

# Quantum-correlated measurements of $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$ and consequences for the determination of $\gamma$

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Quantum-correlated measurements of the decay  $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$  are performed with a data sample corresponding to an integrated luminosity of  $818 \text{ pb}^{-1}$  collected at the  $\psi(3770)$  resonance by the CLEO-c detector. Preliminary results are presented for the  $CP$ -even fraction  $F_+$  and the strong-phase differences of this decay. The value of  $F_+$  is measured to be  $0.246 \pm 0.018$ . The strong-phase differences are measured in different regions of  $K_S^0 \pi^+ \pi^- \pi^0$  phase space by binning around the intermediate resonances present. The potential sensitivity of the results for determining the CKM angle  $\gamma$  from  $B^\pm \rightarrow D(K_S^0 \pi^+ \pi^- \pi^0)K^\pm$  decays using data collected by the Belle detector is also shown.

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## 1. Introduction

Among the three CKM [1] angles  $\gamma$  is measured least precisely. This is due to the small branching fraction ( $\mathcal{O}(10^{-4})$ ) of decays sensitive to  $\gamma$ . An improved measurement of  $\gamma$  is essential for testing the standard model description of  $CP$  violation. The decays  $B^\pm \rightarrow DK^\pm$ , where  $D$  indicates a neutral charm meson reconstructed in a final state common to both  $D^0$  and  $\bar{D}^0$ , provide  $CP$ -violating observables that in turn can be used for measuring  $\gamma$  using data collected by detectors such as BaBar, Belle, LHCb or the upcoming Belle II experiment. The additional inclusion of multibody  $D$  meson final states will reduce the statistical uncertainty on  $\gamma$ . However, such final states require knowledge of the strong-phase difference between the  $D^0$  and  $\bar{D}^0$  that varies over the phase space. The required strong-phase information can be obtained by studying quantum-correlated  $D\bar{D}$  pairs produced in  $e^+e^-$  collisions at an energy corresponding to the  $\psi(3770)$  resonance at CLEO-c.

We present herein preliminary results for the decay  $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$ , which has a large branching fraction of 5.2% [2]. This decay mode has not been used so far to determine  $\gamma$ . The mode is potentially useful in a quasi-GLW [3] analysis along with other  $CP$  eigenstates if its  $CP$ -even fraction  $F_+$  is known [4]. Further, this multibody self-conjugate decay occurs via many intermediate resonances, such as  $K_S^0 \omega$  and  $K^{*\pm} \rho^\mp$ . Hence if the strong-phase difference variation over the phase space is known, a GGSZ-style [5, 6] analysis to determine  $\gamma$  from this final state alone is possible.

## 2. Quantum-correlated $D$ mesons

The wave function for the decay of the vector meson  $\psi(3770)$  to a pair of  $D$  mesons is anti-symmetric as the two daughters are produced in a  $P$ -wave state. Integrating over the whole phase space, the double-tagged yield, where the decays of both the  $D$  mesons are identified, for a signal (tag) decay  $f$  ( $g$ ) can be written in terms of the  $CP$ -even fraction  $F_+^f$  ( $F_+^g$ ) and the branching fractions  $\mathcal{B}(f)$  ( $\mathcal{B}(g)$ ) as

$$M(f|g) = \mathcal{N} \mathcal{B}(f) \mathcal{B}(g) \varepsilon(f|g) \left[ 1 - (2F_+^f - 1)(2F_+^g - 1) \right], \quad (2.1)$$

where  $\mathcal{N}$  is the overall normalization factor and  $\varepsilon$  is the reconstruction efficiency. If  $f$  or  $g$  is a  $CP$  eigenstate, then the value  $(2F_+ - 1)$  becomes the  $CP$  eigenvalue  $\lambda_{CP}$ . So there is two-fold enhancement in the yield if  $f$  and  $g$  have opposite  $CP$  eigenvalue whereas the yield becomes zero if  $f$  and  $g$  have the same  $CP$  eigenvalue. Thus the rate of the decays of the two  $D$  mesons are correlated to each other.

