Penguin pollution in β and β_s

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The mixing-induced CP asymmetries in $B_d \to J/\psi K_S$ and $B_s \to J/\psi\phi$ are essential to detect or constrain new physics in the $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing amplitudes, respectively. To this end one must control the penguin contributions to the decay amplitudes, which affect the extraction of fundamental CP phases from the measured CP asymmetries. Although the "penguin pollution" is doubly Cabibbo-suppressed, it could compete in size with current experimental errors. In this talk I present a calculation of the penguin contributions treating QCD effects with soft-collinear factorisation and compare method and results with the alternative approach employing flavour-SU(3) symmetry. As a novel feature, I present results for the penguin pollution in $b \to c\bar{c}d$ modes.

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[*]Speaker.

Figure 1: Box diagrams describing $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing in the Standard Model.

1. Introduction

In this talk I discuss time-dependent CP asymmetries

$$A_{\mathrm{CP}}^{B_q \to f}(t) \equiv \frac{\Gamma(\bar{B}_q(t) \to f) - \Gamma(B_q(t) \to f)}{\Gamma(\bar{B}_q(t) \to f) + \Gamma(B_q(t) \to f)}, \qquad q = d \text{ or } s, \qquad (1.1)$$

for $B_{d,s}$ decays into final states f consisting of a charmonium and a light pseudoscalar or vector boson. Prime examples are the decays $B_d \to J/\psi K_S$ and $B_s \to J/\psi\phi$, which are both triggered by the quark decay $\bar{b} \to \bar{c}c\bar{s}$. I only consider the case that f is a CP eigenstate; if f comprises two vector mesons (as in $B_s \to J/\psi\phi$) it is understood that the CP-even and CP-odd components are properly separated through an angular analysis. Precise measurements of these mixing-induced CP asymmetries serve to determine the CP phases related to the $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing amplitudes. Within the Standard Model these are

$$2\beta \equiv \arg\left(\frac{V_{tb}V_{td}^*}{V_{cb}V_{cd}^*}\right)^2 \qquad \text{and} \qquad 2\beta_s \equiv \arg\left(\frac{V_{tb}^*V_{ts}}{V_{cb}^*V_{cs}}\right)^2. \qquad (1.2)$$

Here $(V_{tb}V_{tq}^*)^2$ stems from the box diagrams shown in Fig. 1. The $B_q - \bar{B}_q$ mixing amplitudes probe virtual effects of new particles with masses as high as 100 TeV, if new physics enters $B_q - \bar{B}_q$ mixing at tree level. It is therefore of utmost importance to control the theoretical uncertainties in the relation between the measured $A_{\mathrm{CP}}^{B_q \to f}(t)$ and the fundamental CP phases in Eq. (1.2) as precisely as possible.

The CP asymmetry in Eq. (1.1) reads

$$A_{\mathrm{CP}}^{B_q \to f}(t) = \frac{S_f \sin(\Delta m_q t) - C_f \cos(\Delta m_q t)}{\cosh(\Delta \Gamma_q t / 2) + A_{f,\Delta\Gamma_q} \sinh(\Delta\Gamma_q t / 2)}. \qquad (1.3)$$

Here Δm_q and $\Delta\Gamma_q$ are the mass and width difference, respectively, between the mass eigenstates of the $B_q - \bar{B}_q$ system. Δm_q and $\Delta\Gamma_q$ are CP-conserving quantities calculated from the box diagrams in Fig. 1. In B_d decays we can set the denominator in Eq. (1.3) to 1, because $\Delta\Gamma_d$ is very small. The coefficients S_f, C_f, and $A_{f,\Delta\Gamma_q}$ depend on the decay amplitude $A(B_q \to f)$. For the $\bar{b} \to \bar{c}c\bar{s}$ amplitudes of interest one usually writes:

$$A(B_q \to f) = V_{cb}^*V_{cs}T_f + V_{ub}^*V_{us}P_f. \qquad (1.4)$$

The "tree" and "penguin" amplitudes read

$$T_f = \frac{G_F}{\sqrt{2}} \langle f | C_1 Q_1^c + C_2 Q_2^c + \sum_j C_j Q_j | B_q \rangle, \qquad (1.5)$$

$$P_f = \frac{G_F}{\sqrt{2}} \langle f | C_1 Q_1^u + C_2 Q_2^u + \sum_j C_j Q_j | B_q \rangle. \qquad (1.6)$$

