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Experimental mini-review on exclusive $|V_{ub}|$ (Belle II)

Matic Lubej[*†]
Jozef Stefan Institute, Ljubljana, Slovenia
E-mail: matic.lubej@ijs.si

We give a short overview of existing branching fraction measurements of charmless semileptonic B meson decays of the form $B \to X_u \ell \nu$, with X_u being π, ρ, ω, η or η'. We focus on results from Belle and BABAR collaborations. We report prospects for measurements and estimates for $|V_{ub}|$ determinations in $B \to \pi \ell \nu$ decays at Belle II. With the full expected Belle II dataset, 50 ab^{-1} of integrated luminosity, the estimated errors on $|V_{ub}|$, including expected improvements in Lattice QCD calculations in the next decade, are 1.7 % and 1.3 % obtained with the tagged and untagged method of companion B meson reconstruction, respectively.

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*Speaker.
†On behalf of the Belle II Collaboration

1. Introduction

Precise measurements of CKM matrix elements are necessary to probe the quark mixing mechanism of the Standard Model (SM) [1, 2] and to search for possible physics beyond the SM. Semileptonic decays of B mesons involving low-mass charged leptons e or μ are expected to be free of non-SM contributions and therefore play a crucial role in precise determinations of CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$. Eq. 1.1 and 1.2 represent the differential branching fractions for B meson decays to pseudoscalar* and vector states, respectively

$$\frac{\mathrm{d}\mathscr{B}(B \to P\ell\nu)}{\mathrm{d}q^2} = |V_{ub}|^2 \frac{G_F^2 \tau_B}{24\pi^3} p_P^3 |f_+^P(\mathrm{q}^2)|^2, \tag{1.1}$$

$$\frac{\mathrm{d}\mathscr{B}(B \to V\ell\nu)}{\mathrm{d}q^2} = |V_{ub}|^2 \frac{G_F^2 p_V q^2 \tau_B}{96\pi^3 m_B^2} \left[|H_0(\mathrm{q}^2)|^2 + |H_+(\mathrm{q}^2)|^2 + |H_-(\mathrm{q}^2)|^2\right], \tag{1.2}$$

where q^2 is the squared value of the momentum transferred to the lepton pair, f_+^P is one of the pseudoscalar form-factors, H_i are the vector form-factors in helicity basis, m_B, τ_B and p the mass, lifetime and momentum of the B meson, respectively, and G_F the Fermi constant. From Eq. 1.1 and 1.2 it can be seen that in order to determine the value of the $|V_{ub}|$ element, one needs a good measurement of the branching fraction and a good understanding of the form-factors that come into play.

Of all the CKM matrix elements, $|V_{ub}|$ is the least precisely known due to the limited theoretical understanding of the form-factors and challenges with experimental measurements, so additional studies are needed in order to constrain the apex of the Unitarity Triangle even further. With Belle II we are entering an era of precision measurements, where we will be able to determine the value of $|V_{ub}|$ with precision at the percent level [3].

The most commonly studied decay mode to determine the magnitude of the CKM matrix element $|V_{ub}|$ is $B \to \pi\ell\nu$, because it offers a clean experimental measurement of the branching fraction and a precise theoretical calculation of the $B \to \pi$ form-factors. One of the problems with existing determinations of $|V_{ub}|$, the so-called V_{ub} puzzle, is a persistent discrepancy between $|V_{ub}|$ measurements based on inclusive and exclusive charmless B meson decays. In inclusive measurements we focus on all decays of the form $B \to X_u\ell\nu$ at the same time and try to reconstruct them inclusively, and in exclusive measurements we focus on specific decays, as in this case. Furthermore, the $B \to \pi\ell\nu$ decay accounts for only about 7 % of all charmless semileptonic B meson decays [4], so this motivates us to study additional modes.

