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Theory of lepton universality, flavor and number violation

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Lepton flavour universality (LFU) in B-decays is revisited by considering a class of semileptonic operators defined at a scale Λ above the electroweak scale v. The importance of quantum effects is emphasised [1]. We construct the low-energy effective Lagrangian taking into account the running effects from Λ down to v through the one-loop renormalization group equations (RGE) in the limit of exact electroweak symmetry and QED RGEs from v down to the 1 GeV scale. The most important quantum effects turn out to be the modification of the leptonic couplings of the vector boson Z and the generation of a purely leptonic effective Lagrangian. Large LFU breaking effects in Z and τ decays as well as visible lepton flavour violating (LFV) effects in τ decays are induced.

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1. Introduction

Lepton flavour universality (LFU) tests are among the most powerful probes of the Standard Model (SM) and, in turn, of New Physics (NP) effects. In recent years, experimental data in B physics hinted at deviations from the SM expectations, both in charged-current as well as neutral-current transitions. The statistically most significant data are:

- A 3.9σ violation from the τ/ℓ universality ($\ell = \mu, e$) in the charged-current $b \to c$ decays [2, 3, 4, 5]:

$$R^{\tau/\ell}_{D^{(*)}} = \frac{\mathscr{B}(\bar{B} \to D^{(*)}\tau\bar{\nu})_{\rm exp}/\mathscr{B}(\bar{B} \to D^{(*)}\tau\bar{\nu})_{\rm SM}}{\mathscr{B}(\bar{B} \to D^{(*)}\ell\bar{\nu})_{\rm exp}/\mathscr{B}(\bar{B} \to D^{(*)}\ell\bar{\nu})_{\rm SM}}, \tag{1.1}$$

$$R^{\tau/\ell}_{D} = 1.37 \pm 0.17, \qquad R^{\tau/\ell}_{D^*} = 1.28 \pm 0.08\,. \tag{1.2}$$

- A 2.6σ deviation from μ/e universality in the neutral-current $b \to s$ transition [6]:

$$R^{\mu/e}_{K} = \frac{\mathscr{B}(B \to K\mu^+\mu^-)_{\rm exp}}{\mathscr{B}(B \to K e^+ e^-)_{\rm exp}} = 0.745^{+0.090}_{-0.074} \pm 0.036\,, \tag{1.3}$$

  while $(R^{\mu/e}_{K})_{SM} = 1$ up to few % corrections [7].

As argued in [8] by means of global-fit analyses, the explanation of the $R^{\mu/e}_{K}$ anomaly favours an effective 4-fermion operator involving left-handed currents, $(\bar{s}_L\gamma_\mu b_L)(\bar{\mu}_L\gamma_\mu\mu_L)$. This naturally suggests to account also for the charged-current anomaly through a left-handed operator $(\bar{c}_L\gamma_\mu b_L)(\bar{\tau}_L\gamma_\mu\nu_L)$ which is related to $(\bar{s}_L\gamma_\mu b_L)(\bar{\mu}_L\gamma_\mu\mu_L)$ by the $SU(2)_L$ gauge symmetry [9]. Clearly, this picture might work only provided NP couples much more strongly to the third generation than to the first two.

In ref. [1], we revisited the LFU in B-decays focusing on a class of semileptonic operators defined above the electroweak scale v and invariant under the full SM gauge group, along the lines of Refs. [9, 10, 11, 12, 13]. The main new development of our study is the construction of the low-energy effective Lagrangian taking into account the running of the Wilson coefficients of a suitable operator basis and the matching conditions when mass thresholds are crossed. The running effects from the NP scale Λ down to the electroweak scale are included through the one-loop renormalization group equations (RGE) in the limit of exact electroweak symmetry [14]. From the electroweak scale down to the 1 GeV scale we use the QED RGEs. The most important quantum effects turn out to be the modification of the leptonic couplings of the vector boson Z and the generation of a purely leptonic effective Lagrangian. As a result, large LFV and LFU breaking effects in Z and τ decays are induced.

