

On the impact of off-shell contributions on the top quark mass extraction in $t\bar{t}j$ events

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In this contribution we present a systematic comparison of the full off-shell calculation for $pp \rightarrow t\bar{t}j$ in the dileptonic decay channel with a description based on the narrow width approximation. The aim of this study is to estimate the size of off-shell and non-resonant contributions to the shape of differential distributions, that will be utilized in the top quark mass extraction using the template fit method.

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1. Introduction

With the advent of the LHC Run 2 at a center-of-mass energy of $\sqrt{s} = 13$ TeV top quark physics entered the precision era. The main goals of the top quark physics program are the precise determination of top quark properties, such as its mass, its coupling to the Higgs boson as well as gauge bosons. Other key measurements include differential distributions, fiducial cross sections and the measurement of spin correlations of the top quark decay products.

For now the most precise calculations for stable top quarks are the total cross section at NNLO+NNLL [1] and differential distributions at NNLO QCD + NLO EW [2]. However, the top quark is a highly unstable particle and decays before it hadronizes and can thus be studied via its decay products. Including the top quark decay in the calculation allows the precise study of fiducial phase space regions and offers a closer modelling of the experimentally accessible final states. To this end, the decay has been incorporated in the narrow width approximation at NLO in Refs. [3, 4] and at approximate NNLO in Ref. [5]. Contrary to this approach, in Refs. [6, 7, 8, 9, 10, 11] the on-shell treatment of top quarks has been abandoned and the complete NLO QCD corrections for the process $pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b \bar{b} + X$ were calculated and are now also consistently matched with parton showers [12]. Recently also the NLO EW corrections have become available [13] as well as the NLO QCD corrections for the semi-leptonic decay channel [14].

However, at the energies that the LHC is operating, top quarks are abundantly produced in association with either additional jets or electroweak bosons as illustrated in Fig 1. As one can see,

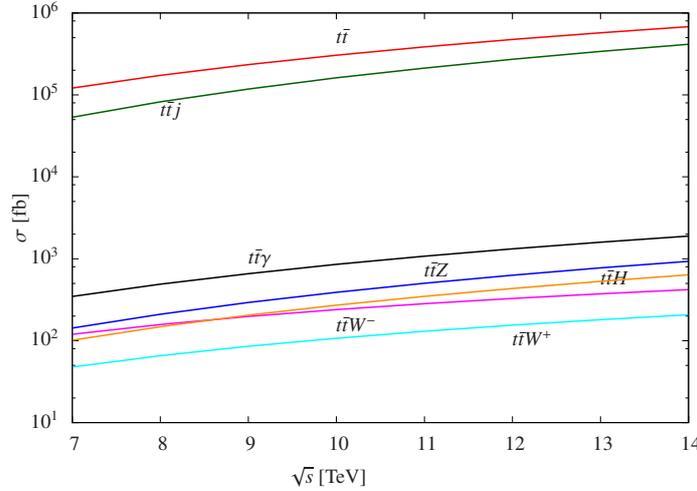


Figure 1: Production cross section of $t\bar{t}$ and associated $t\bar{t}$ production as a function of the center-of-mass energy \sqrt{s} .

a considerable amount of top quark pair events are actually accompanied by an additional hard jet. Therefore, we want to focus on the $pp \rightarrow t\bar{t}j$ process which has already been studied in great detail in recent years [15, 16, 17, 18, 19, 20, 21, 22, 23]. The $pp \rightarrow t\bar{t}j$ process is particularly interesting because it can be used to extract the top quark mass parameter. In Refs. [24, 25] an observable has been designed for this process that has larger m_t sensitivity than the inclusive top quark pair production process. This observable has been also already successfully used by the experimental

collaborations [26, 27] to extract the top quark mass. Here, we present parts of our recent study [28] that extends the aforementioned $t\bar{t}j$ studies where we address the impact of off-shell effects on the extraction of the top quark mass parameter.