2. Reconstruction methods at B factories

Branching fraction measurements of leptonic and semileptonic B mesons are possible using several different experimental techniques that differ only in the way how the companion B meson is reconstructed. In this review we will only focus on the untagged (inclusive tag) method or the tagged method with hadronic tag decays.

*This form of the differential branching fraction has been simplified for low mass charged leptons e and μ.

In untagged analyses [5] we first reconstruct our signal B meson, with the exception of the escaped neutrino. Since the detector hermetically covers a relatively large portion of the full solid angle, we can inclusively determine the 4-momentum of the companion B meson by adding up the 4-momenta of all the remaining charged tracks and clusters in the event, as shown in Eq. 2.1.

$$\mathrm{p}_{B_{comp}} = \sum_{i\,=\,\text{tracks and clusters}}^{\text{rest of event}} (E_i, \mathbf{p}_i)\,. \tag{2.1}$$

Due to the well known initial $\Upsilon(4S)$ state, we can determine the missing 4-momentum as

$$\mathrm{p}_{miss} = \mathrm{p}_{\Upsilon(4S)} - \mathrm{p}_{B_{sig}} - \mathrm{p}_{B_{comp}}, \tag{2.2}$$

which is equal to the 4-momentum of the missing neutrino, if neutrino is the only missing particle in the event. The untagged method is more efficient, with a reconstruction efficiency $\mathcal{O}(10\,\%)$, so it is the recommended approach with smaller data samples. The downside is the lower q^2 resolution, since 4-momentum of the companion B meson is determined in an inclusive way.

In tagged analyses with the hadronic tag reconstruction [6] we first fully reconstruct the companion B meson in one out of many hadronic decay modes, as opposed to not reconstructing it at all in the case of untagged analyses. After having a good companion B meson candidate, we require that the rest of event is consistent with the signature of the signal decay. The missing 4-momentum can be imposed in the same way as in Eq. 2.2, taking into account the different method for $\mathrm{p}_{B_{comp}}$ calculation. Knowing exactly how the companion B meson was reconstructed, this enables us to calculate q^2 with better precision. Unfortunately the efficiency in this case is $\mathcal{O}(0.1\ \%)$, so it is recommended to use this method with abundant data samples.

In the case of $B \to \pi\ell\nu$ decay modes, the difference in q^2 resolution can be seen by comparing the mean width of the q^2 distributions, which is $\sigma_{q^2}^{tagged} \approx 0.25\ \mathrm{GeV}^2/\mathrm{c}^2$ in the tagged analysis [6] and about a factor of 2 larger in the case of the untagged analysis [5].

3. Experimental overview

In this section we focus on exclusive branching fraction measurement of semileptonic B meson decays with specific charmless hadronic final states $\pi,\ \rho,\ \omega,\ \eta$ or η', and determinations of the $|V_{ub}|$ CKM matrix element. We will not cover all existing measurements, only the most prominent and precise ones which are, in most cases, also the most recent ones.

3.1 $B \to \pi\ell\nu$

In the case of the $B \to \pi\ell\nu$ decay mode we mention three untagged measurements performed by Belle and BABAR , and one tagged measurement performed by Belle. In the case of BABAR there is an older measurement which uses a smaller dataset of about 349 fb^{-1}, taken at the energy of the $\Upsilon(4S)$ resonance [7], and a later one, using the full dataset of 416.1 fb^{-1} [8]. In both cases the untagged method was pursued, which can be seen from the large values of extracted signal yield of about $N_{sig} = 10.5\times10^3 \pm 400$ and $12.5\times10^3 \pm 400$. The same can be said for the untagged measurement at Belle with a dataset of about 605 fb^{-1} and signal yield of $N_{sig} = 21.5\times10^3 \pm 500$ [5]. In case of the tagged measurement at Belle the full dataset of 711 fb^{-1} was used with a signal

