Theory update on the inlcusive determination of $|V_{cb}|$

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In this talk I update the precision determination of the CKM element $|V_{cb}|$ from the inclusive decay $B \to X_c \ell \nu_\ell$.

CKM, 2016

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1. Motivation

One of the primary goals of the study of B meson decays and mixing is to construct the unitarity triangle (UT), which is defined by the relation $V_{cd}V_{cb}^* + V_{ud}V_{ub}^* + V_{td}V_{tb}^* = 0$. The Cabbibo-Kobayashi-Masakawa (CKM) elements V_{ij} ($i = u, c, t$ and $j = d, s, b$) are parametrized in terms of four independent parameters: η, ρ, λ and A. In order to construct the UT, various measurements which are sensitive to the these CKM elements are projected into the (ρ, η) plane. The precise measurements of V_{ub} and V_{cb} are important since they play an important role in finding out the regions in the (ρ, η) plane where the apex of the triangle should lie. In this regard, the loop induced and CP violating B decays are also important. For details, see [1].

The tree level semileptonic decays $b \to c\ell\nu_\ell$ ($\ell = e, \mu$) are crucial for the determination of V_{cb}. It can be extracted from both exclusive decays, like $B \to D^{(*)}\ell\nu$, and inclusive decays, like $B \to X_c\ell\nu_\ell$. These decays are expected to be free from any new physics (NP) effects, hence provide a clean environment for the measurement of the V_{cb}. The inclusive channels are relatively clean, and the decay rates have a solid description via operator product expansion (OPE) or heavy quark expansion (HQE) [2]. In these decays, the non perturbative unknowns can be extracted using the final state lepton and hadron energy distribution [3, 4]. These are also useful to extract the b-quark masses, and for a consistency check of the OPE/HQE and other effective theory approaches. As per the measurement is concerned, it has small statistical and systematic errors, and highly sensitive to the theoretical uncertainties, for details, see [5]. Therefore, precise predictions in the SM including reliable uncertainties are possible.

The exclusive semileptonic decays have similar solid descriptions in terms of heavy quark effective theory (HQET) [6]. On contrary to the inclusive decays, the non-perturbative unknowns in the exclusive decays can not be extracted experimentally. One needs to calculate them and that is where the major challenges lie.

2. Framework

The decay rate distribution for the decay $B \to X_c\ell\nu_\ell$ is given by

$$\frac{d\Gamma}{dq^2 dE_e dE_\nu} = 2G_F^2 V_{cb}^2 W_{\mu\nu} L^{\mu\nu}, \tag{2.1}$$

where $L^{\mu\nu}$ and $W_{\mu\nu}$ are the leptonic and hadronic tensors respectively, and they are defined as

$$L^{\mu\nu} = 2\left(p_e^\mu p_{\bar\nu}^\nu + p_e^\nu p_{\bar\nu}^\mu - g^{\mu\nu} p_e \cdot p_{\bar\nu} - i\varepsilon^{\eta\nu\lambda\mu}(p_e)_\eta (p_{\bar\nu})_\lambda\right), \tag{2.2}$$

$$W^{\mu\nu} = \frac{1}{2m_B}\sum_X (2\pi)^3 \delta^4(p_B - q - p_X) \times \langle B(p_B)|J_L^{\dagger\mu}|X_c(p_X)\rangle\langle X_c(p_X)|J_L^\nu|B(p_B)\rangle, \tag{2.3}$$

with $J_L^\mu = \bar{c}\gamma^\mu P_L b$, for details see the reviews [5, 7, 8] and the references therein.

