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Experimental determination of V_{us} from kaon decays

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The status of the experimental determination of V_{us} from $K_{\ell 3}$ and $K_{\mu 2}$ decays is reviewed. Factors currently limiting the precision of the evaluation of V_{us} from kaon decays and prospects for new measurements within the next few years are discussed.

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1. Introduction

At present, the most stringent test of CKM unitarity is obtained from the first-row condition $V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1 + \Delta_{\rm CKM}$. Between 2003 and 2010, a wealth of new measurements of $K_{\ell 3}$ and $K_{\ell 2}$ decays, together with steady theoretical progress, made possible precision tests of the Standard Model based on this relation. In a 2010 review, the FlaviaNet Working Group on Kaon Decays set bounds on $\Delta_{\rm CKM}$ at the level of 0.1% [1]. The 2010 FlaviaNet evaluation was most recently updated in 2014 [2]. Since the 2014 update, experimental progress has been minor, but some issues have come to the fore that illustrate both the potential for future progress and the obstacles to overcome.

The determination of V_{us} from $K_{\ell 3}$ and $K_{\ell 2}$ decays requires values for the hadronic constants $f_+(0)$ and f_K/f_π, respectively. At the moment, the uncertainties on the lattice QCD values for these constants contribute slightly more than those for the experimental data to the overall uncertainty on V_{us}. In recent years, however, the pace of theoretical progress has exceeded that of experiment. Advances in algorithmic sophistication and computing power have lead to a number of new lattice QCD results with total uncertainties at the level of 0.3%. The recently released edition of the biannual review from the Flavor Lattice Averaging Group (FLAG) provides a critical overview and recommended values of the lattice constants entering into the evaluation of V_{us} [3]. Progress on lattice results for the evaluation of V_{us} was also reviewed at this conference [4].

2. Experimental inputs

2.1 Branching ratios and lifetimes

Existing branching ratio (BR) and lifetime measurements allow V_{us} to be evaluated from the rates for the K_{e3} decays of the K_S, K_L, and $K^\pm$ and for the $K_{\mu 3}$ decays of the K_L and K^+; the rate for $K^\pm_{\mu 2}$ provides an independent evaluation of V_{us}/V_{ud}. Most of the available data is in the form of ratios of BRs, and the few absolute BR measurements in the data set have residual dependence on the kaon lifetimes via the experimental acceptance. Therefore, the best values for the leptonic and semileptonic decay rates are obtained from fits to the measured values of the lifetimes and of all of the BRs for the major decay modes, with the sum of the BRs constrained. As a result, new BR measurements of any of the major decay modes are potentially interesting inputs to the analysis, not just the $K_{\ell 3}$ and $K^\pm_{\mu 2}$ modes.

There have been a handful of new measurements entering into the fits since 2010, but none since the 2014 update. The most significant recent development is the 2014 measurement of $\mathrm{BR}(K^\pm \to \pi^\pm\pi^+\pi^-)$ from KLOE-2 [5] which filled a significant gap in the $K^\pm$ data set—there was very little previous constraint on this BR. The inclusion of this measurement significantly reduced the uncertainty on the fit result for $\mathrm{BR}(K^\pm_{\mu 2})$.

Table 1 summarizes the results of the BR and lifetime fits used for the analysis. The input data sets for the K_L and especially the K^+ contain some inconsistent measurements, and the fits have $\chi^2/\mathrm{ndf} = 19.8/12$ ($P = 7.0\%$) and $\chi^2/\mathrm{ndf} = 25.5/11$ ($P = 0.78\%$), respectively. These fits are more selective in the use of older measurements than the fits by the PDG [6], and there are some differences in the handling of correlations and dependence on external parameters. Relative to the PDG fits, the results of the $K^\pm$ fit for the $K_{\ell 3}$ BRs have central values that are 0.3–0.4%