Decay mode	$\mathscr{B}$ $[\times 10^{-4}]$	Method	Dataset	Experiment
$B^0 \to \pi^- \ell^+ \nu$	$1.45 \pm 0.04_{\text{stat}} \pm 0.06_{\text{sys}}$	Untagged	416.1 fb^{-1}	BABAR [8]
$B^0 \to \pi^- \ell^+ \nu$	$1.49 \pm 0.04_{\text{stat}} \pm 0.07_{\text{sys}}$	Untagged	605 fb^{-1}	Belle [5]
$B^0 \to \pi^- \ell^+ \nu$	$1.49 \pm 0.08_{\text{stat}} \pm 0.07_{\text{sys}}$	Tagged	711 fb^{-1}	Belle [6]

Table 1: Results of branching fraction measurements for $B \to \pi\ell\nu$ decay modes. All results are presented for the shown decay mode or are combined using isospin symmetry relations.

yield of $N_{sig} = 500 \pm 30$ [6]. Table 1 shows the results of total branching fraction measurements for all mentioned cases, except the older BABAR measurement. The positive effects of a larger efficiency can be seen in the smaller statistical error of the results obtained with the untagged method of reconstruction compared to the tagged one.

Even though the results are relatively old, there is still activity in the field from the theoretical point of view. Calculations of form-factors are getting more precise with the use of model independent parametrizations. A new fit to extract the $|V_{ub}|$ parameter was performed over an average of the existing $B \to \pi\ell\nu$ data [9] from Belle and BABAR , which uses an averaged value of lattice QCD (LQCD) form-factor parameters [10] calculated from [11] and [12], and the most recent result from sum rules calculations on the light-cone (LCSR) [13]. Since LQCD is most precise in the higher q^2 region and LCSR in the lower one, it is most optimal to include both results. The most recent fit to extract $|V_{ub}|$ from the $B \to \pi\ell\nu$ mode is shown in Figure 1 with the result in Eq. 3.1.

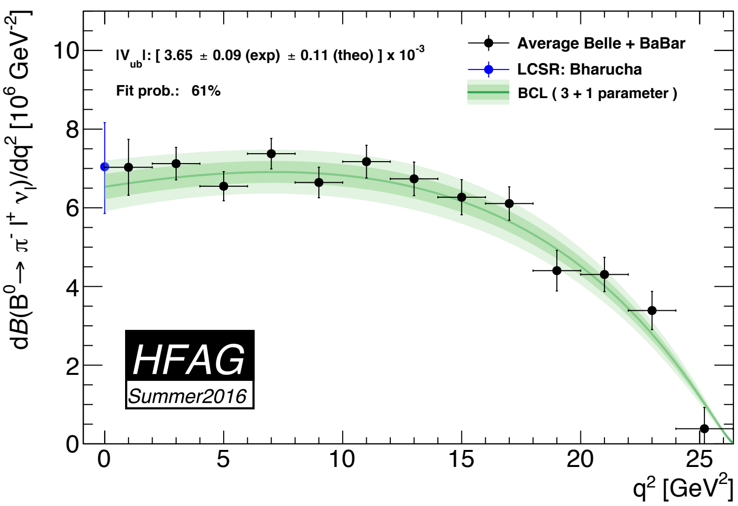

Figure 1: Extraction of $|V_{ub}|$ from the $B \to \pi\ell\nu$ mode with a simultaneous fit to averaged Belle and BABAR data, LCSR point and LQCD form-factor parameters, using a model independent parametrization of the form-factors.

$$|V_{ub}|^{\pi\ell\nu} = (3.65 \pm 0.09_{\text{exp}} \pm 0.11_{\text{theo}}) \times 10^{-3}. \tag{3.1}$$