2. Outline of the calculation

We want to address the size of off-shell effects on the top quark mass extraction from differential distributions. To this end, we compare the full off-shell calculation for the $pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b\bar{b}j$ final state with different approximations of the calculation. The NLO QCD calculation, as described in detail in Refs. [22, 23], is used and takes into account all double, single and non-resonant contributions as well as their interference contributions at $\mathcal{O}(\alpha_s^4 \alpha^4)$. This calculation has been performed within the HELAC-NLO framework [29], which consists of the two building blocks HELAC-DIPOLES [30, 31] and HELAC-1LOOP [32].

The full off-shell calculation is compared to the narrow-width-approximation (NWA) as presented in Ref. [18], where the matrix element can be factorized into the following on-shell top quark contributions

$$\begin{aligned} \lim_{\Gamma_t/m_t \rightarrow 0} \left| M^{WWb\bar{b}j} \right|^2 &= \left| M^{t\bar{t}j} \right|^2 \otimes Br(t \rightarrow Wb) \otimes Br(\bar{t} \rightarrow W\bar{b}) \\ &+ \left| M^{t\bar{t}} \right|^2 \otimes Br(t \rightarrow Wbj) \otimes Br(\bar{t} \rightarrow W\bar{b}) \\ &+ \left| M^{t\bar{t}} \right|^2 \otimes Br(t \rightarrow Wb) \otimes Br(\bar{t} \rightarrow W\bar{b}j) + \mathcal{O}\left(\frac{\Gamma_t}{m_t}\right), \end{aligned} \quad (2.1)$$

where we ignored for brevity the leptonic decays of the W bosons. We also compare the full off-shell calculation with an approximation dubbed NWA_{prod} that employs NLO QCD corrections only to the production part while taking only leading order (LO) top quark decays into account and represents the calculation presented in Ref. [17].

The calculations are performed for the LHC at a center-of-mass energy of $\sqrt{s} = 13$ TeV, while the detailed list of standard model input parameters, phase space cuts, etc. can be found in Ref. [28]. The CT14 [33], MMHT14 [34] and the NNPDF 3.0 [35] PDF sets have been employed in the calculation, while a common renormalization and factorization scale $\mu_R = \mu_F = \mu_0$ has been chosen. We consider three different scale choices, a fixed scale $\mu_0 = m_t$ and two dynamical ones, namely $\mu_0 = E_T/2$ and $\mu_0 = H_T/2$, with

$$E_T = \sqrt{m_t^2 + p_T^2(t)} + \sqrt{m_{\bar{t}}^2 + p_T^2(\bar{t})}, \quad (2.2)$$

where the top quark momenta are reconstructed from the final state momenta, i.e. the top quark momentum is given by $p(t) = p(e^+) + p(\nu_e) + p(j_b)$. On the other end, H_T is independent of the underlying process and defined as

$$H_T = p_T(e^+) + p_T(\mu^-) + p_T(j_b) + p_T(j_{\bar{b}}) + p_T(j_1) + p_T^{miss}. \quad (2.3)$$

As already mentioned in the introduction, we focus in this contribution on two particular observables, namely $\mathcal{R}(m_t^{pole}, \rho_s)$ and M_{be^+} . Before, we discuss the top quark mass extraction from these

distributions in detail, we will investigate them closer and discuss shape differences between the different approaches.

Our first observable under investigation, $\mathcal{R}(m_t^{pole}, \rho_s)$, is defined by

$$\mathcal{R}(m_t^{pole}, \rho_s) = \frac{1}{\sigma_{i\bar{i}j}} \frac{d\sigma_{i\bar{i}j}}{d\rho_s}(m_t^{pole}, \rho_s), \quad \text{with} \quad \rho_s = \frac{2m_0}{M_{i\bar{i}j}}, \quad (2.4)$$

where $m_0 = 170$ GeV is an arbitrary scale and $M_{i\bar{i}j}$ the invariant mass of the $t\bar{t}$ system and the leading hard jet.

