Theory of Inclusive $b \to s\ell^+\ell^-$ Decays

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We briefly review recent analyses of the inclusive mode $\bar{B} \to X_s\ell^+\ell^-$ and then discuss the subleading contributions which are in the focus of the present theoretical studies of this inclusive decay mode. In particular, we analyse the so-called resolved contributions which represent an irreducible uncertainty.

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1. Introduction

The $b \to s\ell + \ell^-$ transitions are in the focus of present research within flavour physics due to the so-called LHCb anomalies in the exclusive $b \to s\ell^+\ell^-$ decay modes: The LHCb collaboration has presented the angular analysis of the $B^0 \to K^{*0}\mu^+\mu^-$ decay with the 3 fb^{-1} data set and has announced a 3.4σ tension with predictions based on the SM within a global fit to the complete set of CP-averaged observables [1, 2]. They indicate that this deviation could be due to contributions from physics beyond the SM or unexpectedly large hadronic effects that are underestimated in the SM predictions.

In contrast to these exclusive modes, the inclusive decay mode $\bar{B} \to X_s\ell^+\ell^-$ is one of the most important theoretically clean modes of the indirect search for new physics via flavour observables (for reviews see Refs. [3, 4, 5]). It also allows for a nontrivial crosscheck of the so-called LHCb anomalies within the recent LHCb data on the corresponding exclusive mode [1, 2]. As was shown in Refs. [7, 8], the future measurements of the inclusive mode will be able to resolve these puzzles. Compared with the $\bar{B} \to X_s\gamma$ decay, the inclusive $\bar{B} \to X_s\ell^+\ell^-$ decay presents a complementary and more complex test of the SM, given that different perturbative electroweak contributions add to the decay rate; as a three body decay process it offers also more observables. Due to the presence of the lepton-antilepton pair, more structures contribute to the decay rate and some subtleties in the formal theoretical description arise which one needs to scrutinize. It is generally assumed that this inclusive mode is dominated by perturbative contributions like the inclusive $\bar{B} \to X_s\gamma$ decay if one eliminates $c\bar{c}$ resonances with the help of kinematic cuts. These perturbative contributions are well explored and have already reached a highly sophisticated level. The most recent analysis of all angular observables in the $\bar{B} \to X_s\ell^+\ell^-$ decay was given in Ref. [6] ; it includes all available perturbative NNLO QCD, NLO QED corrections and also the *known* subleading power corrections. In particular, the logarithmically enhanced electromagnetic corrections of all angular observables in inclusive $\bar{B} \to X_s\ell^+\ell^-$ are investigated in Ref [6]: Analytical results are presented, which are supplemented by a dedicated Monte Carlo study on the treatment of collinear photons in order to determine the size of the electromagnetic logarithms. Since the structure of the double differential decay rate is modified in the presence of QED corrections, one is able to propose new observables which vanish if only QCD corrections are taken into account.

For the inclusive modes $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_s\ell^+\ell^-$, one is able to show that, if only the leading operator in the effective Hamiltonian ($\mathcal{O}_7$ for $\bar{B} \to X_s\gamma$, $\mathcal{O}_9$ for $\bar{B} \to X_s\ell^+\ell^-$) is considered, the heavy mass expansion (HME) makes it possible to calculate the inclusive decay rates of a hadron containing a heavy quark, especially a b quark [10, 11]. In this case one is led to a local operator product expansion (OPE) based on the optical theorem. The free quark model is the first term in the constructed expansion in powers of $1/m_b$ and, therefore, is the dominant contribution. In the applications to inclusive rare B decays, one finds no correction of order Λ/m_b to the free quark model approximation within this OPE due to the equations of motion. As a consequence, the corrections to the partonic decay rate begin with $1/m_b^2$ only, which implies the rather small numerical impact of the nonperturbative corrections on the decay rate of inclusive modes.

