PROCEEDINGS OF SCIENCE

V_{ud} from Nuclear Beta Decays

J.C. Hardy[*]†**and I.S. Towner**
Cyclotron Institute, Texas A&M University, College Station, TX 77843, U.S.A.
E-mail: hardy@comp.tamu.edu, towner@comp.tamu.edu

At present, the best value for $V_{\rm ud}$, 0.97420(21), comes from superallowed $0^+ \rightarrow 0^+$ nuclear β transitions, of which fourteen have now been measured with high precision. The current status of world data is presented and shown to be robust. The derived $V_{\rm ud}$ value is consistent with the considerably less precise results obtained from β decays of the neutron, the pion and $T = 1/2$ mirror nuclei. Nuclear isospin symmetry-breaking corrections appear to be well under control, having been tested by on-going experiments.

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1. Introduction

Superallowed $0^+ \rightarrow 0^+$ nuclear β decays between T = 1 analog states currently provide the most precise determination of V_{ud}, the up-down element of the Cabibbo-Kobayashi-Maskawa matrix. These decays offer several important advantages: First, being between 0^+ states, they depend uniquely on the vector part of the weak interaction; second, being between analog states, their transition intensity is nearly independent of nuclear structure; and third, there being many such transitions accessible to experiment, agreement among the results from all the individual transitions offers a valuable consistency check.

Nuclear β decays are characterized by an experimentally determined ft value, which depends on three separately measured quantities: the total transition energy Q_{EC}, the half-life $t_{1/2}$ of the parent state, and the branching ratio R for the particular transition of interest. The Q_{EC}-value is required to determine the statistical rate function, f, while the half-life and branching ratio combine to yield the partial half-life, t. To accord with Conservation of the Vector Current (CVC), the ft value should be directly related to the vector coupling constant, $G_{\rm V}$, a fundamental constant which, not being renormalized in the nuclear medium, is the same for all such transitions. In practice, the expression for ft includes several small ($\sim$1%) correction terms, so it is convenient to combine some of these terms with the ft value and define a “corrected" $\mathscr{F}t$ value. Thus, we write [1]

$$\mathscr{F}t \equiv ft(1+\delta_R')(1+\delta_{NS}-\delta_C) = \frac{K}{2G_{\rm V}^2(1+\Delta_{\rm R}^{\rm V})} \tag{1.1}$$

where $K/(\hbar c)^6 = 2\pi^3\hbar \ln 2/(m_e c^2)^5 = 8120.2776(9)\times 10^{-10}\,\mathrm{GeV}^{-4}\mathrm{s}$, δ_C is the isospin-symmetry-breaking correction and $\Delta_{\rm R}^{\rm V}$ is the transition-independent part of the radiative correction. The terms δ_R' and δ_{NS} comprise the transition-dependent part of the radiative correction, the former being a function only of the electron's energy and the Z of the daughter nucleus, while the latter, like δ_C, depends in its evaluation on the details of nuclear structure.

From this equation, it can be seen that each measured transition establishes an individual value for $G_{\rm V}$. If multiple transitions all lead to statistically consistent values for $G_{\rm V}$ then this simultaneously confirms the expectations of CVC and verifies the calculated nuclear-structure-dependent correction terms δ_C and δ_{NS}. In that case, an average value for $G_{\rm V}$ can be determined and V_{ud} obtained from the relation $V_{\rm ud} = G_{\rm V}/G_{\rm F}$, where $G_{\rm F}$ is the well known weak interaction constant for purely leptonic muon decay [2]. If, instead, the $\mathscr{F}t$ values were to show significant inconsistencies, then it would be impossible to determine whether the correction terms were incorrect or, less likely, CVC had been violated. Most importantly, the remaining steps could not be taken: Without consistency, there is no coupling “constant" and there can be no justification for extracting a value for $V_{\rm ud}$.