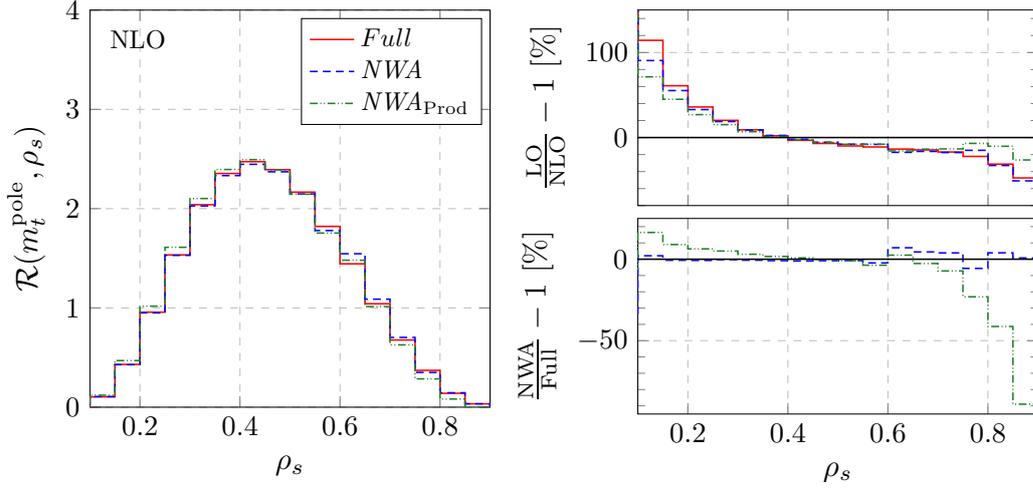


Figure 2: The normalized differential distribution for $\mathcal{R}(m_t^{pole}, \rho_s)$ for the LHC at $\sqrt{s} = 13$ TeV with $\mu_0 = m_t = 173.2$ GeV.

In Fig. 2 the $\mathcal{R}(m_t^{pole}, \rho_s)$ distribution for the three calculations mentioned above together with their NLO K-factors and the relative deviation from the full off-shell calculation is shown. Let us first note, that in all three cases the NLO K-factor is not flat and the shape is altered up to 50% for *Full* and *NWA* in the threshold region ($\rho_s \approx 1$). On the other hand, the *NWA* result approximates the *full* off-shell calculation reasonably well over the whole spectrum of the distribution and differences of at most 15% are visible in the region of $\rho_s > 0.6$. Contrary, *NWA_{prod}* yields substantial differences of up to 85% in the threshold region. These differences will have a sizeable impact on the top quark mass parameter as we will show later.

The second observable under investigation, a rather standard one, is the normalized differential distribution of M_{be^+} , which is defined as

$$\frac{1}{\sigma_{i\bar{i}j}} \frac{d\sigma_{i\bar{i}j}}{dM_{be^+}}, \quad \text{where} \quad M_{be^+} = \min\{M_{b_1e^+}, M_{b_2e^+}\}. \quad (2.5)$$

This observable has a kinematical endpoint at $M_{be^+}^{max} = \sqrt{m_t^2 - m_W^2} \approx 154$ GeV, if top quarks and W bosons are considered on-shell. Therefore, only additional radiation as well as off-shell effects can smear this boundary. The observable is presented in Fig. 3 where we see that neglecting

the QCD corrections to the top quark decay amounts to shape differences between NWA_{prod} and $Full$ of the order of 15% at the kinematical endpoint. On the other hand, NWA accounts for all dominant contributions below the kinematical endpoint. Thus, below the endpoint NWA and $Full$ are essentially identical. However, above the endpoint we see deviations from the full off-shell calculation of the order of 50%. Nonetheless, due to the severe drop of the cross section above the kinematical endpoint these large off-shell effects do not impact the top quark mass extraction substantially.