Decay mode	$\mathcal{B}$ [$\times 10^{-4}$]	Method	Dataset	Experiment
$B^0 \to \rho^- \ell^+ \nu$	$1.75 \pm 0.15_{\text{stat}} \pm 0.27_{\text{sys}}$	Untagged	349 fb^{-1}	BABAR [7]
$B^0 \to \rho^- \ell^+ \nu$	$3.34 \pm 0.16_{\text{stat}} \pm 0.17_{\text{sys}}$	Tagged	711 fb^{-1}	Belle [6]
$B^+ \to \omega \ell^+ \nu$	$1.19 \pm 0.16_{\text{stat}} \pm 0.09_{\text{sys}}$	Untagged	416.1 fb^{-1}	BABAR [8]
$^\dagger B^+ \to \omega \ell^+ \nu$	$1.21 \pm 0.14_{\text{stat}} \pm 0.08_{\text{sys}}$	Untagged	426 fb^{-1}	BABAR [14]
$B^+ \to \omega \ell^+ \nu$	$1.07 \pm 0.16_{\text{stat}} \pm 0.07_{\text{sys}}$	Tagged	711 fb^{-1}	Belle [6]
$B^+ \to \eta \ell^+ \nu$	$0.38 \pm 0.05_{\text{stat}} \pm 0.05_{\text{sys}}$	Untagged	416.1 fb^{-1}	BABAR [8]
$B^+ \to \eta \ell^+ \nu$	$0.42 \pm 0.11_{\text{stat}} \pm 0.03_{\text{sys}}$	Tagged	711 fb^{-1}	Belle [15]
$B^+ \to \eta' \ell^+ \nu$	$0.24 \pm 0.08_{\text{stat}} \pm 0.03_{\text{sys}}$	Untagged	416.1 fb^{-1}	BABAR [8]
$B^+ \to \eta' \ell^+ \nu$	$0.36 \pm 0.27_{\text{stat}} \pm 0.03_{\text{sys}}$	Tagged	711 fb^{-1}	Belle [15]

Table 2: Results of branching fraction measurements for $B \to X_u \ell \nu$ decay modes, with the exception of $B \to \pi \ell \nu$. All results are presented for the shown decay mode or are combined using isospin symmetry relations.

The total precision of the discussed parameter, obtained with the fit above, is about 4 %. With the most recent LQCD calculations the theoretical errors achieve the same region of precision as the experimental errors. Such balance is optimal and one hopes that with increasing statistics and decreasing experimental errors the decrease of theoretical errors will also follow due to improved calculations in the future.

3.2 Other $B \to X_u \ell \nu$

Decay modes with charmless meson states heavier than the charged pion were studied to a lesser extent. The reasons for this are the experimental challenges in reconstruction and incomplete understanding of the form-factors for such complex states. In case of measurements of the ρ meson the branching fractions are even more impacted by the irreducible $B \to X_u \ell \nu$ decays due to the meson's relatively wide decay width. The decay mode with the ω meson is another vector meson decay mode which was measured at B factories. The $B \to X_u \ell \nu$ background can be more suppressed in this case, since the ω width is about 15 times smaller than that of the ρ [4]. However, this decay mode brings different experimental challenges due to generally higher backgrounds and more complex form-factor dependencies. The last modes that we mention are the $B \to \eta \ell \nu$ and $B \to \eta' \ell \nu$. Precise measurements of these modes are important because they represent one of the largest contributions to the systematic uncertainty in the inclusive signal modelling. Table 2 shows the results of branching fraction measurements mentioned above, with the corresponding studied data samples and articles.

Unlike in the case of $B \to \pi \ell \nu$, LQCD calculations don't exist for these decay modes. The only theoretical handle are the LCSR calculations which give reliable form-factor information only at the low q^2 region. Due to this reason the theoretical errors on $|V_{ub}|$ are about 2 – 3 times larger than the experimental ones. Official determinations for $|V_{ub}|$ in other $B \to X_u \ell \nu$ decays exist only

† The principal difference between this analysis and [8] is that in this one the combinatorial ω background is taken from the side-band of the data rather than from the MC simulation.