Here, the major theoretical challenges are the calculation of the tensor $W_{\mu\nu}$ which represents the hadronic contribution to the decay width. Using the optical theorem, the hadronic tensor can be calculated from the imaginary part of the forward scattering amplitude,

$$W_{\mu\nu} \propto Im(T_{\mu\nu}), \tag{2.4}$$

where $T_{\mu\nu}$ is defined as the forward matrix element of the time ordered product of the two currents,

$$T^{\mu\nu} = -i \int d^4x e^{-iq.x} \frac{\langle B|T[J_L^{\mu\dagger}(x)J_L^{\nu}(0)]|B\rangle}{2m_B}. \tag{2.5}$$

If the energy released in the decays of b-quark mediated by weak interactions is large, then such decays will take place on a time scale which is much shorter than the time it takes the quarks in the final state to form physical hadronic states. Hence, the inclusive decay rates may be modeled simply by the decay of free b quark. Also, since the energy released in such decays are much larger than the hadronic scale, they are largely insensitive to the details of the initial hadronic structure. This intuitive picture is formalized by OPE. In the limit $M_W >> m_b >> \Lambda_{QCD}$, we can organize an expansion in Λ_{QCD}/m_b, with the leading term corresponding to the free quark decay. Therefore, the right hand side of eq. 2.5 can be written as an infinite sum of local operators (O_i) of increasing dimension
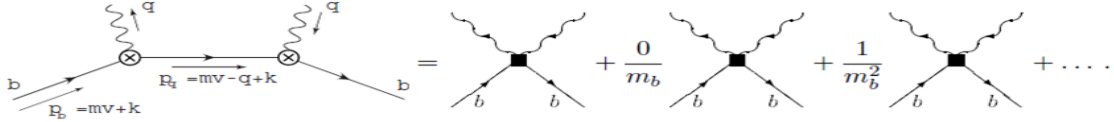

Figure 1: Tree level matching conditions

$$-i \int d^4x e^{-iq.x} T[J_L^{\mu\dagger}(x)J_L^{\nu}(0)] = \sum_i C_i O_i, \tag{2.6}$$

where the Wilson coefficients C_i can be expressed as perturbative series in α_s. The lowest dimensional term will dominate in the limit $m_b \to \infty$. These Wilson coefficients at tree level and at the loop level can be obtained from the matching conditions. As an example, the tree level matching is diagrammatically shown in Fig. 1 with $q = c$. In order to get matching conditions at $\mathcal{O}(\alpha_s)$ diagrams with one loop need to be considered. The two-index amplitude $T^{\mu\nu}$ can be decomposed into five tensor structures

$$T^{\mu\nu} = -g^{\mu\nu}T_1 + v^{\mu}v^{\nu}T_2 - i\varepsilon^{\mu\nu\alpha\beta}v_{\alpha}q_{\beta}T_3 + q^{\mu}q^{\nu}T_4 + (v^{\mu}v^{\nu} + v^{\nu}v^{\mu})T_5, \tag{2.7}$$

where the T_is are known as the projectors. They are the functions of Lorentz invariant quantities q^2 and $q.v$, where $v = P_B/M_B$ is the four velocity of the decaying B meson. These projectors can be expressed as a double series expansion: perturbative in α_s and non-perturbative in Λ_{QCD}/m_b, such as

$$T_i = \sum_{n\geq 3}\sum_{j\geq 0}\left(\frac{\Lambda_{QCD}}{m_b}\right)^{n-3}\left(\frac{\alpha_s}{\pi}\right)^j c_{ij}^{(n)}\langle B|O^n|B\rangle. \tag{2.8}$$

Therefore, the decay width for $B \to X_c\ell\nu_\ell$ can be written as [9]

$$\Gamma_{SL} = |V_{cb}|^2 \frac{G_F^2 m_b^5}{192\pi^3}(1+A_{EW}) \times \left[C_0^0 + \frac{0}{m_b} + C_2(r, \frac{\mu_\pi^2}{m_b^2}, \frac{\mu_G^2}{m_b^2}) + C_3(r, \frac{\rho_D^3}{m_b^3}, \frac{\rho_{LS}^3}{m_b^3}) + ...\right.$$
$$\left. + ... + \alpha_s\left(C_0^1 + C_2^1(r, \frac{\mu_\pi^2}{m_b^2}, \frac{\mu_G^2}{m_b^2}) + C_3^1(r, \frac{\rho_D^3}{m_b^3}, \frac{\rho_{LS}^3}{m_b^3}) + ...\right) + ...\right], \tag{2.9}$$