2. Effective Lagrangians

If the NP contributions originate at a scale $\Lambda \gg v$, in the energy window above v and below Λ the NP effects can be described by an effective Lagrangian $\mathscr{L} = \mathscr{L}_{\rm SM} + \mathscr{L}_{\rm NP}$ invariant under the SM gauge group. Here we assume that NP is dominated by

$$\mathscr{L}_{\rm NP} = \frac{C_1}{\Lambda^2}\,(\bar{q}_{3L}\gamma^\mu q_{3L})\left(\bar{\ell}_{3L}\gamma_\mu\ell_{3L}\right) + \frac{C_3}{\Lambda^2}\,(\bar{q}_{3L}\gamma^\mu\tau^a q_{3L})\left(\bar{\ell}_{3L}\gamma_\mu\tau^a\ell_{3L}\right). \tag{2.1}$$

We move from the interaction to the mass basis through the unitary transformations $u_L \to V_u u_L$, $d_L \to V_d d_L$, $V_u^\dagger V_d = V$, $\nu_L \to U_e \nu_L$, $e_L \to U_e e_L$ where V is the CKM matrix and neutrino masses have been neglected. For future convenience we define

$$\lambda_{ij}^q = V_{q3i}^* V_{q3j} \qquad \lambda_{ij}^e = U_{e3i}^* U_{e3j} \qquad \lambda_{ij}^{ud} = V_{u3i}^* V_{d3j}\,, \tag{2.2}$$

with $q = u,d$. Starting from the effective Lagrangian $\mathscr{L}_{\rm NP}$ at the scale Λ, at lower energies an effective Lagrangian is induced by RGE and by integrating out the heavy degrees of freedom.

The effective Lagrangian describing the semileptonic processes $b \to s\ell\ell$ and $b \to s\nu\nu$ is

$$\mathscr{L}_{\rm eff}^{\rm NC} = \frac{4G_F}{\sqrt{2}}\lambda_{bs}\left(C_\nu^{ij}\mathscr{O}_\nu^{ij} + C_9^{ij}\mathscr{O}_9^{ij} + C_{10}^{ij}\mathscr{O}_{10}^{ij}\right) + h.c.\,, \tag{2.3}$$

where $\lambda_{bs} = V_{tb}V_{ts}^*$ and the operators $\mathscr{O}_\nu$ and $\mathscr{O}_{9,10}$ read

$$\mathscr{O}_\nu^{ij} = \frac{e^2}{(4\pi)^2}(\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_i\gamma^\mu(1-\gamma_5)\nu_j)\,, \tag{2.4}$$

$$\mathscr{O}_9^{ij} = \frac{e^2}{(4\pi)^2}(\bar{s}_L\gamma_\mu b_L)(\bar{e}_i\gamma^\mu e_j)\,, \qquad \mathscr{O}_{10}^{ij} = \frac{e^2}{(4\pi)^2}(\bar{s}_L\gamma_\mu b_L)(\bar{e}_i\gamma^\mu\gamma_5 e_j)\,. \tag{2.5}$$

By matching $\mathscr{L}_{\rm eff}^{\rm NC}$ with $\mathscr{L}_{\rm NP}$, we obtain:

$$C_9^{ij} \simeq -C_{10}^{ij} \simeq \frac{4\pi^2}{e^2\lambda_{bs}}\frac{v^2}{\Lambda^2}(C_1 + C_3)\,\lambda_{23}^d\lambda_{ij}^e\,, \qquad C_\nu^{ij} \simeq \frac{4\pi^2}{e^2\lambda_{bs}}\frac{v^2}{\Lambda^2}(C_1 - C_3)\,\lambda_{23}^d\lambda_{ij}^e\,, \tag{2.6}$$

The effective Lagrangian relevant for charged-current processes like $b \to c\ell\nu$ is given by

$$\mathscr{L}_{\rm eff}^{\rm CC} = -\frac{4G_F}{\sqrt{2}}V_{cb}\,(C_L^{cb})_{ij}\left(\bar{c}_L\gamma_\mu b_L\right)\left(\bar{e}_{Li}\gamma^\mu\nu_{Lj}\right) + h.c.\,, \tag{2.7}$$

where the coefficient $(C_L^{cb})_{ij}$ reads

$$(C_L^{cb})_{ij} = \delta_{ij} - \frac{v^2}{\Lambda^2}\frac{\lambda_{23}^{ud}}{V_{cb}}C_3\,\lambda_{ij}^e\,. \tag{2.8}$$