Decay mode	$\lvert V_{ub}\rvert$ $[\times 10^{-3}]$	Dataset
$B \to \rho \ell \nu$	$3.36 \pm 0.17_{\text{exp}} \pm 0.34_{\text{th}}$	Belle, $q^2 < 8$ GeV^2
$B \to \rho \ell \nu$	$3.25 \pm 0.14_{\text{exp}} \pm 0.34_{\text{th}}$	Belle, $q^2 < 12$ GeV^2
$B \to \rho \ell \nu$	$2.52 \pm 0.42_{\text{exp}} \pm 0.56_{\text{th}}$	BABAR , $q^2 < 8$ GeV^2
$B \to \omega \ell \nu$	$2.49 \pm 0.34_{\text{exp}} \pm 0.32_{\text{th}}$	Belle, $q^2 < 7$ GeV^2
$B \to \omega \ell \nu$	$3.25 \pm 0.36_{\text{exp}} \pm 0.53_{\text{th}}$	BABAR , $q^2 < 8$ GeV^2
$B \to \omega \ell \nu$	$3.25 \pm 0.29_{\text{exp}} \pm 0.46_{\text{th}}$	BABAR , $q^2 < 12$ GeV^2

Table 3: Predictions for $|V_{ub}|$ from $B \to \rho \ell \nu$ and $B \to \omega \ell \nu$, calculated using form-factor calculations from LCSR and several different Belle and BABAR datasets [16].

in the case of $B \to \rho \ell \nu$ and $B \to \omega \ell \nu$ [16], with results cited in Table 3. There is no official determination for $|V_{ub}|$ from the η and η' channels. Although there are some LCSR form-factor calculations [17], the $|V_{ub}|$ is not determined since they are probably still not reliable enough to be used.

4. Belle II prospects

The Belle II collaboration is planning to start operating the Belle II detector in 2018. Due to a new beam scheme, which will increase the luminosity for a factor of 40 compared to that of Belle, we hope to acquire around 50 ab^{-1} of data, which is 50 times more than in the case of its predecessor. In addition to more statistics, the detector itself is being upgraded, with improvements in almost all aspects, such as tracking and particle identification, such as K/π separation. Improvements were also made on the software side, with smarter and more precise algorithms. In order to study the impact of Belle II in the future, we are able to make some predictions using Monte Carlo (MC) simulations. In this section we will cover the Belle II prospects for measurements of charmless semileptonic B meson decays. The MC data sample used in these studies was a mixture of generic $B\overline{B}$ events and continuum $q\overline{q}$ events, where $q = u,\ d,\ s,\ c$, which were produced in a Belle II Monte Carlo campaign in 2015.

4.1 $B \to \pi \ell \nu$

In this subsection we will present the results of a MC study for $B \to \pi \ell \nu$ prospects at Belle II.

In the case of the tagged MC study, the most relevant improvement, besides the detector upgrade, is the better tagging algorithm with significantly higher tagging reconstruction efficiency. From the MC study it was concluded that the overall reconstruction efficiency for $B \to \pi \ell \nu$ is about 0.55 % [3], which is significantly higher than the overall reconstruction efficiency of 0.3 % of the tagged measurement reported by Belle [6]. Figures 2 and 3 show the distributions of the missing mass squared, M^2_{miss}, and the extra energy in the calorimeter, E_{extra}, obtained with the MC study for the case of the tagged analysis at Belle II.

In the case of the untagged MC study, improvements were made in the treatment process of the tracks and clusters not used in signal side reconstruction. As was mentioned in Section 2, these tracks and clusters are assigned to the rest of event and collectively used as the untagged companion

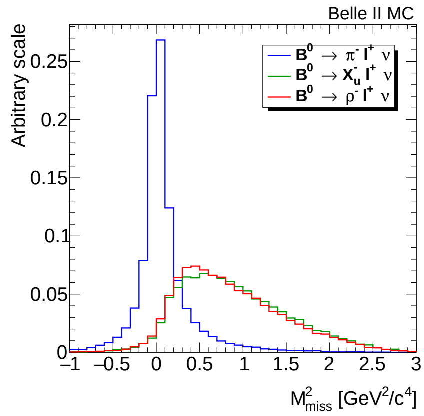

Figure 2: The $M^2_{miss} = p^2_{miss}$ distribution of tagged $B^0 \to \pi^- \ell^+ \nu_\ell$ candidates. The signal is expected to be located in a narrow peak near zero, while background from other $b \to u$ transitions populates a wider region towards higher missing mass, due to missing particles.