However, there are more subtleties to consider if other than the leading operators are taken into account. As already noted in Ref. [9], there is no OPE for the inclusive decay $\bar{B} \to X_s\gamma$ if one considers operators beyond the leading electromagnetic dipole operator $\mathcal{O}_7$. Indeed the so-called resolved photon contributions contain subprocesses in which the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex [12, 13] Within the inclusive decay $\bar{B} \to X_s\gamma$ a systematic analysis [14] of all resolved photon contributions related to other operators in the weak Hamiltonian establishes this breakdown of the local OPE within the hadronic power corrections as a generic result. Such linear power corrections can be analysed within soft-collinear effective theory (SCET). Clearly, estimating such nonlocal matrix elements is very difficult, and an irreducible theoretical uncertainty of $\pm(4-5)\%$ for the total CP averaged decay rate, defined with a photon-energy cut of $E_\gamma = 1.6$ GeV, remains [14].

There are also resolved contributions to the inclusive decay $\bar{B} \to X_s\ell^+\ell^-$ on which we focus in the following. Within the inclusive decay $\bar{B} \to X_s\ell^+\ell^-$, the hadronic ($m_X$) and dilepton invariant (q^2) masses are independent kinematical quantities. In order to suppress potential huge backgrounds one needs an invariant mass cut on the hadronic final state system ($m_X \lesssim 2\,$GeV). This cut poses no additional constraints in the high-dilepton-mass region, but in the low-dilepton the cut on the hadronic mass implies a specific kinematics in which the standard OPE breaks down and nonperturbative b-quark distributions, so-called shape functions, have to be introduced. The specific kinematics of low dilepton masses q^2 and of small hadronic masses m_X leads to a multi-scale problem for which soft-collinear effecive theory (SCET) is the appropriate tool.

A former SCET analysis uses the universality of the leading shape function to show that the reduction due to the m_X-cut in all angular observables of the inclusive decay $\bar{B} \to X_s\ell^+\ell$ can be accurately computed. The effects of subleading shape functions lead to an additional uncertainty of 5% [16, 17]. A later analysis [18] estimates the uncertainties due to subleading shape functions more conservatively. In the future it may be possible to decrease such uncertainties significantly by constraining both the leading and subleading shape functions using the combined $B \to X_s\gamma$, $B \to X_u\ell\bar{\nu}$ and $B \to X_s\ell^+\ell^-$ data [18].

However, in all these previous analyses a problematic assumption is made, namely that q^2 represents a hard scale in the kinematical region of low q^2 and of small m_X. The SCET analysis below shows that the hadronic cut implies the scaling of q^2 being not hard but (anti-) hard-collinear in the low-q^2 region.

2. SCET analysis of the resolved contributions

The effective operator basis for the underlying parton interaction of the semileptonic flavour changing neutral current decay $\bar{B} \to X_s\ell^+\ell^-$ is well-known [19]. Many higher-order calculations have led to the availability of NNLO precision and NNLL resummation in the strong coupling α_s. At the relevant scale m_b of the b-quark, all heavier fields are integrated out, and the effective operator basis contains only active flavours.

We face two problems, while calculating the inclusive decay mode $\bar{B} \to X_s\ell^+\ell^-$: On the one hand, the integrated branching fraction is dominated by resonant $q\bar{q}$ background, especially with $q = c$, i.e. resonant $J/\psi \to \ell^+\ell^-$ intermediate states for the (virtual) photon, which exceeds the nonresonant charm-loop contribution by two orders of magnitude. This feature should not be misinterpreted as a striking failure of global parton-hadron duality as shown in Ref. [20]. However, $c\bar{c}$ resonances that show up as large peaks in the dilepton invariant mass spectrum are removed by appropriate kinematic cuts – leading to so-called 'perturbative q^2-windows', namely the low-dilepton-mass region $1\,\mathrm{GeV}^2 < q^2 = m_{\ell\ell}^2 < 6\,\mathrm{GeV}^2$, and also the high-dilepton-mass region with $q^2 > 14.4\,\mathrm{GeV}^2$.