Parameter	Value	S	Correlation coeff. (%)		
K_S					
BR(K_{e3})	$7.05(8) \times 10^{-4}$				
τ_{K_S}	89.58(4) ps				
K_L					
BR(K_{e3})	40.56(9)%	1.3			
BR($K_{\mu3}$)	27.04(10)%	1.5	−28		
τ_{K_L}	51.16(21) ns	1.1	+8	+14	
$K^{\pm}$					
BR($K_{\mu2}$)	63.58(11)%	1.1			
BR(K_{e3})	5.088(27)%	1.2	−68		
BR($K_{\mu3}$)	3.366(30)%	1.9	−52	+38	
$\tau_{K^{\pm}}$	12.384(15) ns	1.2	+5	+1	0

Table 1: Summary of branching ratio and lifetime measurements from the fits to K_S, K_L, and $K^{\pm}$ BR and lifetime measures. Scale factors are calculated using the PDG prescription [6].

higher and slightly smaller uncertainties. The PDG fits for the K_L and K^+ have $\chi^2/\text{ndf} = 37.4/17$ ($P = 0.30\%$) and $\chi^2/\text{ndf} = 53/28$ ($P = 0.26\%$), respectively.

2.2 Form-factor parameters

The 2010 analysis included measurements of the $K_{\ell 3}$ form-factor parameters for K_L decays from KLOE, KTeV, and NA48, and for K^- decays from ISTRA+. Because of the advantages described in [1], the results from fits using the dispersive representation of [7] were used to evaluate the phase-space integrals. In the dispersive representation, the vector and scalar form factors are each specified by a single parameter: Λ_+, which is essentially the slope of the vector form factor at $t = 0$, and $\ln C$, which is related to the value of the scalar form factor at the unphysical Callan-Treiman point, $t = m_K^2 - m_\pi^2$. In 2012, the NA48/2 experiment released preliminary results for the form factors for $K_{\ell 3}^{\pm}$ decays obtained with the quadratic-linear $(\lambda'_+, \lambda''_+, \lambda_0)$ and polar (M_V, M_S) parameterizations [8]. However, it is possible to obtain approximately equivalent values of Λ_+ and $\ln C$ from a fit to the NA48/2 measurements of $(\lambda'_+, \lambda''_+, \lambda_0)$ using the expressions in the appendix of [7] and the observation that $\lambda_0 \approx \lambda'_0 + 3.5\lambda''_0$ [9]. Including the preliminary NA48/2 results, appropriately converted for the current purposes, the dispersive average becomes $\Lambda_+ = (25.75 \pm 0.36) \times 10^{-3}$, $\ln C = 0.1985(70)$, with $\rho = -0.202$ and $P(\chi^2) = 55\%$ (see Fig 1). The central values of the phase-space integrals barely change with this inclusion; the uncertainties are reduced by 20%.

The above situation is unchanged from 2014. In 2016, the OKA experiment, the successor to ISTRA+, updated its preliminary results on the K_{e3} form-factor parameters from polynomial, pole, and dispersive fits, obtained with a sample of 3.2M K^+ decays [10]. The OKA results will be included in the averages for the form-factor parameters in a future update, once the systematic uncertainties have been fully evaluated.

