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On the theory prediction of R_K and radiative corrections

Marzia Bordone*
University of Zurich
E-mail: mbordone@physik.uzh.ch

We review the theoretical prediction for the ratio R_K which tests LFU in $B \to K\ell\ell$ with $\ell = e, \mu$. In particular we are interested in investigating the size of the uncertainty. To achieve this we perform a semi-analytical calculation of radiative corrections for the decay $B \to K\ell^+\ell^-$, and apply the results to R_K. We find out that the radiative corrections can be sizeable. However, applying the same cuts used in LHCb measurement, the overall effect of soft QED corrections diminishes. We perform also a comparison with PHOTOS, the Montecarlo (MC) tool used by LHCb to correct for QED correction. We find good agreement between our result and the MC with difference only at the at the per mill level. As an outlook we present also the effect of QED corrections on R_{K^*}, for which a measurement by LHCb is expected soon.

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*Speaker.

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1. Introduction

Lepton Flavour Universality (LFU) is one of the strongest predictions of Standard Model (SM). It is based on the fact that the interaction between the gauge vectors and leptons is independent of the lepton family number.
In order to constrain the SM, LFU can be tested. One way of doing that is using ratios of semileptonic $b \to c\ell\nu$ and $b \to s\ell\ell$ decays, where we look at different lepton species in the final state. The ratios are very powerful instruments because through them the hadronic uncertainties are much lower than in the branching ratios and also they allow directly to compare different leptonic families.
In the $b \to c\ell\nu$ channel, the observables $R_{D^{(*)}}$ show about 4σ [1] discrepancy with the SM prediction. They are defined as:

$$R_{D^{(*)}} = \frac{\mathscr{B}(B \to D^{(*)}\tau\nu)}{\mathscr{B}(B \to D^{(*)}\mu\nu)}$$

Exploting instead the $b \to s\ell\ell$ decay we can construct the ratio R_K, whose definition is

$$R_K = \frac{\mathscr{B}(B \to K\mu^+\mu^-)}{\mathscr{B}(B \to Ke^+e^-)},$$

Also for R_K there is a tension between the measurements [2] and the SM prediction of 2.6σ.
From the theoretical point of view, the structure of these observables is quite different. In fact, the decay $b \to c\ell\nu$ arises through a charged channel transition at tree level, while the decay $b \to s\ell\ell$ is mediated by a neutral current at 1 loop level. Moreover, $R_{D^{(*)}}$ probes LFU violation in the tau channel compared to the muons one, while R_K compares the muons versus electrons channel.
New Physics (NP) was considered as a source of the apparent LFU violation, although, given the different size of the SM contributions to $R_{D^{(*)}}$ and R_K, it is not easy to build a consistent UV theory that addresses both the anomalies. To be sure that these discrepancies are signals of NP we should be convinced of the corresponding theory prediction. Therefore, we performed a semi-analytical calculation of radiative correction on R_K.

2. Setup and Calculation

In the SM, in absence of QED corrections, the theory prediction for R_K is[3]:

$$R_K = 1.0003 \pm 0.0001. \tag{2.1}$$

In order to try to explain the discrepancy between theory prediction and measurements one can ask which are the sources of LFU violation. First, we can think about the effects due to the fact that both electrons and muons masses are not identical. However, if we stick to the kinematic region considered for the measurement where $q^2 \in [1,6]$ GeV^2, this effects are very small because they scale like $\frac{m_\ell^2}{q^2}$ and they are already taken in account in the uncertainty budget in Eq.2.1.
Another possibility is instead looking at QED corrections: indeed naively we expect that those corrections can be of the order $\sim \frac{\alpha}{\pi}\log^2(\frac{m_\ell^2}{q^2})$, which can represent a sizeable contribution, especially in the electron case. In the literature there have been already calculation of QED corrections [4, 5, 6],

but they are carried out in the case in which the photon is fully inclusive, and hence not directly applicable to R_K.