On the other hand, in a realistic experimental environment we need to suppress potential huge backgrounds by an invariant mass cut on the hadronic final state system ($m_X \lesssim 2\,\mathrm{GeV}$). This cut possesses no additional constraints in the high-dilepton-mass region. But in the low-dilepton mass region we have in the B meson rest frame due to $q = p_B - p_X$

$$2\,m_B\,E_X \;= m_B^2 + M_X^2 - q^2\,. \tag{2.1}$$

Thus, for low enough q^2 in combination with $m_X^2 \ll E_X^2$ the X_s system is jet-like with $E_X \sim m_B$. This further implies hat p_X is near the light cone.

Within these kinematic constraints, soft-collinear-effective theory (SCET) [21] is the appropriate tool to study the factorization properties of inclusive B-meson decays in this region and to analyse the multi-scale problem. Thus, the cuts in the two independent kinematic variables, namely the hadronic and dilepton invariant masses, force us to study the process in the so-called shape function region with a large energy E_X of order m_B and low invariant mass $m_X \sim \sqrt{m_b \Lambda_{\mathrm{QCD}}}$ of the hadronic system. SCET enables us to systematically obtain a scaling law of the momentum components. In our set-up the scales Λ_{QCD}, m_X, q^2 and m_B are relevant. For the ratio of these scales, one finds the following hierarchy:

$$\Lambda_{\mathrm{QCD}}/m_B \ll m_X/m_B \ll 1\,. \tag{2.2}$$

Hence, resumming logarithms between these scales becomes important. SCET allows to systematically resum the logarithms of these scale ratios, and more importantly factorizes the effects stemming from the different regions. This enables us to calculate the process in a consistent expansion, and to factorize off effects that can be calculated perturbatively. This reduces the non-perturbative quantities to a limited set of soft functions. Defining $\lambda = \Lambda_{\mathrm{QCD}}/m_B$, we numerically have $m_X \lesssim \sqrt{m_B \Lambda_{\mathrm{QCD}}} \sim m_B\sqrt{\lambda}$. This sets the power-counting scale for the possible momentum components in light-cone coordinates $n^\mu = (1,0,0,1)$ and $\bar{n}^\mu = (1,0,0,-1)$. And any four-vector may be decomposed according to $a^\mu = n\cdot a\,\bar{n}^\mu/2 + \bar{n}\cdot a\,n^\mu/2 + a_\perp^\mu$. We define the short-hand notation $a \sim (n\cdot a, \bar{n}\cdot a, a_\perp)$ to indicate the scaling of the momentum components in powers of λ. Within the validity of SCET, we have a hard momentum region $p_{\mathrm{hard}} \sim (1,1,1)$, a hard-collinear region $p_{\mathrm{hc}} \sim (\lambda, 1, \sqrt{\lambda})$, an anti-hard-collinear region $p_{\overline{\mathrm{hc}}} \sim (1, \lambda, \sqrt{\lambda})$ and a soft region $p_{\mathrm{soft}} \sim (\lambda,\lambda,\lambda)$.

As far as the two-body radiative decay is concerned, the kinematics imply $q^2 = 0$ and $E_\gamma \sim m_b/2$, and the scaling including the invariant mass and photon energy requirement is fixed to be a hard-collinear hadronic jet recoiling against an anti-hard collinear photon.

In the case of a lepton-antilepton pair in the final state, we need to restrict the momentum transfer to the leptons around the mass window of the $c\bar{c}$ resonances as described above. In Fig. 1 we compare the momentum scaling of the lepton-antilepton pair in terms of the light-cone coordinate decomposition and the experimental cuts. The gray band corresponds to the hadronic invariant mass cut in order to suppress background, while the red band is the q^2 constraint to reject the $c\bar{c}$ resonances. The blue lines show the validity of SCET in terms of the momentum component scaling, on the left figure for an anti-hard-collinear scaling, while on the right one for a hard momentum