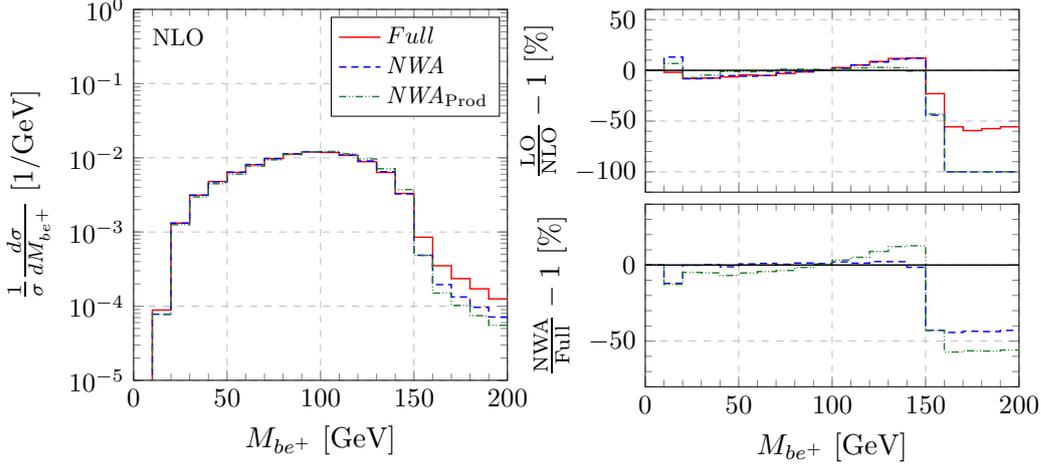


Figure 3: The normalized differential distribution for M_{be^+} for the LHC at $\sqrt{s} = 13$ TeV with $\mu_0 = m_t = 173.2$ GeV.

3. Top quark mass extraction

Let us now discuss the extraction of the top quark mass parameter from the shapes of the above mentioned differential distributions. To illustrate their sensitivity to the m_t parameter we show in Fig. 4 the two observables for five different values of the top quark mass. Thus, fitting the shape of the distribution as a function of the top quark mass to a measured distribution allows one to extract the top quark mass parameter. We redirect the reader to Ref. [28] for a description of the statistical analysis. We employ the full off-shell calculation using a top quark mass of $m_t^{in} = 173.2$ GeV, the scale $\mu_0 = H_T/2$ and the CT14 PDF set in order to generate pseudo-data sets for an assumed integrated luminosity of $\mathcal{L} = 2.5 \text{ fb}^{-1}$ and $\mathcal{L} = 25 \text{ fb}^{-1}$. Then, template distributions for five different top quark masses between 168.2 GeV and 178.2 GeV for different scales and PDF sets are generated for the three different approaches discussed earlier: $Full$, NWA and NWA_{prod} . These templates will be used to fit the shape of the pseudo-data set as a function of m_t . The minimum of the χ^2 distribution of the fit yields the extracted top quark mass m_t^{out} . In order to avoid statistical fluctuations we repeat this procedure 1000 times.

In Tab. 1 the results of the top quark mass extraction from the $\mathcal{B}(m_t^{pole}, \rho_s)$ distribution for $\mathcal{L} = 2.5 \text{ fb}^{-1}$ and $\mathcal{L} = 25 \text{ fb}^{-1}$ are shown. Let us first note that the fit using templates from the full off-shell calculation with the $H_T/2$ scale reproduces exactly the input top quark mass of