The single-tagged yield, where only one of the  $D$  mesons is reconstructed without any constraints on the other, is given by

$$S(g) = \mathcal{N} \mathcal{B}(g) \varepsilon(g). \quad (2.2)$$

Assuming  $\varepsilon(f|g) = \varepsilon(f) \varepsilon(g)$ , we write the ratios between the double and single-tagged yields,  $N^+$  and  $N^-$ , when mode  $g$  is  $CP$ -odd ( $\lambda_{CP}^g = -1$ ) or even ( $\lambda_{CP}^g = 1$ ), as

$$N^\pm = \frac{M(f|g)}{S(g)} = \mathcal{B}(f) \varepsilon(f) \left[ 1 \mp (2F_+^f - 1) \right], \quad (2.3)$$

which leads to the definition of  $F_+^f$  in terms of  $N^+$  and  $N^-$ :

$$F_+^f \equiv \frac{N^+}{N^+ + N^-}. \quad (2.4)$$

In addition, we can also use some tag modes whose  $CP$ -even fraction  $F_+^g$  is already known to determine  $F_+^f$ . For this, we define a quantity  $N^g$  as the ratio of double and single-tagged yields as

$$N^g = \mathcal{B}(f) \varepsilon(f) \left[ 1 - (2F_+^f - 1)(2F_+^g - 1) \right]. \quad (2.5)$$

This is used along with  $N^+$  to extract  $F_+^f$  as

$$F_+^f = \frac{N^+ F_+^g}{N^g - N^+ + 2N^+ F_+^g}. \quad (2.6)$$

The  $g$  mode can also be self-conjugate modes like  $K_S^0 \pi^+ \pi^-$  or  $K_L^0 \pi^+ \pi^-$ . The phase space of these multibody states can be divided into different bins. The  $K_{S,L}^0 \pi^+ \pi^-$  Dalitz plot is studied and binned according to the Equal  $\delta_D$  scheme [7] based on the amplitude model reported in Ref. [8]. The double-tagged yield in each of these bins is

$$M_i(K_S^0 \pi^+ \pi^- \pi^0 | K_{S,L}^0 \pi^+ \pi^-) = h_{K_{S,L}^0 \pi^+ \pi^-} \left[ K_i^{K_{S,L}^0 \pi^+ \pi^-} + K_{-i}^{K_{S,L}^0 \pi^+ \pi^-} - 2c_i \sqrt{K_i^{K_{S,L}^0 \pi^+ \pi^-} K_{-i}^{K_{S,L}^0 \pi^+ \pi^-}} (2F_+^f - 1) \right], \quad (2.7)$$

where  $K_i$  and  $K_{-i}$  are the fraction of flavour-tagged  $D^0$  and  $\bar{D}^0$  decays in each bin,  $c_i$  is the cosine of the strong phase difference for  $K_{S,L}^0 \pi^+ \pi^-$ , and  $h_{K_{S,L}^0 \pi^+ \pi^-}$  is the normalization factor. With these  $F_+^f$  can be determined if the double-tagged yields in each of the  $K_{S,L}^0 \pi^+ \pi^-$  bins are measured.

To perform a GGSZ analysis with a self-conjugate multibody final state  $f$ , the amplitude-weighted averages of  $\cos \Delta \delta_D$  and  $\sin \Delta \delta_D$  over regions of phase space [5, 6], referred to as  $c_i$  and  $s_i$ , respectively are required. Here  $\Delta \delta_D$  is the strong-phase difference between  $CP$  conjugate points in the phase space. The values of  $c_i$  and  $s_i$  are obtained by tagging with  $CP$  and quasi- $CP$  eigenstates and other self-conjugate modes. For  $CP$  eigenstate tag modes, the double-tagged yield is given by

$$M_i^\pm = h_{CP} \left[ K_i + \bar{K}_i \mp 2\sqrt{K_i \bar{K}_i} c_i \right], \quad (2.8)$$

where  $h_{CP}$  is the normalization constant. If the tag is a quasi- $CP$  eigenstate of known  $F_+$ , the  $c_i$  sensitive term is scaled by  $(2F_+ - 1)$  rather than 1. For the self-conjugate tag mode  $K_S^0 \pi^+ \pi^-$  [9, 10], the double-tagged yield is

$$M_{i\pm j}^{K_S^0 \pi^+ \pi^-} = h_{K_S^0 \pi^+ \pi^-} \left[ K_i K_{\mp j}^{K_S^0 \pi^+ \pi^-} + \bar{K}_i K_{\pm j}^{K_S^0 \pi^+ \pi^-} - 2\sqrt{K_i K_{\pm j}^{K_S^0 \pi^+ \pi^-} \bar{K}_i K_{\mp j}^{K_S^0 \pi^+ \pi^-}} (c_i c_j^{K_S^0 \pi^+ \pi^-} + s_i s_j^{K_S^0 \pi^+ \pi^-}) \right], \quad (2.9)$$