where $r = m_c/m_b$. Here, A_{EW} represents the electroweak corrections the ultraviolet renormalization of the Fermi interaction, and $1 + A_{EW} \approx 1.014$ [10]. The C_i^js are the perturbatively calculable functions. The parameters like μ_π^2, μ_G^2, and ρ_D^3, ρ_{LS}^3 are the matrix elements of the non-perturbative operators at order $(\Lambda/m_b)^2$ (kinetic and chromomagnetic) and $(\Lambda/m_b)^3$ (Darwin and spin-orbit) respectively. They are defined as given below

$$m_B\mu_\pi^2 \sim \langle B|\bar{b}_v D^\mu D_\mu b_v|B\rangle, \qquad m_B\mu_G^2 \sim \langle B|\bar{b}_v g_s G_{\mu\nu}\sigma^{\mu\nu}b_v|B\rangle, \qquad (2.10)$$

and

$$m_B\rho_D^3 \sim \langle B|\bar{b}_v \left[iD_\mu, [iD_\sigma, iD_\nu]\right] b_v|B\rangle \, \Pi^{\mu\nu}v^\sigma,$$
$$m_B\rho_{LS}^3 \sim \langle B|\bar{b}_v \left\{iD_\mu, [iD_\sigma, iD_\nu]\right\} (-i\sigma_{\alpha\beta})b_v|B\rangle \, \Pi^{\alpha\mu}\Pi^{\beta\nu}v^\sigma, \qquad (2.11)$$

where $\Pi_{\mu\nu} = (g_{\mu\nu} - v_\mu v_\nu)$. We note that the decay rate is sensitive to the CKM element $|V_{cb}|$. Also, the main sources of uncertainties are: (i) mass of the b quark and the ratio r, (ii) higher order QED and QCD radiative corrections, (iii) higher order of the $1/m_b$ corrections, (iv) extractions of HQE parameters, and (v) parton hadron duality [11].

The OPE/HQE parameters can be extracted from the moments of the differential distributions, like the leptonic energy moments are defined as [4]

$$M_1^\ell = \frac{1}{\Gamma}\int dE_\ell E_\ell \frac{d\Gamma}{dE_\ell}; \qquad M_n^\ell = \frac{1}{\Gamma}\int dE_\ell (E_\ell - M_1^\ell)^n \frac{d\Gamma}{dE_\ell} \quad (n > 1). \qquad (2.12)$$

with

$$M_n^\ell = \left(\frac{m_b}{2}\right)^n \left[\phi_n(r) + \bar{a}_n(r)\frac{\alpha_s}{\pi} + \bar{b}_n(r)\frac{\mu_\pi^2}{m_b^2} + \bar{c}_n(r)\frac{\mu_G^2}{m_b^2} + \bar{d}_n(r)\frac{\rho_D^3}{m_b^3} + \bar{s}_n(r)\frac{\rho_{LS}^3}{m_b^3} + ...\right]. \qquad (2.13)$$

Similarly, the moments of the hadronic mass is given by

$$M_1^X = \frac{1}{\Gamma}\int dM_X^2 (M_X^2 - \bar{M}_D^2)\frac{d\Gamma}{dM_X^2}; \qquad M_n^X = \frac{1}{\Gamma}\int dM_X^2 (M_X^2 - \langle M_X^2\rangle)^n \frac{d\Gamma}{dM_X^2} \quad (n > 1). \qquad (2.14)$$

with

$$M_n^X = m_b^{2n}\sum_{l=0}\left[\frac{M_B - m_b}{m_b}\right]^l \left[E_{nl}(r) + \bar{a}_{nl}(r)\frac{\alpha_s}{\pi} + \bar{b}_{nl}(r)\frac{\mu_\pi^2}{m_b^2} + \bar{c}_{nl}(r)\frac{\mu_G^2}{m_b^2} + \bar{d}_{nl}(r)\frac{\rho_D^3}{m_b^3} + \bar{s}_{nl}(r)\frac{\rho_{LS}^3}{m_b^3} + ...\right].$$
$$(2.15)$$

Instead of the linear moments, which are highly correlated, the central moments are more useful in the extraction of physical information. Here, the functions $\phi_n(r)$, $\bar{a}_{n(l)}(r)$, $\bar{b}_{n(l)}(r)$, $\bar{c}_{n(l)}(r)$, $\bar{d}_{n(l)}(r)$, $\bar{s}_{n(l)}(r)$ are calculable perturbatively. We note that these moments are highly sensitive to the quark masses and the OPE parameters. Therefore, a global fit to decay rate and moments allows us to extract $|V_{cb}|$, m_b, m_c, μ_π^2, μ_G^2, ρ_D^3 and ρ_{LS}^3.