Here G_F is the Fermi constant and $Q_1^q = \bar{q}^\alpha \gamma_\mu (1-\gamma_5) s^\beta \bar{b}^\beta \gamma^\mu (1-\gamma_5) q^\alpha$ and $Q_2^q = \bar{q}^\alpha \gamma_\mu (1-\gamma_5) s^\alpha \bar{b}^\beta \gamma^\mu (1-\gamma_5) q^\beta$ are the current-current operators generated by W-boson exchange. The sum over j comprises the penguin operators Q_{3-6} and the chromomagnetic operator Q_{8G} (see Ref. [1] for the definitions). The C_k's are the Wilson coefficients which encode the short-distance physics; the top-quark penguin loops (entering C_{3-6} and C_{8G}) appear in both T_f and P_f, because the CKM unitarity relation $V_{tb}^* V_{ts} = -V_{cb}^* V_{cs} - V_{ub}^* V_{us}$ is used to eliminate $V_{tb}^* V_{ts}$ from Eq. (1.6). Expanding to first order in $\varepsilon = |V_{us} V_{ub}/(V_{cs} V_{cb})| \approx 0.02$ one has

$$S_f \simeq -\eta_f \sin(\phi_q + \Delta\phi_q) \qquad \text{with} \qquad \tan(\Delta\phi_q) \simeq 2\varepsilon \sin\gamma \, \text{Re} \frac{P_f}{T_f}, \qquad (1.7)$$

where $CP|f\rangle = \eta_f |f\rangle$ with $\eta_f = \pm 1$, $\phi_d = 2\beta$, and $\phi_s = -2\beta_s$. Furthermore, $C_f \simeq 2\varepsilon \sin\gamma \, \text{Im}\,(P_f/T_f)$ quantifies direct CP violation.

$\Delta\phi_q$ in Eq. (1.7) is the *penguin pollution* which obscures a clean extraction of ϕ_q from the measured S_f. The size of the penguin pollution depends on the considered decay mode through $\text{Re}\,(P_f/T_f)$ in Eq. (1.7). A standard way to estimate $\Delta\phi_q$ employs the flavour-SU(3) symmetry of QCD or its SU(2) subgroup U-spin. The latter connects pairs of hadronic matrix elements related by the interchange of down and strange quarks. In the case of $B_d \to J/\psi K_S$ one can extract the desired P_f/T_f from control channels such as $B_s \to J/\psi K_S$ or $B_d \to J/\psi \pi^0$, which are induced by the quark decay $\bar{b} \to \bar{c} c \bar{d}$. In these control channels the CKM factor ε is replaced by $|V_{ud} V_{ub}/(V_{cd} V_{cb})| \approx 0.38$ which permits to determine P_f/T_f from the coefficients C_f and S_f measured in these modes. In this way one finds the values $-3.9° \le \Delta\phi_d \le -0.8°$ [2], $|\Delta\phi_d| \le 1.6°$ [3], $|\Delta\phi_d| \le 0.8°$ [4], and $\Delta\phi_d = -1.1°{}^{+0.85°}_{-0.7°}$ [5] for $f = J/\psi K_S$. The values (listed in chronological order) become more accurate with more precise data the on the control channels. A general drawback of the method is the unknown size of $\text{SU}(3)_f$ breaking caused by unequal strange and down quark masses. $\text{SU}(3)_f$ symmetry can be very accurate, as e.g. in semileptonic $B_{d,s}$ decays, but may also fail completely: for example, a b quark fragments into a B_d meson almost four times more often than into a B_s. In the case of $B_s \to J/\psi \phi$ one faces the problem that the ϕ meson is an equal mixture of an octet and a singlet of $\text{SU}(3)_f$ symmetry. It is not clear how to treat $\text{SU}(3)_f$ breaking in such a case of maximal symmetry violation and the method may fail in this case.