One of the effects due to $\mathscr{L}_{\rm NP}$ is the modification of the leptonic couplings of the vector bosons W and Z. Focusing on the Z couplings, we find that

$$\mathscr{L}_Z = \frac{g_2}{c_W}\bar{e}_i\left(\not{Z}\,g_{\ell L}^{ij}P_L + \not{Z}\,g_{\ell R}^{ij}P_R\right)e_j + \frac{g_2}{c_W}\bar{\nu}_{Li}\not{Z}\,g_{\nu L}^{ij}\nu_{Lj}\,, \tag{2.9}$$

where $g_{fL,R} = g_{fL,R}^{\rm SM} + \Delta g_{fL,R}$, $c_W = \cos\theta_W$ and

$$\Delta g_{\ell L}^{ij} \simeq \frac{v^2}{\Lambda^2}\left(3y_t^2 c_-\lambda_{33}^u L_t + g_2^2 C_3 L_z + \frac{g_1^2}{3}C_1 L_z\right)\frac{\lambda_{ij}^e}{16\pi^2}, \tag{2.10}$$

$$\Delta g_{\nu L}^{ij} \simeq \frac{v^2}{\Lambda^2}\left(3y_t^2 c_+\lambda_{33}^u L_t - g_2^2 C_3 L_z + \frac{g_1^2}{3}C_1 L_z\right)\frac{\lambda_{ij}^e}{16\pi^2}, \tag{2.11}$$

with $L_t = \log(\Lambda/m_t)$, $L_z = \log(\Lambda/m_Z)$ and $\Delta g_{\ell R} = 0$. Quantum effects generate also a purely leptonic effective Lagrangian, as well as corrections to the semileptonic interactions:

$$\mathscr{L}_{\rm eff}^{\ell} = -\frac{4G_F}{\sqrt{2}}\lambda_{ij}^e\Big[(\bar{e}_{Li}\gamma_\mu e_{Lj})\sum\nolimits_\psi \overline{\psi}\gamma^\mu\psi\left(2g_\psi^Z c_t^e - Q_\psi c_\gamma^e\right)$$
$$+ \; + c_t^{cc}(\bar{e}_{Li}\gamma_\mu\nu_{Lj})(\overline{\nu}_{Lk}\gamma^\mu e_{Lk} + \overline{u}_{Lk}\gamma^\mu V_{kl}d_{Ll}) + h.c.\Big]\,, \tag{2.12}$$

where $\psi = \{\nu_{Lk}, e_{Lk,Rk}, u_{L,R}, d_{L,R}, s_{L,R}\}$ and $g^Z_\psi = T_3(\psi) - Q_\psi \sin^2\theta_W$. Finally, the coefficients $c_t^{e,cc}$ and c_γ^e are given by

$$c_\gamma^e = \frac{e^2}{48\pi^2}\frac{v^2}{\Lambda^2}\left[(3C_3 - C_1)\log\frac{\Lambda^2}{\mu^2} - (C_1+C_3)\lambda^d_{33}\log\frac{m_b^2}{\mu^2} + 2(C_1 - C_3)\left(\lambda^u_{33}\log\frac{m_t^2}{\mu^2} + \lambda^u_{22}\log\frac{m_c^2}{\mu^2}\right)\right],$$
$$c_t^e = \frac{3y_t^2}{32\pi^2}\frac{v^2}{\Lambda^2}(C_1 - C_3)\,\lambda^u_{33}\log\frac{\Lambda^2}{m_t^2}, \qquad c_t^{cc} = \frac{3y_t^2}{16\pi^2}\frac{v^2}{\Lambda^2}C_3\,\lambda^u_{33}\left[\log\frac{\Lambda^2}{m_t^2} + \frac{1}{2}\right]. \tag{2.13}$$

The residual scale dependence is removed by evaluating the matrix elements in the low energy theory. For simplicity, we have done this within the quark model, by assuming for u, d and s a common constituent mass $\mu \approx 1$ GeV.

3. Observables

In our model, $R_K^{\mu/e}$ is approximated by the expression

$$R_K^{\mu/e} \approx \frac{|C_9^{\mu\mu} + C_9^{\rm SM}|^2}{|C_9^{ee} + C_9^{\rm SM}|^2} \approx 1 - 0.28\,\frac{(C_1 + C_3)}{\Lambda^2(\rm TeV)}\frac{\lambda^d_{23}\lambda^e_{22}}{10^{-3}}. \tag{3.1}$$

while the expression for $R^{\tau/\ell}_{D^{(*)}}$ reads

$$R^{\tau/\ell}_{D^{(*)}} = \frac{\sum_j |(C_L^{cb})_{3j}|^2}{\sum_j |(C_L^{cb})_{\ell j}|^2} \approx 1 - \frac{0.12\,C_3}{\Lambda^2(\rm TeV)}\left(\lambda^d_{33} + \frac{V_{cs}}{V_{cb}}\lambda^d_{23}\right). \tag{3.2}$$

