, *Power corrections to $\bar{B} \rightarrow X_u \ell \bar{\nu}$ ($X_s \gamma$) decay spectra in the ‘shape-function’ region*, *JHEP* **06** (2005) 071, [hep-ph/0411395].
- [11] C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, *An Effective field theory for collinear and soft gluons: Heavy to light decays*, *Phys. Rev.* **D63** (2001) 114020, [hep-ph/0011336].
- [12] C. W. Bauer, D. Pirjol and I. W. Stewart, *Soft collinear factorization in effective field theory*, *Phys. Rev.* **D65** (2002) 054022, [hep-ph/0109045].
- [13] A. J. Larkoski, D. Neill and I. W. Stewart, *Soft Theorems from Effective Field Theory*, *JHEP* **06** (2015) 077, [1412.3108].
- [14] I. Feige, D. W. Kolodrubetz, I. Moulton and I. W. Stewart, *A Complete Basis of Helicity Operators for Subleading Factorization*, 1703.03411.
- [15] F. E. Low, *Bremsstrahlung of very low-energy quanta in elementary particle collisions*, *Phys. Rev.* **110** (1958) 974–977.
- [16] T. H. Burnett and N. M. Kroll, *Extension of the Low soft photon theorem*, *Phys. Rev. Lett.* **20** (1968) 86.
- [17] A. V. Manohar, T. Mehen, D. Pirjol and I. W. Stewart, *Reparameterization invariance for collinear operators*, *Phys. Lett.* **B539** (2002) 59–66, [hep-ph/0204229].