The setup we used for our calculation is described in the following:

- we restrict the kinematic region such that $m_\ell^2 \ll q^2$;
- we focus on the terms associated with soft and collinear divergences that arise when a photon is emitted by the particles in the final state, to extract log-enhanced terms;
- we neglect finite correction, since they are very small compared to the log-enhanced terms;
- we explicitly verify that the effects due to the emission of a photon by both mesons are not log-enhanced, and therefore negligible.

By means of the previous approximations it is possible to calculate the radiation function $\omega(x, x_\ell)$. It represents the probability density function of the dilepton system in the final state to retain a fraction $\sqrt{x}$ of its original invariant mass q_0^2 after bremsstrahlung. We obtain [7]:

$$\omega(x, x_\ell) = \omega_1(x, x_\ell)\theta(1-x-x_*) + \omega_2(x, x_\ell, x_*)\delta(1-x)$$

where $\omega_1(x, x_\ell)$ represent real emission above the infrared cutoff, wthile $\omega_2(x, x_\ell, x_*)$ encodes the virtual correction and the emission below the infrared cutoff x_*. For completeness we report the definition of the following quantities involved in the radiator function: $x = \frac{q^2}{q_0^2}$ and $x_\ell = \frac{m_\ell^2}{q_0^2}$.
The decay width at NLO is obtained by convoluting the radiator itself and the LO decay width. In order to be able to make any comparison with the measurements, the integration limits of the convolution respect the experimental setup, included the cutoff for the reconstructed mass of the B meson m_B^{rec}.
To complete our calculation, we need also to introduce the resonances. Given the kinematic region of interest, the only one we care about is the J/ψ. Using the parametrization in [8], the J/ψ can modelled introducing the following effective Wilson coefficient:

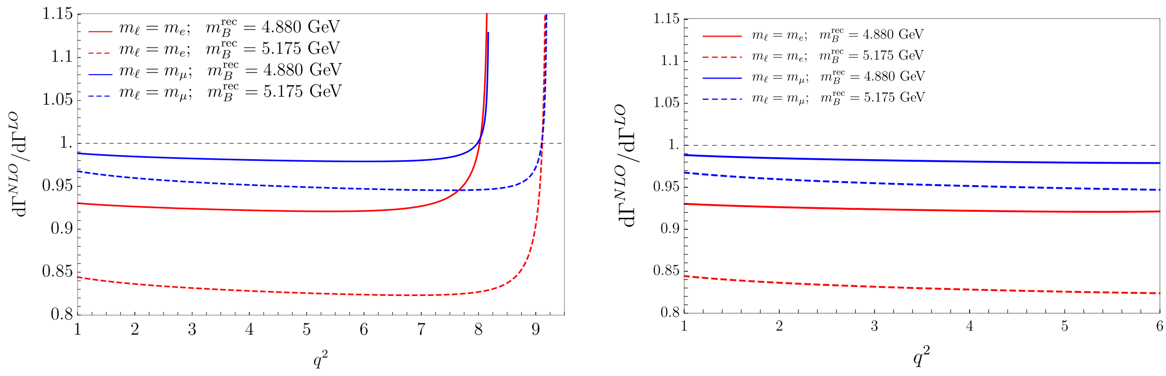
$$a_9(q^2) = a_9^{pert} + \kappa_\psi \frac{q^2}{q^2 - m_{J/\psi}^2 + i m_{J/\psi}\Gamma_{J/\psi}},$$

where a_9^{pert} ensures the behavior at low q^2, while $m_{J/\psi}$ and $\Gamma_{J/\psi}$ represents respectively the mass and the width of the J/ψ state and the coefficient κ_ψ is fixed by the branching ratio of the process $B \to K\, J/\psi$.