The experimental world average $2\beta + \Delta\phi_d = 43.8° \pm 1.4°$ [7] is dominated by $B_d \to J/\psi K_S$, so that $\Delta\phi_d$ here can be identified with the penguin pollution in this mode. The experimental error is comparable in size with the expected penguin pollution. The situation is similar with the experimental value $2\beta_s + \Delta\phi_s = 1.7° \pm 1.9°$ [7] which dominantly stems from LHCb data on $B_s \to J/\psi K^+ K^-$ and $B_s \to J/\psi f_0[\to \pi^+ \pi^-]$, with an experimental error of $2.2°$ on $2\beta_s + \Delta\phi_s$ [9]. The statistical powers of $B_s \to J/\psi \phi[\to K^+ K^-]$, non-resonant $B_s \to J/\psi K^+ K^-$, and $B_s \to J/\psi f_0[\to \pi^+ \pi^-]$ on the determination of $2\beta_s + \Delta\phi_s$ are 52%, 8%, and 42%, respectively [10]. The value of $2\beta_s$ inferred from a global fit to the CKM unitarity triangle is $2\beta_s = 2.12° \pm 0.04°$ [8].

In this talk I present calculations of the penguin contributions to CP asymmetries which do not use $\text{SU}(3)_F$ symmetry, but instead employ soft-collinear factorisation in QCD [6].
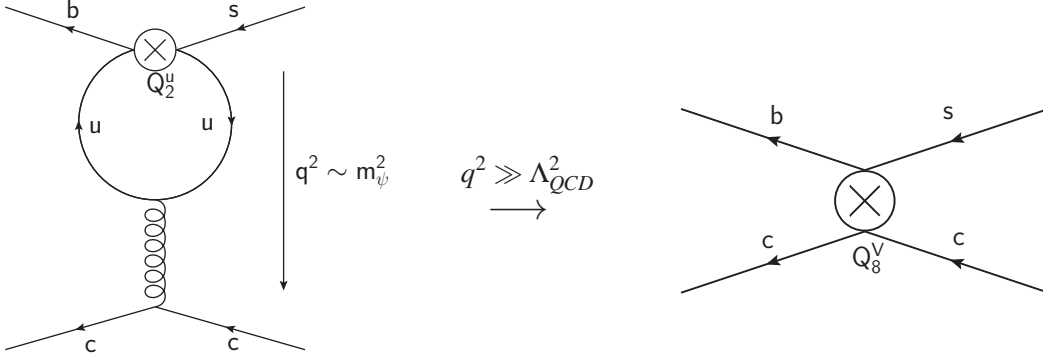
Figure 2: Pictorial representation of the OPE for the up-quark loop: Since the momentum transfer q to the charmonium is large, we can express the left diagram as the product of a perturbative Wilson coefficient and the effective four-quark operator on the right.

2. Operator Product Expansion

Many physical problems involve a hard scale $\sqrt{q^2}$ which is much larger than the fundamental scale $\Lambda_{\text{QCD}} \sim 0.4\,\text{GeV}$ of QCD. The operator product expansion (OPE) is a calculational tool to express the quantity of interest in terms of a series in $\Lambda_{\text{QCD}}/\sqrt{q}$. In our case we a apply the OPE to P_f in Eq. (1.6) and $\sqrt{q^2} \sim m_\psi \sim 3\,\text{GeV}$ is the hard scale. The troublesome contribution to P_f stems from $Q_{1,2}^u$ in Eq. (1.6); the corresponding one-loop contribution is shown in Fig. 2. The OPE for the contribution of Q_j^u, $j = 1,2$, to P_f for $B_d \to J/\psi K_S$ reads

$$\langle J/\psi K_S | Q_j^u | B_d \rangle = \sum_k \widetilde{C}_{j,k} \langle J/\psi K_S | Q_k | B_d \rangle + \dots \tag{2.1}$$