3. Measurements of $|V_{cb}|$: State of the art

So far a lot of progress has been made in improving the precision in the extractions of V_{cb} and the OPE parameters. At tree level, i.e at leading order in α_s, all the corrections up to order $1/m_b^5$

have been estimated [12, 13]. A large number of parameters are associated with the $\mathcal{O}(1/m_b^{4,5})$ corrections, and hence, they can not be fitted directly from the experimental data. Therefore, these parameters are estimated using ground state saturation approximation. Only the parameters relevant up to the $\mathcal{O}(1/m_b^{2,3})$ are fitted directly from the experimental data. The corrections to the partonic rate and leptonic and hadronic mass distributions are fully known at order α_s [14] and $\mathcal{O}(\alpha_s^2 \beta_0)$ [15]. The two loop corrections of order $\mathcal{O}(\alpha_s^2)$ to the width and first few moments are calculated [16]. The $\mathcal{O}(\alpha_s)$ corrections to the kinetic operator have been estimated only numerically in [17]. In the last few years, a complete analytical calculation of the $\mathcal{O}(\alpha_s \Lambda_{QCD}^2/m_b^2)$ corrections have been performed. For details, see the references [18]. Similar calculations to the Wilson coefficients of the dimension-6 operators, defined in 2.11, are ongoing [19].

After incorporating all the known corrections to the decay width and moments, and fitting all the relevant parameters with the available data [20] on width and moments, one obtains [21],

$$\frac{\Gamma}{z(r)\Gamma_0} = 1 - 0.116_{\alpha_s} - 0.030_{\alpha_s^2} - 0.042_{1/m_b^2} - 0.002_{\alpha_s/m_b^2} - 0.030_{1/m_b^3} + 0.005_{1/m_b^4} + 0.005_{1/m_b^5}.$$
(3.1)

In this expression $z(r) = 1 - 8r + 8r^3 - r^4 - 12r^2 ln(r)$ and $\Gamma_0 = (1 + A_{EW})|V_{cb}|^2 G_F^2 m_b^5/192\pi^3$. We note that the width depends on the fifth power of the mass of b quark. Hence, the uncertainties associated with this mass has great impact in the precision extractions of $|V_{cb}|$. In the fitting, the b quark mass and the non-perturbative matrix elements are expressed in the kinetic scheme [3], setting the cutoff μ^{kin} at 1 GeV. Details of the fitting procedure and various inputs can be seen from ref. [4]. The extracted value of V_{cb} without incorporating the recently calculated α_s/m_b^2 and $1/m_b^{4,5}$ corrections is given by [4]

$$|V_{cb}| = (42.42 \pm 0.86) \times 10^{-3},$$
(3.2)

the estimated error is $\approx 2\%$. After the inclusion of α_s/m_b^2 corrections the value is [22]

$$|V_{cb}| = (42.21 \pm 0.78) \times 10^{-3}.$$
(3.3)

The error is reduced to 1.8% and the central value is reduced by 5% too. Including all the known corrections given in eq. 3.1, one obtains [21]

$$|V_{cb}| = (42.11 \pm 0.74) \times 10^{-3}.$$
(3.4)

Here we also note that the central value has reduced by only 0.25% after the inclusion of $\mathcal{O}(1/m_b^{4,5})$ effects, and the estimated error is 1.7%.

4. Conclusions

The onset of Belle-II experiment will bring us to a high precision era. Considerable progress has been made towards improving the precision $|V_{cb}|$. A more precise extraction of $|V_{cb}|$ is necessary in order to understand the SM, QCD approaches, and for an implicit search of NP. There is much more to do in order to improve the precision.

5. Acknowledgement

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POS(CKM2016)048