2. Survey of World Data

Early in 2015, we published a new critical survey of all half-life, decay-energy and branching-ratio measurements related to 20 superallowed $0^+ \rightarrow 0^+$ β decays [1]. Included were 222 individual measurements of comparable precision obtained from 177 published references. We obtained world-average ft values for each of the 18 transitions that had a complete set of data, and then

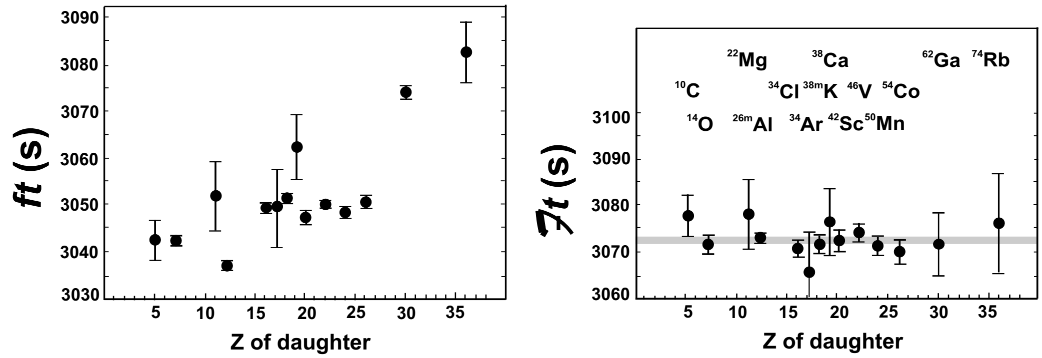

Figure 1: Results from the 2015 survey [1] updated to 2016: uncorrected ft values for the 14 best known superallowed decays on the left; the same results but incorporating the δ_R', δ_C and δ_{NS} correction terms on the right. The grey band in the right panel is the average $\mathscr{F}t$ value and its uncertainty.

applied radiative and isospin-symmetry-breaking corrections to extract corrected $\mathscr{F}t$ values. We have now updated our survey to incorporate all results published since September 2014, the original closing date. The most consequential of the new results are the Q_{EC}-value [3] and branching ratio [4] of the superallowed decay branch from ^{14}O and the half-life of ^{10}C [5]. As it now stands, a total of 9 of the average ft values have a precision of order 0.05% or better, and 5 more have precisions between 0.14% and 0.27%. The uncorrected ft values and the corrected $\mathscr{F}t$ values are shown in Fig. 1.

It is immediately evident from the figure that the $\mathscr{F}t$ values are all consistent with one another from A=10 to A=74. This simultaneously confirms the CVC expectation of a constant value for G_V and demonstrates the absence of any significant scalar current, which would introduce an upward or downward curve into the $\mathscr{F}t$-value locus at low Z [1]. It also goes a long way towards validating the particular set of calculated transition-dependent corrections that were used in the analysis. These calculations of δ_C and δ_{NS} were an updated version of those presented in Ref. [6] and employed the best available shell-model wave functions, which in each case had been based on a wide range of spectroscopic data for nuclei in the same mass region. They were further tuned to agree with measured binding energies, charge radii and coefficients of the isobaric multiplet mass equation for the specific states involved. This means that the origins of these correction terms are completely independent of the superallowed decay data, so consistency in the corrected $\mathscr{F}t$ values gives powerful support to the calculated corrections used in the derivation of those $\mathscr{F}t$ values. We will return later to the question of alternative calculations for these correction terms.

With a mutually consistent set of $\mathscr{F}t$ values, one is then justified in proceeding to determine the value of G_V and, from it, V_{ud}. The result we obtain from our updated survey is

$$|V_{ud}| = 0.97420(21) \qquad \text{[nuclear superallowed]}.$$

This represents an insignificant change of 0.00003 from the value we recommended in 2015 [1], which is further evidence for the robustness of the world data set.