Here $k = 0V, 0A, 8V, 8A$ labels different local four-quark operators with flavour structure $\bar{b}s\bar{c}c$:

$$\begin{aligned}
Q_{0V} &\equiv \bar{b}\gamma_\mu(1 - \gamma_5)s\,\bar{c}\gamma^\mu c, \\
Q_{0A} &\equiv \bar{b}\gamma_\mu(1 - \gamma_5)s\,\bar{c}\gamma^\mu \gamma_5 c, \\
Q_{8V} &\equiv \bar{b}\gamma_\mu(1 - \gamma_5)T^a s\,\bar{c}\gamma^\mu T^a c, \\
Q_{8A} &\equiv \bar{b}\gamma_\mu(1 - \gamma_5)T^a s\,\bar{c}\gamma^\mu \gamma_5 T^a c.
\end{aligned} \tag{2.2}$$

These operators suffice to reproduce P_f at the leading power of $\Lambda_{\text{QCD}}/\sqrt{q}$. Sub-leading powers involve additional operators , which are indicated by the dots in Eq. (2.1). The Wilson coefficients $\widetilde{C}_{j,k}$ in Eq. (2.1) are found by calculating $\bar{b} \to \bar{c}c\bar{s}$ Feynman diagrams with Q_j^u in the desired order of α_s and comparing the result with Feynman diagrams involving the operators Q_k in Eq. (2.2) in the corresponding order of QCD. The leading non-vanishing order, shown in Fig. 2, only involves the operator Q_{8V} on the right-hand side (RHS) of the OPE in Eq. (2.1). The coefficient is

$$\widetilde{C}_{2,8V}^{(0)} = \frac{2}{3}\frac{\alpha_s}{4\pi}\left[\ln\left(\frac{q^2}{\mu^2}\right) - i\pi - \frac{2}{3}\right], \tag{2.3}$$

where μ is the renormalisation scale. The idea to factorise the one-loop diagram in this way was proposed by Bander, Silverman, and Soni (BSS) in Ref. [11] and applied to $B_d \to J/\psi K_S$
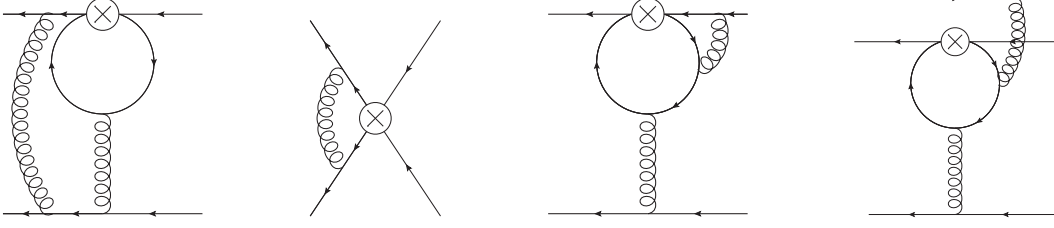
Figure 3: The soft IR divergence of the first diagram (contributing to the LHS of Eq. (2.1)) factorises with the corresponding diagram of the local operator (RHS of Eq. (2.1)) shown next. The third diagram has a collinear IR divergence and finally a spectator-scattering diagram is shown.