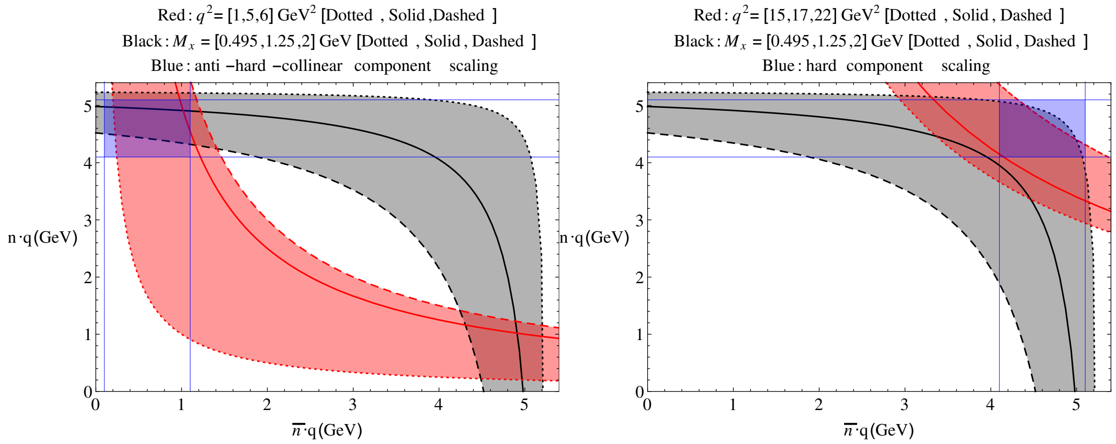

Figure 1: $q^2 = (n \cdot q)(\bar{n} \cdot q)$ with $q_\perp = 0$ for the two perturbative mass windows. The gray band shows the experimental hadronic invariant mass cut with the K as the lowest mass state, and the red band corresponds to the q^2 cut. The blue lines indicate the scaling of the two light-cone components. Left: Low invariant mass window. Scaling of $q_{\overline{\mathrm{hc}}}$ is indicated. Right: High invariant mass window, with the maximally allowed value of M_B. Scaling of q_{hard} is indicated.

scaling. Note that there exist two solutions for the left figure, as we may view the leptons to be anti-hard-collinear and the hadronic jet collinear and vice versa. Obviously, the high mass window corresponds to hard leptons and is outside of the validity of a description in terms of SCET. It can be readily seen that the current mass cuts do not have an impact on this scenario. That is in contrast to the low q^2 region. The overlap of the red and gray band is the allowed region after experimental cuts, and it is in good agreement with our assumptions for the effective theory, which is approximately given by the blue rectangle. Therefore with assigning an anti-hard-collinear momentum to the virtual photon and a hard-collinear one to the hadronic system, we are in a good approximation in the validity window of both the experimental requirement and the effective theory.

To show this more explicitly, we can introduce the two light-cone components of the hadronic momentum with $p_X^- p_X^+ = m_X^2$ and $p_X^\perp = 0$

$$\begin{aligned} \bar{n} \cdot p_X &= p_X^- = E_X + |\vec{p}_X| \sim O(m_B) \\ n \cdot p_X &= p_X^+ = E_X - |\vec{p}_X| \sim O(\Lambda_{\mathrm{QCD}})\,. \end{aligned} \tag{2.3}$$

And using the kinematical relations, the leptonic light-cone variables are given by

$$\begin{aligned} q^+ &= n \cdot q = m_B - p_X^+ \\ q^- &= \bar{n} \cdot q = m_B - p_X^- = q^2/(m_B - p_X^+)\,. \end{aligned} \tag{2.4}$$

In Fig. 2, we show the scaling of the momentum components of the hadronic system $p_X^+ = n \cdot p_X$ and $p_X^- = \bar{n} \cdot p_X$ (left plot) and of the lepton system $q^+ = n \cdot q$ and $q^- = \bar{n} \cdot q$ (right plot) as function of q^2 for three different values of the hadronic mass cut. It can be clearly seen, that for the experimentally envoked cuts – without further assumptions other than assuming the effective two-body decay system $B \to X_s \gamma^*$ to be aligned along the light-cone axis without a perp component – the hadronic