3. Other methods for determining V_{ud}

Neutron β decay is the simplest β decay to involve both the vector and axial-vector weak interactions. It is an attractive option for determining V_{ud} since its analysis does not require the application of corrections for isospin-symmetry-breaking, δ_{C}, or for nuclear-structure-dependent radiative effects, δ_{NS}. However, it has the distinct disadvantage that it requires a difficult correlation measurement in order to separate the vector-current contribution to its decay from the axial-vector one. Not only that, but neutrons are inherently more difficult to handle and contain than nuclei.

Since the Q_{EC} value and the branching ratio for neutron β decay are very well known, the crucial measurements required to determine V_{ud} are its mean-life and a decay correlation – usually selected to be the β asymmetry from the decay of polarized neutrons. World data for both these quantities are not statistically consistent among themselves, the normalized chi-squared (χ^2/N) for the mean-life average being 3.7 and that for the β asymmetry being 4.1. More alarming still is the fact that the mean-life results from two different measurement techniques appear to be systematically different from one another. The average mean-life obtained when the decay products are recorded from a beam of neutrons is 888.1(20)s; while it is 879.6(7)s when neutrons are confined in a "bottle" and the survivors are counted a known time later. It is difficult to know how to deal with such conflicts so we employ two different methods. With the first, we follow exactly the same procedures as we do for the superallowed decays, averaging all world data for each parameter and increasing its uncertainty by the square root of the normalized chi-squared. For the second we simply assign a range to the mean-life, which encompasses both the conflicting sets of results. The results for V_{ud} are

$$|V_{\mathrm{ud}}| = 0.9757(14) \qquad \text{[neutron average]},$$
$$0.9701 \le V_{\mathrm{ud}} \le 0.9767 \qquad \text{[neutron range]}.$$

Neutron β decay is just a special case of decay between $T = 1/2$ mirror nuclei. Like neutron decay, these nuclear mirror decays are mixed vector and axial-vector decays; so, in addition to Q_{EC} values, half-lives and branching ratios, they also require a β-asymmetry measurement. Of course, unlike the neutron, these decays as well require the corrections δ_{C} and δ_{NS} for small nuclear-structure-dependent effects. There are five mirror decays, ^{19}Ne, ^{21}Na, ^{29}P, ^{35}Ar and ^{37}K, for which sufficient data are known. World data were surveyed first in 2008 [7], then updated in 2014 [8]. If we now include a more recent result for ^{37}K [9] we obtain

$$|V_{\mathrm{ud}}| = 0.9730(14) \qquad \text{[mirror nuclei]}.$$

Finally, the rare pion beta decay, $\pi^+ \to \pi^0 e^+ \nu_e$, which has a branching ratio of $\sim 10^{-8}$, is one of the most basic semi-leptonic electroweak processes. It is a pure vector transition between two spin-zero members of an isospin triplet and is therefore analogous to the superallowed $0^+ \to 0^+$ decays. In principle, it can yield a value of V_{ud} unaffected by nuclear-structure uncertainties. In practice, the branching ratio is very small and has proved difficult to measure with sufficient precision. The most recent, and by far the most precise, measurement of the branching ratio is by the PIBETA group [10]. This leads to the result [11]