in Ref. [12]. In order to establish the OPE in Eq. (2.1) one must prove that the coefficients $\widetilde{C}_{j,k}$ are free from infrared singularities, which involves the study of higher orders in α_s. This proof has been carried out in Ref. [6] and involves the analysis of (i) soft IR divergences of the two-loop diagrams contributing to $\langle Q_j^u \rangle$, (ii) collinear IR divergences of these diagrams, (iii) spectator scattering diagrams, and (iv) higher-order diagrams in which the large momentum bypasses the penguin loop ("long distance penguins"). Sample diagrams are shown in Fig. 3. In Ref. [6] it has been shown that indeed all infrared singularities properly factorise and cancel from the coefficients $\widetilde{C}_{j,k}$, which therefore can be calculated perturbatively order-by-order in α_s. The leading order (LO) contribution to the $\widetilde{C}_{j,k}$ stems from the penguin operators Q_{3-6} in Eq. (1.6), which contribute trivially to Eq. (2.1) as local $\bar{b}s\bar{c}c$ operators. The dependence of C_{3-6} on the unphysical renormalisation scale μ cancels (to order α_s) with the μ-dependent terms of the next-to-leading order (NLO) corrections. The result in Eq. (2.3) belongs to the NLO and depends on the renormalisation scale and scheme. It is only meaningful in combination with the LO contributions involving C_{3-6}, so that these scale and scheme dependences cancel. In Ref. [12] the LO contribution has been omitted and the inferred penguin pollution is substantially smaller than the one found by us.

The standard application of soft-collinear factorisation in flavour physics addresses B decays into two light mesons (QCD factorisation) [13]. In our case instead one of the final-state mesons is heavy and the J/ψ mass is the relevant heavy scale in the problem. As a consequence, we cannot factorise the matrix element of colour-octet four-quark operators into a form factor and a decay constant [14].

3. Matrix elements and numerical results

In order to predict the size of the penguin pollution $\Delta\phi_d$ for $B_d \to J/\psi K_S$ in Eq. (1.7) from the calculated $\mathrm{Re}\,(P_f/T_f)$ we need the (ratios of the) hadronic matrix elements

$$
\begin{aligned}
v_0 V_0 &\equiv \langle J/\psi K^0 | Q_{0V} | B_d \rangle, & a_0 V_0 &\equiv \langle J/\psi K^0 | Q_{0A} | B_d \rangle, \\
v_8 V_0 &\equiv \langle J/\psi K^0 | Q_{8V} | B_d \rangle, & a_8 V_0 &\equiv \langle J/\psi K^0 | Q_{8A} | B_d \rangle.
\end{aligned}
\tag{3.1}
$$

Eq. (3.1) defines the complex parameters $v_{0,8}$ and $a_{0,8}$ with the common normalisation factor

$$
V_0 \equiv \langle J/\psi K^0 | Q_{0V} | B_d \rangle_{\mathrm{fact}} = 2 f_{J/\psi} m_{B_d} p_{cm} F_1^{B \to K}(m_\psi^2) = (4.26 \pm 0.16)\,\mathrm{GeV}^3.
\tag{3.2}
$$

4

V_0 is the factorised matrix element of the colour-singlet operator Q_{0V} involving the J/ψ decay constant $f_{J/\psi}$, the B_d mass m_{B_d}, the magnitude of the K_S center-of mass three-momentum p_{cm}, and the form factor $F_1^{B\to K}$. Next $v_{0,8}$ and $a_{0,8}$ are categorised in terms of $1/N_c$ counting, where $N_c = 3$ is the number of colours. One has $v_0 = 1 + \mathcal{O}(1/N_c^2)$, $v_8, a_8 = \mathcal{O}(1/N_c)$, and $a_0 = \mathcal{O}(1/N_c^2)$. It is well-known that the coefficient of v_0 in T_f is small, so that the branching ratio $B(B_d \to J/\psi K_S)$ is dominated by v_8 and a_8. Therefore we can use the measured $B(B_d \to J/\psi K_S)_{\text{exp}}$ as a cross-check of our colour counting for the peculiar colour-octet matrix elements. With the numerical values of the Wilson coefficients and Eq. (3.2) one finds [6]:

$$\frac{B(B_d \to J/\psi K_S)}{B(B_d \to J/\psi K_S)_{\text{exp}}} = [1 \pm 0.08] \, |0.47 v_0 + 7.8(v_8 - a_8)|^2. \qquad (3.3)$$