3. Results

The results[7] are shown in Fig.1. The first issue that must be investigated is the impact of the tail of the J/ψ on the NLO decay width. Indeed, the presence of the J/ψ is not included in PHOTOS, so it could in principle give a relevant contribution still not taken in account in the measurement. From Fig.1(a) we can see that in the low q^2 region, the overall behaviour of the distributions appears rather smooth, while in the region closer to the resonance a considerable raise appears. The position, in which the J/ψ tail appears, depends uniquely on the cutoff m_B^{rec}. Even

applying the looser cut $m_B^{\rm rec} = 4.880$ GeV in Ref.[2], the kinematic region of $q^2 \in [1,6]$ GeV^2 is free from any effect due to the J/ψ.
We now proceed in analysing closely the $q^2 \in [1,6]$ GeV^2 region. We can see that, as we expected from the beginning, the corrections due to QED effects can be sizeable, especially in the electron channel. Despite that, when we look at the size of the correction in Tbl.1 and we take the benchmark for the different values of $m_B^{\rm rec}$ associated with muons of electrons of Ref.[2], we find that the overall shift on R_K is roughly $\Delta R_K = +3\%$, which is in good agreement with PHOTOS prediction in a $\pm 1\%$ interval.[1]



(a) (b)

Figure 1: Relative impact of radiative correction in $B \to K\ell^+\ell^-$ for $q^2 \in [1,9.5]$ GeV^2 (left) and $q^2 \in [1,6]$ GeV^2 (right), with different cuts on the reconstructed mass and different lepton masses.

$B \to K\ell^+\ell^-$	$\ell = e$	$\ell = \mu$
$m_B^{\rm rec} = 4.880$ GeV	-7.6%	-1.8%
$m_B^{\rm rec} = 5.175$ GeV	-16.9%	-4.6%

Table 1: Relative impact of radiative corrections for $q^2 \in [1,6]$ GeV^2, with different cuts on the reconstructed mass and different lepton masses.

As an outlook, we present also the results for R_{K^*}. From the theory side, the treatment of QED correction is completely analogous to the one of R_K. Since there is not a published analysis so far, we took as benchmark for this case the very same ones that are discussed in the analysis for R_K. The results are shown in Fig.2: the conclusions are similar to the ones we just described, and the overall shift on R_{K^*} due to QED correction can be extracted from the values in Tbl.2 and it is estimated to be $\Delta R_{K^*} = 2.8\%$.

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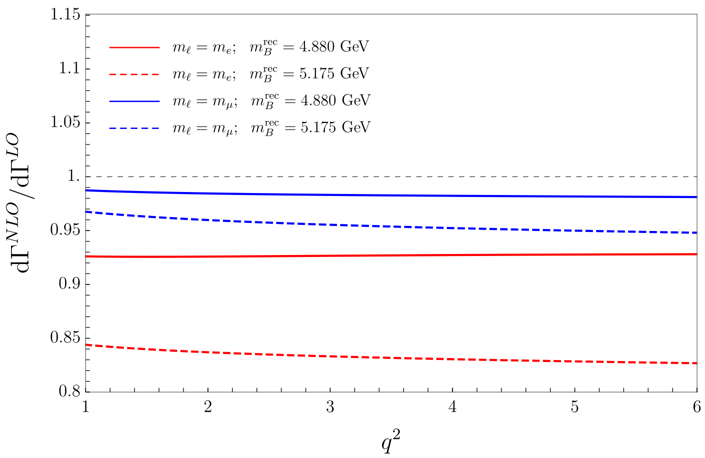

Figure 2: Relative impact of radiative correction in $B \to K^* \ell^+ \ell^-$ for $q^2 \in [1,6]$ GeV^2 (right), with different cuts on the reconstructed mass and different lepton masses.

$B \to K^* \ell^+ \ell^-$	$\ell = e$	$\ell = \mu$
$m_B^{\rm rec} = 4.880$ GeV	-7.3%	-1.7%
$m_B^{\rm rec} = 5.175$ GeV	-16.7%	-4.5%

Table 2: Relative impact of radiative corrections for $q^2 \in [1,6]$ GeV^2, with different cuts on the reconstructed mass and different lepton masses.

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