and for a  $K_L^0 \pi^+ \pi^-$  tag, the double-tagged yield is

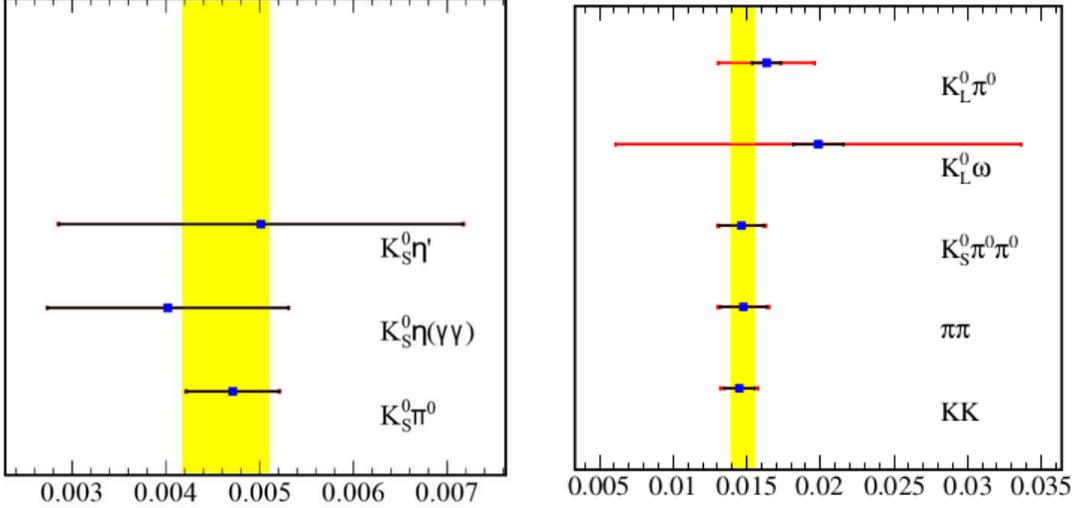
$$M_{i\pm j}^{K_L^0 \pi^+ \pi^-} = h_{K_L^0 \pi^+ \pi^-} \left[ K_i K_{\mp j}^{K_L^0 \pi^+ \pi^-} + \bar{K}_i K_{\pm j}^{K_L^0 \pi^+ \pi^-} + 2\sqrt{K_i K_{\pm j}^{K_L^0 \pi^+ \pi^-} \bar{K}_i K_{\mp j}^{K_L^0 \pi^+ \pi^-}} (c_i c_j^{K_L^0 \pi^+ \pi^-} + s_i s_j^{K_L^0 \pi^+ \pi^-}) \right]. \quad (2.10)$$

If both tag and signal states are the same, then

$$M_{ij} = h_f \left[ K_i \bar{K}_j + \bar{K}_i K_j - 2\sqrt{K_i \bar{K}_j \bar{K}_i K_j} (c_i c_j + s_i s_j) \right], \quad (2.11)$$

where  $h_f$  is the normalization constant.

Type	Modes
CP-even	$K^+ K^-, \pi^+ \pi^-, K_S^0 \pi^0 \pi^0, K_L^0 \omega, K_L^0 \pi^0$
CP-odd	$K_S^0 \pi^0, K_S^0 \eta, K_S^0 \eta'$
Mixed CP	$\pi^+ \pi^- \pi^0, K_S^0 \pi^+ \pi^-, K_L^0 \pi^+ \pi^-$
Flavour	$K^\pm e^\mp \nu_e$

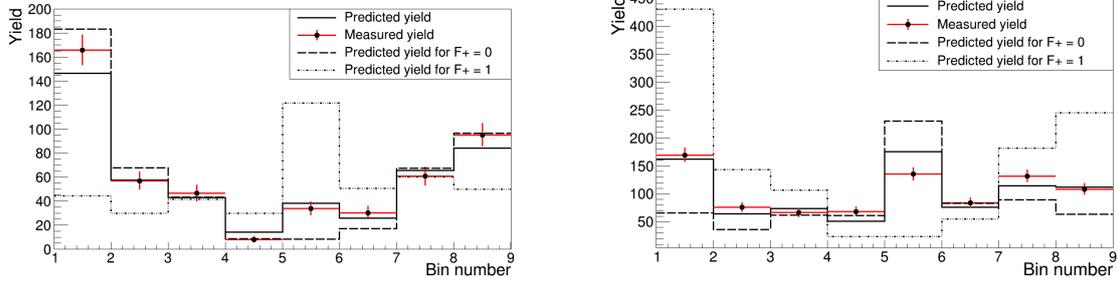
**Table 1:** Different tag modes used in the analysis.**Figure 1:**  $N^+$  values for the CP-odd modes (left) and  $N^-$  values for the CP-even modes (right). The yellow region shows the average value. Horizontal black lines show the statistical uncertainty and red lines the total uncertainty.