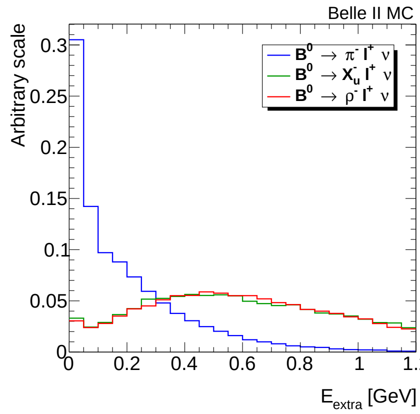

Figure 3: The E_{extra} distribution of tagged $B^0 \to \pi^- \ell^+ \nu_\ell$ candidates. In correctly reconstructed signal events there is little energy deposited in the calorimeter that can not be associated neither to the decay products of the signal nor to the companion B meson.

B meson. In MC studies it was discovered that the rest of event consists of a considerable amount of tracks and clusters which should not be taken into account. These tracks and clusters, dubbed as extra tracks and clusters, are for example produced in secondary interactions of primary particles with detector material, or are fake candidates stemming from imperfect reconstruction. Either way, such cases should be discarded from our selection by cleaning up the rest of event. Taking all improvements into account, the overall reconstruction efficiency of for this channel is around 20 % [3], as opposed to 11 % reported in the untagged measurement of $B \to \pi \ell \nu$ decays performed by Belle [5]. Figures 4 (left) and 5 (left) show the improvement effect of the rest of event clean-up in Belle II MC on the ΔE and M_{BC} distributions, respectively, while Figures 4 (right) and 5 (right) show the final sample composition after all the selection criteria were applied.

The studies were performed on MC samples of about 500 fb^{-1}, which is only half the size of the full Belle dataset. Fortunately, it is possible to extrapolate the errors to larger values of integrated luminosity, where one should keep in mind that the total error also has contributions which do not scale with integrated luminosity. Eq. 4.1 shows the general formula for error scaling with separate scalable and non-scalable contributions to the total error,

$$\sigma_{\text{tot}}(\mathscr{L}) = \sqrt{(\sigma^2_{\text{stat}}(\mathscr{L}_0) + \sigma^2_{\text{sysred}}(\mathscr{L}_0)) \times \frac{\mathscr{L}_0}{\mathscr{L}} + \sigma^2_{\text{sysirred}}(\mathscr{L}_0)}, \tag{4.1}$$

where $\mathscr{L}$ is the arbitrary integrated luminosity which we are interested in, $\mathscr{L}_0$ is the integrated luminosity at which the initial studies/measurements were made, σ_{stat} is the statistical error, σ_{sysred} the reducible part and σ_{sysirred} the irreducible part of the systematic error. Based on the tagged and untagged Belle analyses at 711 fb^{-1} [6] and 605 fb^{-1} [5], it is possible to estimate the precision limit of the irreducible systematic error to 2.0 % and 1.6 % [3], respectively.