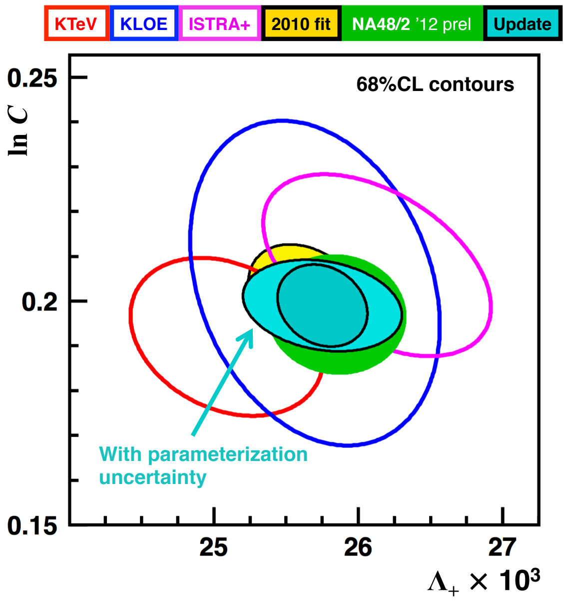

Figure 1: 68% confidence contours for form-factor parameters from dispersive fits, for different experiments (K_{e3}-$K_{\mu3}$ averages) The FlaviaNet 2010 average and the updated average, with the NA48/2 result included, are shown. The confidence contours do not include the contribution to the uncertainties from the dispersive parameterization, except in the case of the outer ellipse for the updated fit.

The dispersive parameterizations are affected by uncertainty in the behavior of the $K\pi$ phase shifts in the high-energy region, which in turn gives rise to uncertainties on constants in the expressions used to fit the experimental data. The contributions of these uncertainties to Λ_+ and $\ln C$ are evaluated by the experiments, and are typically about 0.3×10^{-3} for Λ_+ and 0.004 for $\ln C$ (see, e.g., [7, 11]). Since these contributions are common to all of the measurements, they are removed from the input data before averaging and added back to the uncertainties on the final results. As shown in Fig. 1, until recently, the uncertainties from the parameterization were small on the scale of the overall uncertainties for the individual measurements. However, they are not small compared to the uncertainties for the average values of Λ_+ and $\ln C$, and even the preliminary NA48/2 results are significantly affected. With correlations taken into account, the uncertainties from the parameterization contribute about 0.05% to the uncertainties on the phase-space integrals [11]. As measurements of the form-factor parameters become more precise, it will become increasingly important to reduce the uncertainties arising from the representation of the form factors. This underscores the importance of preserving and presenting the experimental kinematic distributions in a form that will allow refitting with improved parameterizations when they become available.

Alternatively, the t-dependence of the $K_{\ell3}$ form factors may someday soon be taken from lattice QCD calculations. The ETM Collaboration has recently published results from an $N_f = 2+1+1$ calculation of the $K_{\ell3}$ form factors in which synthetic data points representing the t-dependence of the form factors were fit with the dispersive parameterization, giving $\Lambda_+ = (24.22 \pm 1.16) \times 10^{-3}$ and $\ln C = 0.1998(138)$ (with $\rho = +0.376$) [12]. These values are in reasonable agreement with the experimental average; their uncertainties are not too much larger than those for some of the

			Approx contrib to % err			
Mode	$V_{us}\,f_+(0)$	% err	BR	τ	Δ	I
K_{Le3}	0.2163(6)	0.25	0.09	0.20	0.11	0.05
$K_{L\mu3}$	0.2166(6)	0.28	0.15	0.18	0.11	0.06
K_{Se3}	0.2155(13)	0.61	0.60	0.02	0.11	0.05
$K^{\pm}_{e3}$	0.2171(8)	0.36	0.27	0.06	0.22	0.05
$K^{\pm}_{\mu3}$	0.2170(11)	0.51	0.45	0.06	0.22	0.06

Table 2: Values of $V_{us}\,f_+(0)$ from data for different decay modes, with breakdown of uncertainty from different sources: branching ratio measurements (BR), lifetime measurements (τ), long-distance radiative and isospin-breaking corrections (Δ), and phase-space integrals from form-factor parameters (I).

individual measurements contributing to the average.

3. Evaluation of V_{us} and related tests

The evaluations of $V_{us}\,f_+(0)$ for each of the five decay modes (K_{Le3}, $K_{L\mu3}$, K_{Se3}, $K^{\pm}_{e3}$, and $K^{\pm}_{\mu3}$) are presented in Table 2, with a breakdown of the uncertainties from different sources in each case. The most precise values are from the K_L decays, where the dominant uncertainty is from τ_{K_L}. For the other channels, the dominant uncertainties are from the BR measurements. The uncertainties from the phase-space integrals are insignificant, although uncertain knowledge of the form-factor parameters may limit the precision of next-generation BR measurements by entering into the acceptance calculations. The five-channel average is $V_{us}\,f_+(0) = 0.21654(41)$ with $\chi^2/\mathrm{ndf} = 1.54/4$ ($P = 82\%$).