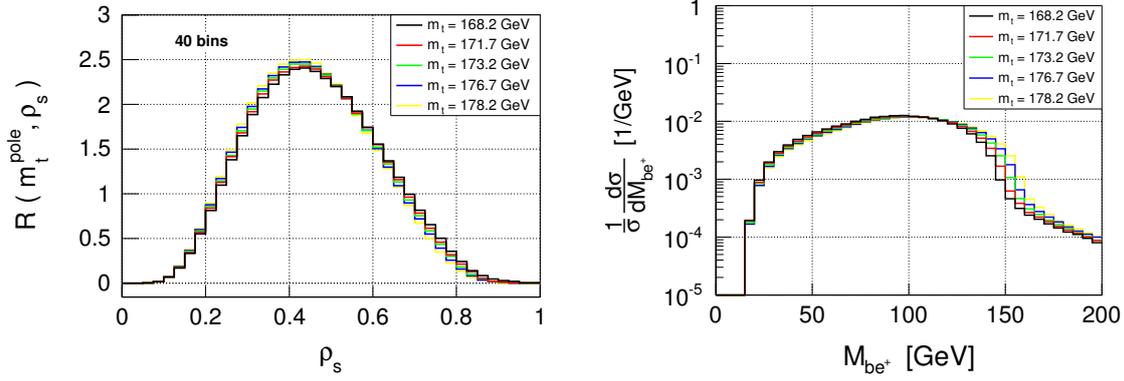


Figure 4: The normalized differential distribution for $\mathcal{R}(m_t^{\text{pole}}, \rho_s)$ and M_{be^+} for $\sqrt{s} = 13$ TeV with $\mu_0 = m_t$ for five different values of the top quark mass.

Theory, NLO QCD CT14 PDF	$m_t^{\text{out}} \pm \delta m_t^{\text{out}}$ [GeV]	Averaged $\chi^2/\text{d.o.f.}$	Probability <i>p-value</i>	$m_t^{\text{in}} - m_t^{\text{out}}$ [GeV]
2.5 fb^{-1}				
<i>Full</i> , $\mu_0 = H_T/2$	173.05 ± 1.31	0.99	0.42 (0.8σ)	+0.15
<i>Full</i> , $\mu_0 = E_T/2$	172.19 ± 1.34	1.05	0.39 (0.9σ)	+1.01
<i>Full</i> , $\mu_0 = m_t$	173.86 ± 1.39	1.42	0.21 (1.2σ)	-0.66
<i>NWA</i> , $\mu_0 = m_t$	175.22 ± 1.15	1.38	0.23 (1.2σ)	-2.02
<i>NWA</i> _{Prod.} , $\mu_0 = m_t$	169.39 ± 1.46	1.12	0.35 (0.9σ)	+3.81
25 fb^{-1}				
<i>Full</i> , $\mu_0 = H_T/2$	173.06 ± 0.44	0.97	0.44 (0.8σ)	+0.14
<i>Full</i> , $\mu_0 = E_T/2$	172.36 ± 0.44	1.38	0.23 (1.2σ)	+0.84
<i>Full</i> , $\mu_0 = m_t$	173.84 ± 0.42	5.12	$1 \cdot 10^{-4}$ (3.9σ)	-0.64
<i>NWA</i> , $\mu_0 = m_t$	175.23 ± 0.37	5.28	$7 \cdot 10^{-5}$ (4.0σ)	-2.03
<i>NWA</i> _{Prod.} , $\mu_0 = m_t$	169.43 ± 0.50	2.61	0.02 (2.3σ)	+3.77

Table 1: Results of the template fits for the $\mathcal{R}(m_t^{\text{pole}}, \rho_s)$ distribution for the LHC at $\sqrt{s} = 13$ TeV. m_t^{out} and δm_t^{out} refer to the mean and the 1σ statistical uncertainty of the 1000 pseudo-data experiments. The averaged χ^2/dof and the *p-value* indicate the quality of the performed fit. In the last column, the mass shift $m_t^{\text{in}} - m_t^{\text{out}}$ with $m_t^{\text{in}} = 173.2$ GeV is presented.

173.2 GeV, which yields a good cross check that the fitting procedure works. Using the full off-shell calculation but different central scales μ_0 for the templates yields a shift in the extracted top quark mass of the order of 1 GeV, as can be seen in the last column of Tab. 1. Furthermore, if we compare the obtained mass shifts for *Full* and *NWA* for the fixed scale $\mu_0 = m_t$ we see, independent of the considered integrated luminosity, a difference of 1.4 GeV, which has to be attributed to off-shell effects and continuum contributions. Addressing scale uncertainties by a simultaneous rescaling of the renormalization and factorization scale yields an uncertainty of 0.6 – 1.2 GeV for