### 3. Measurement of $F_+$ in $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$ decays

A data sample corresponding to an integrated luminosity of  $818 \text{ pb}^{-1}$ , collected by the CLEO-c detector at the interaction point of CESR  $e^+e^-$  collider, consisting of  $D\bar{D}$  pairs coming from the  $\psi(3770)$  resonance is used in this analysis. The  $D\bar{D}$  final state is reconstructed for the signal state  $K_S^0 \pi^+ \pi^- \pi^0$  along with the tag modes listed in Table 1. All charged tracks and energy deposits associated with both the  $D$  mesons are reconstructed; the selection criteria for the tag modes are identical to those presented in Ref. [4]. Modes involving  $K_L^0$  or  $\nu$  are reconstructed partially using a missing-mass squared technique [11].

With the double-tagged yields measured and single-tagged yields taken from Ref. [12], we calculate  $N^+$  and  $N^-$  from the CP-odd and even modes, respectively. They are shown in Fig. 1. With the quasi-CP mode  $\pi^+ \pi^- \pi^0$ , we calculate  $F_+$  using Eqn. 2.6 with the input value  $F_+^{\pi^+ \pi^- \pi^0} = 0.973 \pm 0.017$  [12]. The value of  $F_+$  obtained with CP and quasi-CP modes is  $0.244 \pm 0.021$ . This suggests that the mode  $K_S^0 \pi^+ \pi^- \pi^0$  is significantly CP-odd. Using  $K_{S,L}^0 \pi^+ \pi^-$  modes,  $F_+$  is calculated with Eqn. 2.7. The values of  $K_i, K_{-i}, c_i,$  and  $s_i$  for  $K_{S,L}^0 \pi^+ \pi^-$  are taken from Ref. [7]. The values of predicted and measured double-tagged yields in each of the  $K_{S,L}^0 \pi^+ \pi^-$  bins are shown in Fig. 2; from these data  $F_+$  is determined to be  $0.265 \pm 0.029$  in our calculation. With all the three

above mentioned methods, the average  $F_+$  is  $0.246 \pm 0.018$ . The uncertainty includes statistical as well as systematic contributions.



**Figure 2:** The predicted and measured yields for  $K_S^0 \pi^+ \pi^-$  (left) and  $K_L^0 \pi^+ \pi^-$  (right) in each bin obtained from a combined fit of both the modes. The histogram shows the predicted values, points show the measured values, dashed line corresponds to  $F_+ = 0$  and the dotted line shows  $F_+ = 1$ .

#### 4. Determination of $c_i$ and $s_i$

The five-dimensional phase space of  $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$  is studied to extract  $c_i$  and  $s_i$  values. There is no trivial symmetry in the phase space to define the bins and hence the bins are constructed around the resonances present. The lack of an amplitude model for this channel makes a proper optimization difficult. An exclusive eight-bin scheme is followed around the resonances such as  $\omega$ ,  $K^*$  and  $\rho$ . The kinematic regions of the bins are listed in Table 2 along with the fraction of flavour-tagged  $D^0$  and  $\bar{D}^0$  decays in each of them. These values are determined from semileptonic flavour tag  $K^\pm e^\mp \nu_e$ .

Bin number	Specification	$K_i$	$\bar{K}_i$
1	$m(\pi^+ \pi^- \pi^0) \approx m(\omega)$	$0.222 \pm 0.019$	$0.176 \pm 0.017$
2	$m(K_S^0 \pi^-) \approx m(K^{*-})$ & $m(\pi^+ \pi^0) \approx m(\rho^+)$	$0.394 \pm 0.022$	$0.190 \pm 0.017$
3	$m(K_S^0 \pi^+) \approx m(K^{*+})$ & $m(\pi^- \pi^0) \approx m(\rho^-)$	$0.087 \pm 0.013$	$0.316 \pm 0.021$
4	$m(K_S^0 \pi^-) \approx m(K^{*-})$	$0.076 \pm 0.012$	$0.046 \pm 0.009$
5	$m(K_S^0 \pi^+) \approx m(K^{*+})$	$0.057 \pm 0.010$	$0.065 \pm 0.011$
6	$m(K_S^0 \pi^0) \approx m(K^{*0})$	$0.059 \pm 0.011$	$0.092 \pm 0.013$
7	$m(\pi^+ \pi^0) \approx m(\rho^+)$	$0.045 \pm 0.009$	$0.045 \pm 0.009$
8	Remainder	$0.061 \pm 0.011$	$0.070 \pm 0.011$