Since there are no new experimental inputs to the analysis since 2014, the figures in Table 2 are unchanged, except for very small changes to the values of $V_{us}\,f_+(0)$ for the $K^{\pm}$ decays. These changes arise from the correction for strong isospin breaking used in this analysis, $\Delta_{SU(2)} = (2.45 \pm 0.19)\%$, which was updated using the 2016 $N_f = 2+1$ FLAG averages for the quark-mass ratios $Q = 22.5(6)(6)$ and $m_s/\hat{m} = 27.43(13)(27)$, with the isospin-limit meson masses $M_K = 494.2(3)$ MeV and $M_\pi = 134.8(3)$ MeV [3] (see [13] for discussion and notation). Previous to the advent of precise lattice results for the light-quark masses, the uncertainty on $\Delta_{SU(2)}$ was a leading contribution to the uncertainty on $V_{us}\,f_+(0)$ from the charged-kaon modes. As seen from Table 2, it still is, and the value of $\Delta_{SU(2)}$ used here should be confirmed and improved upon. There is continuing progress on the estimation of the light-quark masses. For example, Colangelo *et al.* have performed a dispersion-relation analysis of the $\eta \to 3\pi$ Dalitz plot making use of KLOE data for the charged mode, obtaining $Q = 22.0(7)$ [14], while the BMW Collaboration has obtained the result $Q = 23.4(6)$ from a lattice calculation with $N_f = 2+1$ and partially quenched QED [15]. A systematic review of these and other results would be quite useful to refine the value of $\Delta_{SU(2)}$. Averaging the results for $V_{us}\,f_+(0)$ separately for neutral and charged kaons gives $V_{us}\,f_+(0) = 0.2163(5)$ for K^0 and 0.2224(7) for $K^{\pm}$, with $\chi^2/\mathrm{ndf} = 0.75/3$ and a negligible correlation from the use of the same form-factor parameters for the evaluation of the phase-space

Choice of $f_+(0)$		V_{us}	$\Delta_{\rm CKM}$	
$N_f = 2+1$	0.9677(27)	0.2238(8)	$-0.0009(5)$	-1.6σ
$N_f = 2+1+1$	0.9704(32)	0.2231(9)	$-0.0011(6)$	-2.0σ

Table 3: Results for V_{us} and first-row unitarity test from $K_{\ell 3}$ decays.

integrals in either case. Perfect equality of the uncorrected results for $V_{us} f_+(0)$ from charged and neutral modes would then require $\Delta_{SU(2)} = 2.82(38)\%$.

A value for $f_+(0)$ is needed to obtain the value of V_{us}. The FLAG review provides separate recommended values for $N_f = 2+1$ and $N_f = 2+1+1$. For 2016, the FLAG value for $N_f = 2+1$, $f_+(0) = 0.9677(27)$, was updated with the addition of a new result from RBC/UKQCD [16]; the value for $N_f = 2+1+1$, $f_+(0) = 0.9704(32)$, from [17], is unchanged from 2014. After the FLAG 2016 cutoff, the $N_f = 2+1+1$ result from [12], $f_+(0) = 0.9709(46)$, was published; as noted above, values for Λ_+ and $\ln C$ were also obtained in this analysis. In addition, two ongoing studies were reported at Lattice 2016; one of these, the FNAL/MILC update of the $N_f = 2+1+1$ result quoted by FLAG, expects to obtain an overall precision of $\sim$0.2% [18], such that in the near future, the precision on $f_+(0)$ from the lattice will become competitive with the precision of $V_{us} f_+(0)$ from experiment.