This implies $0.07 \leq |v_8 - a_8| \leq 0.19$ if v_0 is set to 1, illustrating that the colour counting works for the branching ratio. a_0 comes with small coefficients in both T_f and P_f and is negligible. For the prediction of P_f/T_f at NLO we need v_8 and impose $|v_8| \leq 1/3$, complying with colour counting, and vary the phases of the matrix elements between $-\pi$ and π. The result is [6]

$$|\Delta_d| \leq 0.68°, \qquad |C_{J/\psi K_S}| \leq 1.33 \cdot 10^{-2}. \qquad (3.4)$$

The bound on Δ_d is comparable to the one derived from SU(3)$_F$ symmetry (quoted in the introduction), but sharper.

In the case of $B_s \to J/\psi \phi$ one finds

$(J/\psi\phi)^0$	$(J/\psi\phi)^{\|}$	$(J/\psi\phi)^\perp$
$|\Delta\phi_s| \leq 0.97°$	$|\Delta\phi_s| \leq 1.22°$	$|\Delta\phi_s| \leq 0.99°$
$|C_f| \leq 1.89 \cdot 10^{-2}$	$|C_f| \leq 2.35 \cdot 10^{-2}$	$|C_f| \leq 1.92 \cdot 10^{-2}$

for the scalar, parallel, and perpendicular polarisation states, respectively.

As a novel feature, the method of Ref. [6] permits the prediction of the penguin contributions to $\bar{b} \to \bar{c}c\bar{d}$ decays, for example:

$$B_d \to J/\psi \pi^0 : \qquad |S_{J/\psi\pi^0} + \sin(2\beta)| \leq 0.18, \qquad |C_{J/\psi\pi^0}| \leq 0.29. \qquad (3.5)$$

$$B_s \to J/\psi K_S : \qquad |S_{J/\psi K_S} - \sin(-2\beta_s)| \leq 0.26, \qquad |C_{J/\psi K_S}| \leq 0.27. \qquad (3.6)$$

The first result means $-0.86 \leq S_{J/\psi\pi^0} \leq -0.50$. Eq. (3.5) favours the Belle result [15] $S_{J/\psi\pi^0} = -0.67 \pm 0.22$, $C_{J/\psi\pi^0} = -0.08 \pm 0.17$ over the BaBar result [16] $S_{J/\psi\pi^0} = -1.23 \pm 0.21$, $C_{J/\psi\pi^0} = -0.20 \pm 0.19$. Predictions for more $\bar{b} \to \bar{c}c\bar{s}$ and $\bar{b} \to \bar{c}c\bar{d}$ modes can be found in Tab. 1 of Ref. [6].

It is worthwhile to compare the methods and results presented in this talk with those of the alternative approach based on SU(3)$_F$ symmetry: It is gratifying to see that the two completely different methods give compatible results for $\Delta\phi_d$ in the case of $B_d \to J/\psi K_S$. However, the SU(3)$_F$ estimate of $\Delta\phi_d$ depends on the choice for the size of SU(3)$_F$ breaking added to the value of P_f/T_f extracted from the $\bar{b} \to \bar{c}c\bar{d}$ control channels. In analyses of branching fractions (which probe T_f with little sensitivity to P_f) it is possible to include linear SU(3)$_F$ breaking in the T_f amplitudes and thereby test the quality of the method from the data (see Ref. [4] for $B \to J/\psi X$ decays and Refs. [18] and [17] for B, D decays to two light mesons, respectively). However, in