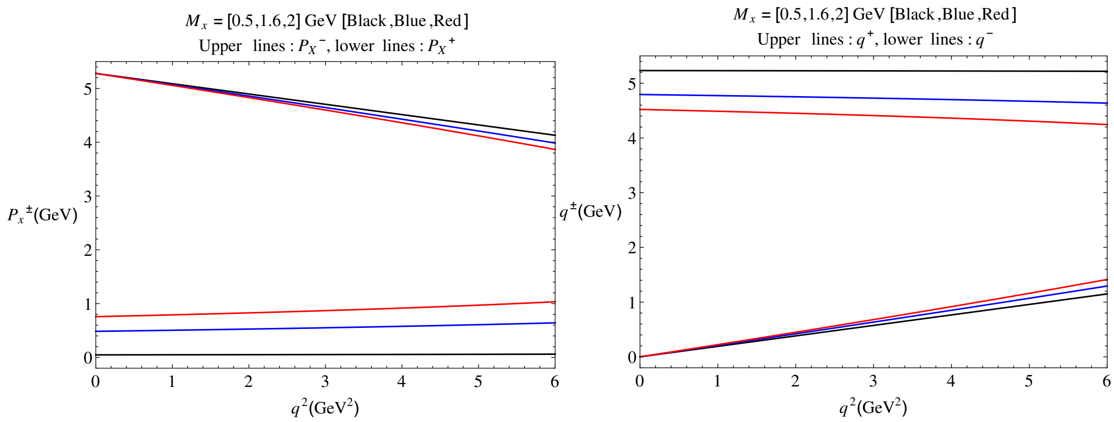

Figure 2: The scaling of the momentum components of the hadronic system $p_X^+ = n \cdot p_X$ and $p_X^- = \bar{n} \cdot p_X$ [left] and the lepton system $q^+ = n \cdot q$ and $q^- = \bar{n} \cdot q$ is plotted as a function of q^2 for each three values of the hadronic invariant mass.

system scales as hard-collinear, while the lepton system scales as anti-hard collinear. However, as also can be extracted from these plots, a lower cut of $q^2 \lesssim 5$ GeV^2 instead of $q^2 \lesssim 6$ GeV^2 is preferred because a higher value of the q^2 cut pushes the small component to values slightly beyond our assumptions of the momentum component scaling and therefore neglected higher order terms may have a more sizeable contribution. Nevertheless, the assumption of a hard q momentum as used in the calculations of Refs. [16, 17, 18] is not appropriate. Moreover, it implies a different scaling and also a different matching of the operators.

We therefore describe the hadronic effects with SCET, corresponding to an expansion of the forward scattering amplitude in non-local operator matrix elements. One derives a factorization formula for the considered process - in complete analogy to the radiative decay in [14]:

$$\begin{aligned} d\Gamma(\bar{B} \to X_s \ell^+ \ell^-) &= \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)} + \\ &+ \sum_{n=1}^{\infty} \frac{1}{m_b^n} \Big[\sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} + \\ &\qquad + \sum_i H_i^{(n)} J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} \otimes \bar{J}_i^{(n)} \Big] . \end{aligned} \tag{2.5}$$

The formula contains the so-called direct contributions in the first line, while the second and third line describe the resolved contributions which occur first only at the order $1/m_b$ in the heavy-quark expansion. Here $H_i^{(n)}$ are the hard functions describing physics at the high scale m_b, $J_i^{(n)}$ are so-called jet functions characterizing the physics of the hadronic final state X_s with the invariant mass in the range described above. The hadronic physics associated with the scale $\Lambda_{\rm QCD}$ is parameterized by the soft functions $S_i^{(n)}$. Similarly to the radiative decay investigated in Ref. [14], we have in addition resolved virtual-photon contributions in the second line, which effects are described by new jet functions $\bar{J}_i^{(n)}$. This occurs due to the coupling of virtual photons with energies of order $\sqrt{m_b \Lambda_{\rm QCD}}$ to light partons instead of the weak vertex directly. Consequently they probe the hadronic substructure at this scale. Resolved effects may occur as a single or double