$$|V_{\mathrm{ud}}| = 0.9749(26) \qquad \text{[pion]}.$$

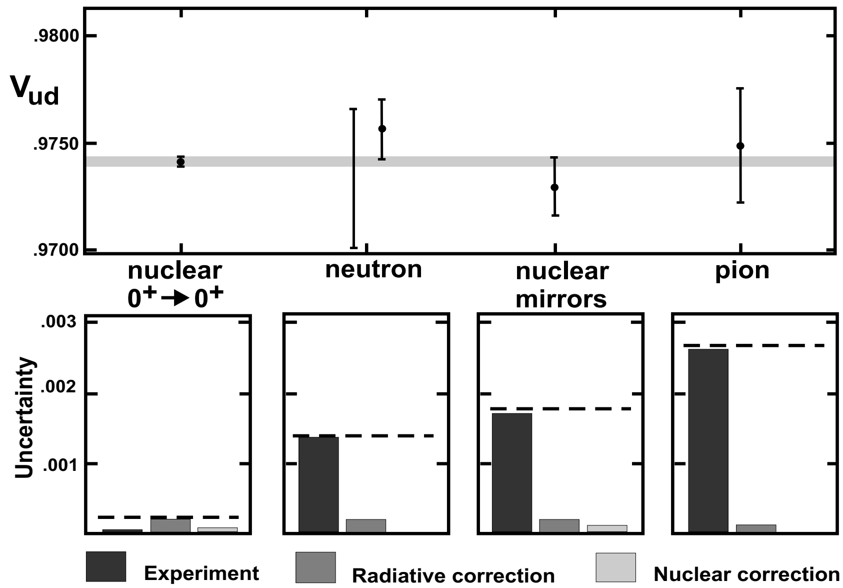

Figure 2: The five values of $|V_{\rm ud}|$ given in the text are shown in the top panel, the grey band being the average value. The four panels at the bottom show the error budgets for the corresponding results with points and error bars at the top.

4. Recommended value for $V_{\rm ud}$

The five results we have quoted for $|V_{\rm ud}|$ are plotted in Fig. 2. Obviously they are consistent with one another but, because the nuclear superallowed value has an uncertainty a factor of 7 to 13 smaller than the other results, it dominates the average. Furthermore, the more precise of the two neutron results can hardly be considered definitive since it ignores a serious systematic uncertainty in the data. Consequently we recommend using the nuclear superallowed result as the best value for $|V_{\rm ud}|$: i.e.

$$|V_{\rm ud}| = 0.97420(21). \tag{4.1}$$

5. Tests of Correction terms and Future Directions

Figure 2 also shows the "uncertainty budgets" for all four methods used to determine $V_{\rm ud}$. Only in the case of the superallowed $0^+ \rightarrow 0^+$ decays is experiment not the principal source of uncertainty. In fact, it is the radiative correction, notably $\Delta_{\rm R}^{\rm V}$, that dominates the uncertainty for our recommended value of $V_{\rm ud}$, with the component that arises from the nuclear-structure dependent corrections, δ_C and δ_{NS}, being about half as large. Experiment is smaller by another factor of two.

To reduce the uncertainty in $\Delta_{\rm R}^{\rm V}$ is a problem purely for theory. However, the calculated nuclear-structure dependent corrections change in magnitude from transition to transition, so they are amenable to experimental tests that can in principle serve to reduce their uncertainty further. We have devised two tests for this purpose, both based on the premise that the CVC hypothesis is valid. The first test considers the corrected $\mathscr{F}t$ values for all measured transitions, which according to CVC should be statistically consistent with one another (*i.e.* with $\chi^2/N{\sim}1$). As part of our survey, we applied this test on the data using each of the various published sets of calculated correction terms. The resultant χ^2/N values spanned a wide range, with only a single set yielding a value near one. In this way, we identified the set [6] we ultimately used in our analysis of the experimental data (see Fig. 1).

The second test involves the measurement of mirror pairs of superallowed transitions. It turns out that the ratio of mirror ft values offers a particularly sensitive test of the correction terms, but it is only recently that the first mirror pair, ^{38}Ca $\rightarrow$ ^{38m}K and ^{38m}K $\rightarrow$ ^{38}Ar, has been precisely measured [12]. This result confirmed the same set of calculated corrections [6] as the first test. Taking account of current capabilities for producing superallowed T_Z = -1 parent nuclei in sufficient quantities for a high-statistics measurement, we conclude that this test can be extended to three more mirror pairs, at A=26, 34 and 42, which can be completed in the near future. We are currently engaged in making these difficult measurements.

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