The test of CKM unitarity requires a value for V_{ud}. A preliminary update of the survey of experimental data on $0^+ \to 0^+$ β decays from Hardy and Towner [19] was presented at this conference, giving $V_{ud} = 0.97420(21)$ [20]. The update includes a few new measurements, including BR and $Q_{\rm EC}$ measurements for ^{14}O. The world data set on $0^+ \to 0^+$ β decays is very robust at this point and additional measurements have small effects on V_{ud}, the dominant uncertainty on which is from the calculation of the short-distance radiative correction, Δ_R.

The results for V_{us} from $K_{\ell 3}$ decays with $N_f = 2+1$ and $N_f = 2+1+1$ lattice values for $f_+(0)$ and the latest value of V_{ud} are listed in Table 3. Because the $N_f = 2+1$ and $N_f = 2+1+1$ lattice values for $f_+(0)$ obtained since 2012 are larger than the pre-2012 values, which are mainly from $N_f = 2$ simulations, the agreement with the expectation from first-row unitarity at the 0.1% level obtained in [1] is no longer observed: $\Delta_{\rm CKM}$ is different from zero by -1.6σ and -2.0σ when the $N_f = 2+1$ and $N_f = 2+1+1$ results for $f_+(0)$ are used, respectively.

Up to kinematic factors and long-distance electromagnetic corrections, $\delta_{\rm EM}$, the ratio of the inner-bremsstrahlung-inclusive rates for $K^\pm_{\mu 2}$ and $\pi^\pm_{\mu 2}$ decays provides access to the quantity $V_{us}/V_{ud} \times f_{K^\pm}/f_{\pi^\pm}$. The addition to the $K^\pm$ fit of the BR($K^\pm \to \pi^\pm\pi^+\pi^-$) measurement from KLOE-2 in 2014 slightly increased BR($K^\pm_{\mu 2}$) and reduced its uncertainty from 0.3% to 0.2%, leading to the result $V_{us}/V_{ud} \times f_{K^\pm}/f_{\pi^\pm} = 0.27599(37)$. This result is obtained using $\delta_{\rm EM} = -0.0069(17)$, from the chiral-perturbation theory analysis of [21]. Note that the quantity $f_{K^\pm}/f_{\pi^\pm}$ includes the effects of strong isospin breaking: $f_{K^\pm}/f_{\pi^\pm} \equiv f_K/f_\pi \times [1+\delta_{SU(2)}]^{1/2}$. The ETM Collaboration has recently obtained the value $\delta_{\rm EM} + \delta_{SU(2)} = -0.0137(13)$ from an $N_f = 2+1+1$ lattice simulation, to be compared with the value from [21], $\delta_{\rm EM} + \delta_{SU(2)} = -0.0112(21)$. The ETM result is preliminary and the uncertainty from the use of the quenched QED approximation is not included, but it highlights the potential role of the lattice for checking and improving on corrections for electromagnetic effects. The 2016 FLAG average of four complete and published

Choice of $f_{K^\pm}/f_{\pi^\pm}$		V_{us}/V_{ud}	$\Delta_{\rm CKM}$	
$N_f = 2+1$	1.192(5)	0.2315(10)	$-0.00004(59)$	-0.06σ
$N_f = 2+1+1$	1.1933(29)	0.2313(6)	$-0.00015(48)$	-0.3σ

Table 4: Results for V_{us}/V_{ud} and first-row unitarity test from $K^\pm_{\mu 2}$ decays.

$N_f = 2+1$ determinations of $f_{K^\pm}/f_{\pi^\pm}$ is 1.192(5), unchanged from 2014. The $N_f = 2+1+1$ average, $f_{K^\pm}/f_{\pi^\pm} = 1.1933(29)$, is dominated by results from HPQCD [22] and Fermilab/MILC [23] which were obtained using in part the same staggered-quark ensembles generated by MILC.