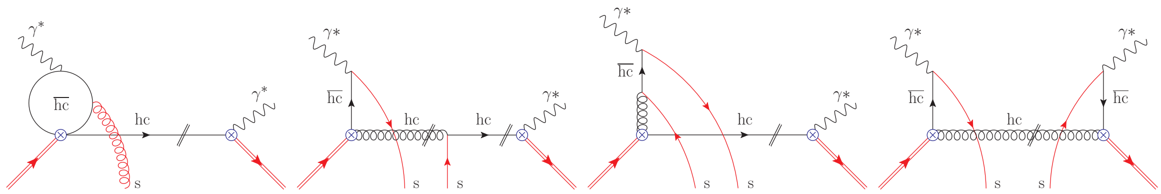

Figure 3: Diagrams arising from the matching of the $\mathcal{O}_1^q - \mathcal{O}_{7\gamma}$ contribution onto SCET, of the two $\mathcal{O}_{7\gamma} - \mathcal{O}_{8g}$ contributions, and of the $\mathcal{O}_{8g} - \mathcal{O}_{8g}$ contribution (from left to right). Red indicates soft fields, black (anti-) hardcollinear fields. Hard fields are already integrated out.

"resolved" contribution due to interference of the various operators, which also have the "direct virtual-photon" contribution. Finally the soft or shape functions are defined in terms of forward matrix elements of non-local heavy-quark effective theory (HQET) operators. This limited set of shape functions cannot be calculated perturbatively, but yet this allows a systematic analysis of hadronic effects in this decay mode.

As far as the resolved contributions are concerned, which we consider here to order $1/m_b$, we need to compute the resolved contributions from $\mathcal{O}_1 - \mathcal{O}_7$, $\mathcal{O}_7 - \mathcal{O}_8$ and $\mathcal{O}_8 - \mathcal{O}_8$. Note that the conversion of a photon to the lepton pair does not introduce a further power suppression. These various contributions to order $1/m_b$ are shown in Fig. 3.

In an exemplary mode, we discuss the structure of the $\mathcal{O}_8 - \mathcal{O}_8$ contribution. For the differential decay rate we find

$$\frac{d\Gamma}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{e_s^2\alpha_s}{m_b}\int d\omega\, \delta(\omega + m_b - n\cdot q)$$
$$\int \frac{d\omega_1}{\omega_1 + \bar{n}\cdot q + i\varepsilon}\int \frac{d\omega_2}{\omega_2 + \bar{n}\cdot q - i\varepsilon}\, g_{88}(\omega,\omega_1,\omega_2),$$

and the shape function has the following structure

$$g_{88}(\omega,\omega_1,\omega_2) = \int \frac{dr}{2\pi}e^{-i\omega_1 r}\int \frac{du}{2\pi}e^{i\omega_2 u}\int \frac{dt}{2\pi}e^{-i\omega t}$$
$$\frac{1}{M_B}\langle \bar{B}|\bar{h}(tn)\ldots s(tn+u\bar{n})\bar{s}(r\bar{n})\ldots h(0)|\bar{B}\rangle\,. \tag{2.6}$$

There are two remarks in order: First, all diagrams in Fig. 3 show that if we considered the lepton momenta as hard, the resolved contributions would not exist: The hard momentum of the leptons would imply also a hard momentum of the intermediate parton. The latter would be integrated out at the hard scale and the virtual photon would be directly connected to the effective electroweak interaction vertex. Secondly, as (2.6) shows the shape function is nonlocal in both light cone directions. Thus, the resolved contributions stay nonlocal even when the hardronic mass cut is relaxed. In this sense the resolved contributions represent an irreducible uncertainty within the inclusive decay $\bar{B} \to X_s\ell^+\ell^-$.

For a complete analysis of all resolved contributions to order $O(1/m_b)$ and their phenomenological impact within the inclusive decay $\bar{B} \to X_s\ell^+\ell^-$ we refer the reader to Ref. [22].

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