To study the total precision on $|V_{ub}|$ with Belle II, the $|V_{ub}|$ parameter was extracted from partial branching fraction distributions for the tagged and untagged study. The LQCD input used

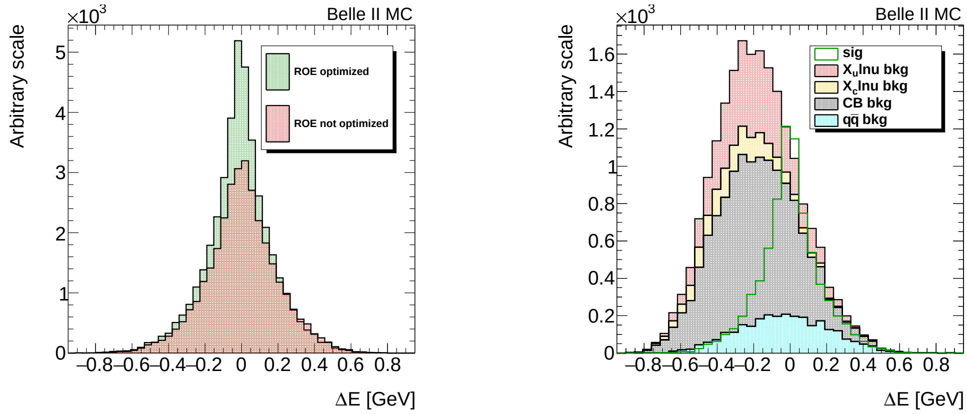

Figure 4: The ΔE distribution of signal $B \to \pi \ell \nu$ candidates before and after rest of event (ROE) clean-up (left). ΔE is a variable which compares the B meson energy to the CMS energy of the initial beam. Signal candidates peak at zero, while background (except continuum) candidates are shifted to the negative values due to an incomplete reconstruction (right).

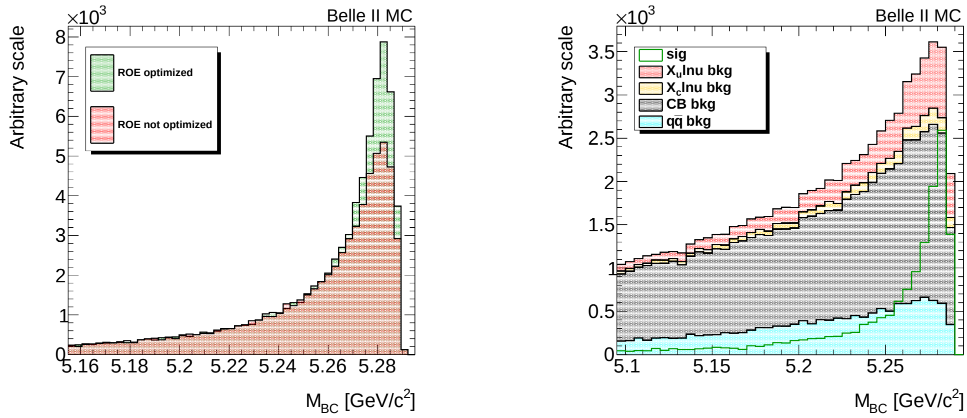

Figure 5: The M_{BC} distribution of signal $B \to \pi \ell \nu$ candidates before and after rest of event (ROE) clean-up (left). M_{BC} is a variable which constrains the B meson to the CMS energy of the initial beam, while obtaining the reconstructed momentum of the particle. Signal candidates peak at nominal B meson mass with a narrow width, while the peak-like distribution of background candidates is not as prominent (right).

in the extraction fit was the average of [11] and [12], performed by [10]. Optimally, LQCD input will become more precise as the Belle II detector acquires data, so it is possible to take LQCD forecasts into account [3, 18]. Figure 6 shows an example of $|V_{ub}|$ parameter extraction using a simultaneous fit to data and theory input at $\mathscr{L} = 5$ ab^{-1}, while Figure 7 shows the precision of $|V_{ub}|$ for multiple values of integrated luminosity from 5 to 50 ab^{-1} for current LQCD and LQCD forecast in the next decade [3]. With the full Belle II dataset and the LQCD forecasts in 10 years we estimate the precision on $|V_{ub}|$ to be 1.7 % and 1.3 % in the case of the tagged and untagged analysis, respectively.