Non trivial constraints arise from the observable $R_K^{\nu\nu} = \mathscr{B}(B\to K\nu\bar\nu)/\mathscr{B}(B\to K\nu\bar\nu)_{\rm SM}$ [13],

$$R_K^{\nu\nu} = \frac{\sum_{ij}|C_\nu^{\rm SM}\delta^{ij} + C_\nu^{ij}|^2}{3|C_\nu^{\rm SM}|^2} \approx 1 + \frac{0.6\,c_-}{\Lambda^2(\rm TeV)}\left(\frac{\lambda^d_{23}}{0.01}\right) + \frac{0.3\,c_-^2}{\Lambda^4(\rm TeV)}\left(\frac{\lambda^d_{23}}{0.01}\right)^2, \tag{3.3}$$

while the experimental bound reads $R_K^{\nu\nu} < 4.3$ [15]. If LFU effects arise from LFV sources, LFV phenomena are unavoidable [11]. In our setting, it turns out that

$$\mathscr{B}(B\to K\tau\mu) \approx 4\times 10^{-8}\,|C_9^{\mu\tau}|^2 \approx 10^{-7}\left|\frac{C_9^{\mu\mu}}{0.5}\frac{0.3}{\lambda^e_{23}}\right|^2, \tag{3.4}$$

which is orders of magnitude below the current bound $\mathscr{B}(B\to K\tau\mu) \le 4.8\times 10^{-5}$ [16].

Modifications of the leptonic Z couplings are constrained by the LEP measurements [17]

$$\frac{v_\tau}{v_e} = 0.959\,(29), \qquad \frac{a_\tau}{a_e} = 1.0019\,(15), \tag{3.5}$$

where $v_\ell = g^{\ell\ell}_{\ell L} + g^{\ell\ell}_{\ell R}$ and $a_\ell = g^{\ell\ell}_{\ell L} - g^{\ell\ell}_{\ell R}$ are the vector and axial-vector couplings, respectively, which in our model read

$$\frac{v_\tau}{v_e} \simeq 1 - \frac{2\Delta g^{33}_{\ell L}}{(1-4s_W^2)} \approx 1 - 0.05\,\frac{(c_- + 0.2C_3)}{\Lambda^2(\rm TeV)},$$
$$\frac{a_\tau}{a_e} \simeq 1 - 2\Delta g^{33}_{\ell L} \approx 1 - 0.004\,\frac{(c_- + 0.2C_3)}{\Lambda^2(\rm TeV)}. \tag{3.6}$$

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Moreover, modifications of the Z couplings to neutrinos affect the extraction of the number of neutrinos N_ν from the invisible Z decay width. We find that

$$N_\nu = 2 + \left(\frac{g^{33}_{\nu L}}{g^{\rm SM}_{\nu L}}\right)^2 \simeq 3 + 4\Delta g^{33}_{\nu L} \approx 3 + 0.008\,\frac{(c_+ - 0.2C_3)}{\Lambda^2({\rm TeV})}\,, \qquad (3.7)$$

to be compared with the experimental result $N_\nu = 2.9840 \pm 0.0082$ [17].

LFU breaking effects in $\tau \to \ell\bar{\nu}\nu$ (with $\ell_{1,2} = e,\mu$) are described by the observables

$$R^{\tau/\ell_{1,2}}_\tau = \frac{\mathscr{B}(\tau\to\ell_{2,1}\nu\bar{\nu})_{\rm exp}/\mathscr{B}(\tau\to\ell_{2,1}\nu\bar{\nu})_{\rm SM}}{\mathscr{B}(\mu\to e\nu\bar{\nu})_{\rm exp}/\mathscr{B}(\mu\to e\nu\bar{\nu})_{\rm SM}} \approx 1 + 2c^{cc}_t\lambda^e_{33} \approx 1 + \frac{0.008\,C_3}{\Lambda^2({\rm TeV})} \qquad (3.8)$$

and are experimentally tested at the few per-mill level [18]

$$R^{\tau/\mu}_\tau = 1.0022 \pm 0.0030\,, \quad R^{\tau/e}_\tau = 1.0060 \pm 0.0030\,. \qquad (3.9)$$

The effective Lagrangian of eq. (2.12) generates LFV processes such as $\tau\to\mu\ell\ell$ and $\tau\to\mu P$ with