the case of the up-quark loop in P_f there is not enough information to disentangle the penguin pollution from the matrix elements parametrising $SU(3)_F$ breaking, no matter how many control channels are included: The $SU(3)_F$ breaking stemming from the $d \rightarrow s$ replacement when linking the $\bar{b} \rightarrow \bar{c}c\bar{d}$ control channel to the $\bar{b} \rightarrow \bar{c}c\bar{s}$ signal process is never constrained by any of these control channel processes. On the contrary, the OPE-based approach of Ref. [6] makes enough redundant predictions to simultaneously test the method and to constrain the penguin pollution in the $\bar{b} \rightarrow \bar{c}c\bar{s}$ decays: Here the litmus test are the predictions for the various $\bar{b} \rightarrow \bar{c}c\bar{d}$ channels (such as those in Eqs. (3.5) and (3.6)), which make the method falsifiable.

The $SU(3)_F$ method utilises the feature that the $SU(3)_F$ symmetry is approximately exact, with corrections treatable as small (i.e. $\mathscr{O}(30\%)$) perturbations. The quality of the symmetry allows us to assign exact or approximate $SU(3)_F$ quantum numbers to the particle states, as we routinely do for the light pseudoscalar mesons. In the case of $B_s \rightarrow J/\psi\phi$ one faces the fact that the ϕ meson is an equal mixture of octet and singlet, so that it does not correspond to an approximate $SU(3)_F$ eigenstate. There are two possible explanations of this observations: (i) $SU(3)_F$ is not a good symmetry for decays into final states with vector mesons. (ii) $SU(3)_F$ breaking is small, but the spectrum of the "unperturbed" strong hamiltonian (corresponding to the limit $m_s = m_d = m_u$) is almost degenerate, so that even a small perturbation can lead to maximal mixing. If case (i) is realised in nature, $SU(3)_F$ cannot be applied to constrain the penguin pollution in $B_s \rightarrow J/\psi\phi$. If (ii) is the correct explanation, a necessary ingredient of an $SU(3)_F$-based assessment of the penguin pollution is the determination of both the octet and singlet matrix elements from the control channels. In addition, one must develop a formalism which permits the treatment of $SU(3)_F$ breaking for the case that the final states of the considered decays cannot be approximated by $SU(3)_F$ eigenstates. In view of this situation it is safe to say that $SU(3)_F$-based estimates of the penguin pollution in $B_s \rightarrow J/\psi\phi$ rest on shaky ground.

4. Conclusions and outlook

In this talk I have presented results of Ref. [6] for the penguin pollution affecting the extractions of the CP phases 2β and $2\beta_s$ from the decays $B_d \rightarrow J/\psi K_S$ and $B_s \rightarrow J/\psi\phi$, respectively. The predictions are based on a new calculational approach which utilises an operator product expansion (OPE) for the penguin amplitude. To establish the OPE the infrared safety of the Wilson coefficients calculated from the up-quark loop contribution to the penguin amplitude had to be proven, which elevates the BSS approach of Ref. [11] to a field-theoretic concept applicable at any order of α_s. (However, we found no justification to apply the OPE to the charm-quark loop, which in our framework resides in the hadronic matrix elements.) Our method can also be applied to CP asymmetries in $\bar{b} \rightarrow \bar{c}c\bar{d}$ decays, in which the penguin-to-tree ratio is much larger. As examples I have quoted bounds on the penguin contributions for the CP asymmetries in the decays $B_d \rightarrow J/\psi\pi^0$ and $B_s \rightarrow J/\psi K_S$. The confrontation of our predictions for $\bar{b} \rightarrow \bar{c}c\bar{d}$ decays with more precise data will be a stringent test of the OPE-based approach. In the future the errors of the predictions may shrink, if effort is put into the calculation of the hadronic parameter v_8, possibly with the help of QCD sum rules. In my talk I have further expressed a critical view of the application of $SU(3)_F$ symmetry to the penguin pollution in $B_s \rightarrow J/\psi\phi$.

Acknowledgements

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