The results for V_{us}/V_{ud} and the first-row unitarity test from $K^\pm_{\mu 2}$ decays obtained with the $N_f = 2+1$ and $N_f = 2+1+1$ lattice values for $f_{K^\pm}/f_{\pi^\pm}$ and V_{ud} from [20] are presented in Table 4. The agreement with first-row unitarity is better for the case of $K^\pm_{\mu 2}$ decays than for $K_{\ell 3}$ decays.

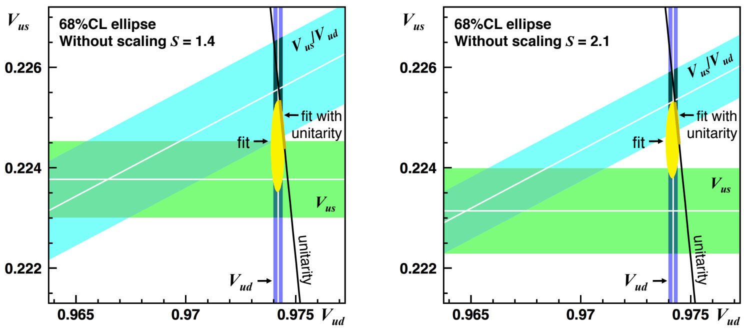

Figure 2: Fits to V_{ud} from $0^+ \to 0^+$ β decays, V_{us} from $K_{\ell 3}$ decays, and V_{us}/V_{ud} from $K^\pm_{\mu 2}$ decays, with lattice values of $f_+(0)$ and $f_{K^\pm}/f_{\pi^\pm}$ from $N_f = 2+1$ (left) and $N_f = 2+1+1$ (right). The yellow ellipses indicate the 68% confidence intervals in the plane of (V_{ud}, V_{us}) for the fits with with no constraints. The yellow line segments indicate the results obtained with the constraint $\Delta_{\rm CKM} = 0$.

The values of V_{ud} from $0^+ \to 0^+$ β decays, V_{us} from $K_{\ell 3}$ decays, and V_{us}/V_{ud} from $K^\pm_{\mu 2}$ decays can be combined in a single fit, as illustrated in Fig. 2. The fit can be performed with or without the unitarity constraint, $\Delta_{\rm CKM} = 0$. The unconstrained fits give $V_{us} = 0.2244(6)$ with $\Delta_{\rm CKM} = -0.0006(5)$ (-1.2σ) for the analysis using $N_f = 2+1$ lattice results, and $V_{us} = 0.2246(5)$ with $\Delta_{\rm CKM} = -0.0005(5)$ (-1.1σ) for the analysis using $N_f = 2+1+1$ results, However, because of the change in the value of V_{us} from $K_{\ell 3}$ following from the increased value of $f_+(0)$ from the recent generation of lattice results, the $K_{\ell 3}$ and $K^\pm_{\mu 2}$ results are not as consistent as previously observed, and the uncertainties quoted for V_{us} do not include scale factors of 1.4 and 2.1 from the χ^2 values of the respective fits.

4. Outlook

All of the post-2010 experimental results combined have a marginal effect on the results of the unitarity test, and good agreement with unitarity is still observed for $K^\pm_{\mu 2}$ decays. The question arises as to whether hidden systematics in the $K_{\ell 3}$ data and/or lattice calculations are becoming important as the stated uncertainties shrink. On the experimental side, the consistency of the fits to $K_{\ell 3}$ rate data is creaky. Yet, the errors on the BRs from these fits are scaled to reflect internal inconsistencies, and after this procedure, the values of $V_{us} f_+(0)$ from K_L, K_S, and $K^\pm$ modes show good agreement. There is also a fair amount of redundancy in the $K_{\ell 3}$ data set, so adding or eliminating single measurements doesn't change the results for $V_{us} f_+(0)$ by much.