Theory, NLO QCD CT14 PDF	$m_t^{out} \pm \delta m_t^{out}$ [GeV]	Averaged $\chi^2/d.o.f.$	Probability p -value	$m_t^{in} - m_t^{out}$ [GeV]
2.5 fb^{-1}				
<i>Full</i> , $\mu_0 = H_T/2$	173.09 ± 0.48	1.05	0.38 (0.9 σ)	+0.11
<i>Full</i> , $\mu_0 = E_T/2$	173.01 ± 0.50	1.06	0.37 (0.9 σ)	+0.19
<i>Full</i> , $\mu_0 = m_t$	173.07 ± 0.49	1.22	0.18 (1.3 σ)	+0.13
<i>NWA</i> , $\mu_0 = m_t$	173.90 ± 0.50	1.11	0.30 (1.0 σ)	-0.70
<i>NWA</i> _{Prod.} , $\mu_0 = m_t$	172.56 ± 0.54	1.64	0.01 (2.6 σ)	+0.64
25 fb^{-1}				
<i>Full</i> , $\mu_0 = H_T/2$	173.18 ± 0.15	1.02	0.42 (0.8 σ)	+0.02
<i>Full</i> , $\mu_0 = E_T/2$	173.23 ± 0.15	1.03	0.41 (0.8 σ)	-0.03
<i>Full</i> , $\mu_0 = m_t$	173.22 ± 0.16	1.78	0.005 (2.8 σ)	-0.02
<i>NWA</i> , $\mu_0 = m_t$	173.98 ± 0.16	2.56	$5 \cdot 10^{-6}$ (4.6 σ)	-0.78
<i>NWA</i> _{Prod.} , $\mu_0 = m_t$	172.62 ± 0.17	8.23	0 ($\gg 5\sigma$)	+0.58

Table 2: Results of the template fits for the M_{be^+} distribution for the LHC at $\sqrt{s} = 13$ TeV. m_t^{out} and δm_t^{out} refer to the mean and the 1σ statistical uncertainty of the 1000 pseudo-data experiments. The averaged χ^2/dof and the p-value indicate the quality of the performed fit. In the last column, the mass shift $m_t^{in} - m_t^{out}$ with $m_t^{in} = 173.2$ GeV is presented.

the dynamical scales and 2.1 – 2.8 GeV for the fixed scale. The uncertainty related to different PDF sets amounts to 0.4 – 0.7 GeV. In Tab. 2, the corresponding results obtained for the normalized M_{be^+} distribution are shown. Overall, one can say that this observable is more sensitive to the top quark mass parameter than the $\mathcal{R}(m_t^{pole}, \rho_s)$ distribution by looking at the resulting statistical uncertainty on the extracted top quark mass δm_t^{out} , the fit quality as well as the obtained mass shifts. The choice of the central scale μ_0 does not have a strong impact on the extracted top quark mass and only amounts to a 10 MeV uncertainty, as can be seen from the mass shifts of the full off-shell calculation. The impact of off-shell and non-resonant contributions on the extracted top quark mass amounts to a mass shift of around 800 MeV which is perfectly consistent with the results obtained in the recent publication [36]. Uncertainties related to missing higher order corrections are estimated to be of the order of 50 MeV for dynamical scales and 1 GeV for the fixed scale. PDF uncertainties are also very small, namely of the order of 30 MeV.

4. Conclusion

We have studied the impact of off-shell effects on the top quark mass extraction using template distributions. To this end we performed a systematic comparison at fixed-order NLO QCD between the full off-shell $t\bar{t}j$ calculation and the description of the process within the NWA. In this contribution, we focused on two observables to highlight the findings of our more detailed study in Ref. [28], where also additional distributions are discussed. Our findings are that off-shell effects can have an impact on the top quark mass extraction but this question has to be answered on the

case-by-case basis. For example, large off-shell effects as in the case of M_{be^+} do not play an important role because they are only visible in kinematic regions which are less important for the top quark mass extraction. On the other hand, we found off-shell effects of less than 15% in the top mass sensitive region of the $\mathcal{R}(m_t^{pole}, \rho_s)$ distribution. Therefore, these effects have an impact on the extracted value of the top quark mass. In this case, we find that fits based on the narrow width approximation lead to large mass shifts.

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