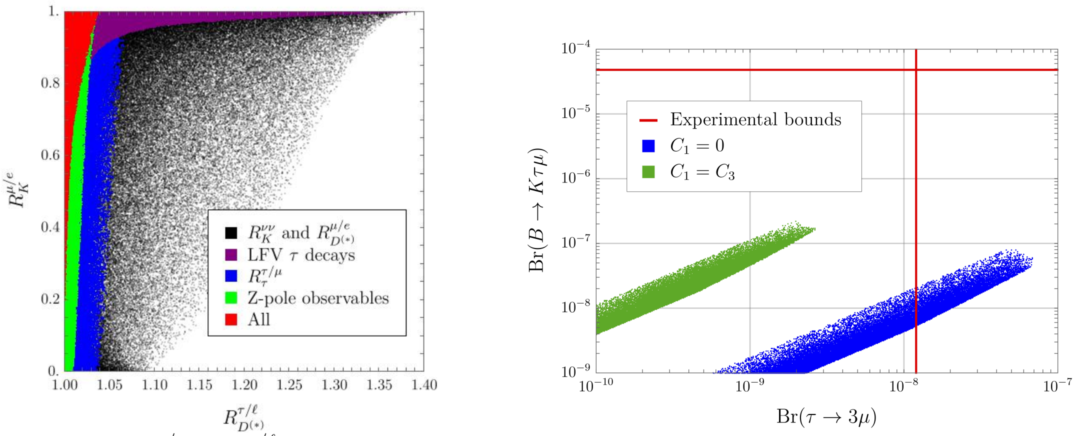

Figure 1: Left: $R^{\mu/e}_K$ vs. $R^{\tau/\ell}_{D^{(*)}}$ for $C_1 = 0$, $|C_3| \leq 3$, $|\lambda^d_{23}| \leq 0.04$ and $|\lambda^e_{23}| \leq 1/2$. The allowed regions are coloured according to the legend. Right: $\mathscr{B}(B\to K\tau\mu)$ vs. $\mathscr{B}(\tau\to 3\mu)$ for $|\lambda^d_{23}| = 0.01$, $C_1 = C_3$ (green points) or $C_1 = 0$ (blue points) imposing all the experimental bounds except $R^{\tau/\ell}_{D^{(*)}}$.

$P = \pi,\eta,\eta',\rho$, etc. The most sensitive channels, taking into account their NP sensitivities and experimental resolutions, are $\tau\to\mu\ell\ell$, $\tau\to\mu\rho$ and $\tau\to\mu\pi$. For instance, we find

$$\mathscr{B}(\tau\to 3\mu) \approx 5\times 10^{-8}\,\frac{c_-^2}{\Lambda^4({\rm TeV})}\left(\frac{\lambda^e_{23}}{0.3}\right)^2\,, \qquad (3.10)$$

where the current bounds are $\mathscr{B}(\tau\to 3\mu) \leq 1.2\times 10^{-8}$ [16].

Most importantly, we find that $R^{\tau/\ell}_\tau$ strongly disfavours an explanation of the $R^{\tau/\ell}_{D^{(*)}}$ anomaly, see the left plot of fig. 1. In the right plot of fig. 1, we show $\mathscr{B}(B\to K\tau\mu)$ vs. $\mathscr{B}(\tau\to 3\mu)$. Considering the current and expected future experimental sensitivities, we conclude that $\tau\to 3\mu$ is a more powerful probe than $B\to K\tau\mu$.

4. Conclusions

Recent experimental data hinting at non-standard LFU breaking effects in semileptonic B-decays stimulated many theoretical investigations of NP scenarios. In ref. [1], we revisited LFU in B-decays assuming a class of gauge invariant semileptonic operators at the NP scale $\Lambda \gg v$, as in Refs. [9, 10, 11, 12, 13]. We constructed the low-energy effective Lagrangian taking into account the running effects from Λ down to v through the one-loop RGEs in the limit of exact electroweak symmetry and QED RGEs from v down to the 1 GeV scale. At the quantum level, we find that the leptonic couplings of the W and Z vector bosons are modified. Moreover, quantum effects generate also a purely leptonic effective Lagrangian, as well as corrections to the semileptonic interactions. The main phenomenological implications of these findings are the generation of large LFU breaking effects in Z and τ decays, which are correlated with the B-anomalies, and τ LFV processes. Overall, the experimental bounds on Z and τ decays significantly constrain LFU breaking effects in B-decays, challenging an explanation of the current non-standard data. Interestingly, if LFU breaking effects arise from LFV sources, the most sensitive LFV channels are not B-decays, as commonly claimed in the literature but, instead, τ decays such as $\tau \to \mu\ell\ell$ and $\tau \to \mu\rho$. Although our results have been obtained in the context of an effective Lagrangian dominated by left-handed operators, the present work shows that electroweak radiative effects should be carefully analysed in any framework addressing the explanation of B-anomalies.

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