4.2 Other charmless semileptonic B meson decays at Belle II

Unfortunately, there were no extensive studies made for decays other than $B \to \pi \ell \nu$. However,

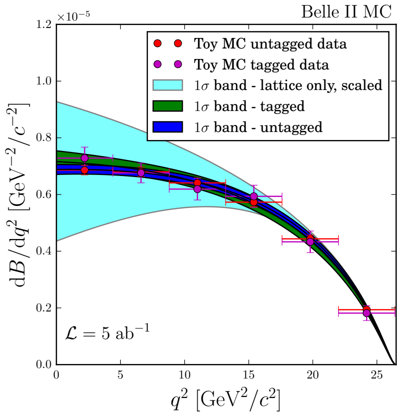

Figure 6: An example of a Belle II simultaneous fit to MC data and LQCD input at 5 ab^{-1} including LQCD forecasts in 5 years for the tagged and untagged analysis.

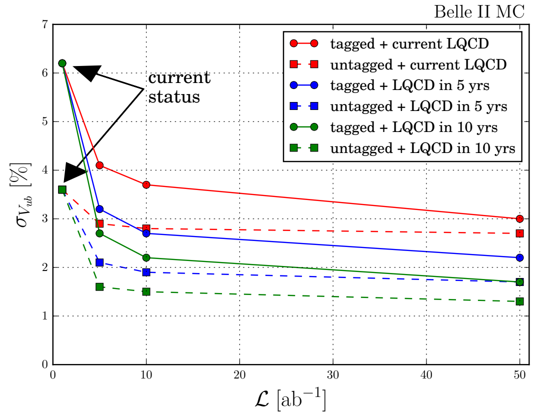

Figure 7: $|V_{ub}|$ precision estimates for the tagged and untagged reconstruction method at 5, 10 and 50 ab^{-1} of integrated luminosity for current LQCD and LQCD forecasts in 5 and 10 years.

it is possible to assume sample sizes which will be acquired with the Belle II detector, based on the tagged analysis by Belle [6]. Taking efficiency improvements into account, we estimate to acquire the following number of candidates with the full Belle II dataset of 50 ab^{-1}:

- $N_{\rho^0} = 8 \times 10^4$, $\delta_{stat} = 0.5$ % - (Belle: $N_{\rho^0} = 621.1 \pm 35.0$)
- $N_{\rho^+} = 4.4 \times 10^4$, $\delta_{stat} = 0.7$ % - (Belle: $N_{\rho^+} = 343.3 \pm 28.3$)
- $N_{\omega(3\pi)} = 1.25 \times 10^4$, $\delta_{stat} = 1.3$ % - (Belle: $N_{\omega(3\pi)} = 96.7 \pm 14.5$)

With such sample sizes it will be possible to perform much more detailed analyses, such as a full helicity angle analysis or a check for right-handed currents in these channels. Further investigations in the future and possible new LQCD calculations for such modes will contribute to our better understanding of the $b \to u$ spectrum.

5. Conclusions

In this paper a short review of existing measurements of charmless semileptonic B meson decays was presented, with a focus on measurements at B factories. While several modes with charmless hadronic final states such as π, ρ, $\omega, \eta^{(\prime)}$ were studied, the most precise and reliable one is the $B \to \pi \ell \nu$ decay mode, with current levels of $|V_{ub}|$ precision at around 4 %.

With the full planned Belle II data sample made detailed studies on MC samples and estimate to measure the $|V_{ub}|$ CKM matrix parameter to at least a 1.7 % and 1.3 % precision in the case of a tagged and untagged reconstruction method of the companion B meson, respectively.

Acknowledgments

I would like to congratulate the organizers for another successful CKM conference with very interesting contributions and excellent hospitality.

Additionally, I would also like to thank the Belle II Speakers Committee for giving me the opportunity to represent the Belle II collaboration at such a prestige conference.

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