Fortunately, a new generation of experiments holds forth the promise of new results that may help to clarify this situation. Since the dominant errors are systematic for most measurements, new, high-statistics data would be principally helpful to the extent that it provides samples for detailed systematic studies. Any of the following experiments could potentially contribute:

- **NA62** The successor experiment to NA48/2, NA62 [24] aims to measure BR$(K^+ \to \pi^+ \nu \bar{\nu})$ to $\sim$10%. When running at low intensity, NA62 collects on the order of 1M K_{e3} decays per week. Relative to NA48/2, NA62 has better π/μ separation and full beam tracking to help with the reconstruction of t for form-factor measurements, though the presence of additional material upstream of the calorimeter may partially offset these advantages. NA48/2 itself has yet to finalize its preliminary measurement of the $K_{\ell 3}$ form-factor parameters, and data acquired by NA62 2007 with the NA48/2 setup could also be analyzed.

- **OKA** A fixed-target experiment at the U-70 synchrotron in Protvino, OKA is a successor to ISTRA+, installed in a new beamline with an RF-separated K^+ beam. OKA can measure K^+ BRs, and as noted above, has an analysis of the K_{e3} form factor in progress. In runs from 2010 to 2013, OKA collected $\sim$17M K_{e3} events; the experiment took data again in late 2016 [10].

- **KLOE-2** Like its predecessor, KLOE-2 can measure the full suite of observables for V_{us}, including BRs, form-factor parameters, and, importantly, lifetimes for the $K^\pm$, K_L, and K_s. KLOE-2 started taking data at the end of 2014 and is expected to collect 5 fb^{-1} total by the end of 2017 [25]. In addition, 2 fb^{-1} of KLOE data have yet to be fully exploited. The original KLOE V_{us} analyses were based on about 0.4 fb^{-1} of data. KLOE measurements related to V_{us} are limited by systematics, but the high statistics KLOE-2 data should allow a precise measurement of the BRs for $K_{S\ell 3}$, particularly K_{Se3}. Because τ_{K_S} is known to 0.04%, K_{Se3} could be the best channel for the determination of V_{us}. The new data might also allow KLOE-2 to improve on KLOE results for the form-factor parameters (including those for $K^\pm$) and on the K_L lifetime.

- **LHCb** 10^{13} K_S/fb^{-1} are produced inside the LHCb acceptance. The recent limit on BR$(K_S \to \mu\mu)$ [26] (and preliminary update [27]) demonstrates LHCb's capability to measure K_S decays to muons. LHCb might be able to measure BR$(K_{S\mu 3})$, although relative to $K_S \to \mu\mu$ there are additional difficulties because of the incomplete reconstruction of the final state, and the presence of $K_{L\mu 3}$ would make lifetime analysis necessary. There is a possible limitation from the trigger, but with a dedicated high-level trigger line, this might be overcome. Like K_{Se3}, $K_{S\mu 3}$ offers high sensitivity because τ_{K_S} is precisely known, though at LHCb this fact may have to be exploited for K_S/K_L separation. On the other hand, BR$(K_{S\mu 3})$ has never been measured and would provide a new channel for the measurement of V_{us}.

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- **TREK** TREK is designed to measure the T-violating transverse muon polarization in $K_{\mu 3}$ decay. A first, low-intensity phase of the experiment took data in 2015 with the goal of measuring $\mathrm{BR}(K_{e2})/\mathrm{BR}(K_{\mu 2})$ to within 0.25%, among others [28]. The experiment makes use of an upgraded KEK-246 setup, moved to J-PARC: K^+s are stopped in an active target surrounded by a toroidal spectrometer, with EM calorimetry and redundant e/μ identification. Since KEK-246 measured $\mathrm{BR}(K_{\mu 3})/\mathrm{BR}(K_{e3})$, TREK should also be able to make BR measurements of interest for V_{us}.

In conclusion, there are good prospects for a new round of experimental results to reduce the uncertainty on $V_{us} f_+(0)$ from 0.18% at present to $\sim$0.12% within 5 years. Perhaps more important than reducing the uncertainty on V_{us} per se, this should allow a critical test of the consistency of the results for $K_{\ell 3}$ and better comparison with the result from $K^{\pm}_{\mu 2}$.

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