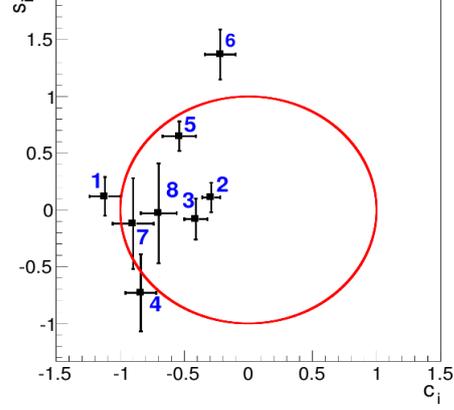
**Table 2:** The specifications for the eight exclusive bins of  $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$  phase space along with the fraction of  $D^0$  and  $\bar{D}^0$  events in each of them.

The yields for  $CP$ , quasi- $CP$  and self-conjugate modes in each of the bins are measured and the  $c_i$  and  $s_i$  values are extracted using Eqns. 2.8-2.11. The migration of events from one bin to another

due to the narrowness of each bin is considered in the fit. The preliminary results are summarized in Table 3 and Fig. 3. The uncertainties mentioned are statistical only.

Bin	$c_i$	$s_i$
1	$-1.12 \pm 0.12$	$0.12 \pm 0.17$
2	$-0.29 \pm 0.07$	$0.11 \pm 0.13$
3	$-0.41 \pm 0.09$	$-0.08 \pm 0.18$
4	$-0.84 \pm 0.12$	$-0.73 \pm 0.34$
5	$-0.54 \pm 0.13$	$0.65 \pm 0.13$
6	$-0.22 \pm 0.12$	$1.37 \pm 0.22$
7	$-0.90 \pm 0.16$	$-0.12 \pm 0.40$
8	$-0.70 \pm 0.14$	$-0.03 \pm 0.44$

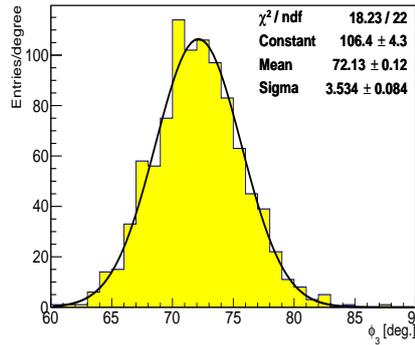
**Table 3:** Preliminary results for  $c_i$  and  $s_i$  values obtained from the fit.



**Figure 3:**  $c_i$  and  $s_i$  values in each bin.

## 5. Estimation of $\gamma$ sensitivity with $B^\pm \rightarrow D(K_S^0 \pi^+ \pi^- \pi^0) K^\pm$

We estimate the sensitivity of  $\gamma$  with the preliminary results of  $c_i$  and  $s_i$  values described in the previous section, in a GGSZ framework with  $B^\pm \rightarrow D(K_S^0 \pi^+ \pi^- \pi^0) K^\pm$  decays from Belle ( $\approx 1 \text{ ab}^{-1}$ ). We run 1000 pseudo-experiments with  $c_i$ ,  $s_i$ ,  $K_i$ , and  $\bar{K}_i$  values as inputs with each experiment consisting of  $\approx 1200$  events. The sample sizes are determined from the Belle sample of  $B^\pm \rightarrow D(K_S^0 \pi^+ \pi^-) K^\pm$  [13]. Here we assume that increase in branching fraction for  $K_S^0 \pi^+ \pi^- \pi^0$  compared to  $K_S^0 \pi^+ \pi^-$  is compensated by loss in efficiency due to a  $\pi^0$  in final state. The estimated uncertainty on  $\gamma$  is  $\sigma_\gamma = 25^\circ$ . The projection of this to a  $50 \text{ ab}^{-1}$  sample of Belle II gives  $\sigma_\gamma = 3.5^\circ$  (see Fig. 4).



**Figure 4:**  $\gamma$  sensitivity with  $50 \text{ ab}^{-1}$  Belle II sample.

## 6. Conclusions

The studies of  $D$  meson final states open up additional avenues for measuring the CKM angle  $\gamma$ .

In particular, the decay  $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$  can serve as an additional mode in quasi-GLW methods, with the  $CP$ -even fraction  $F_+$  measured to be  $0.246 \pm 0.018$ , reducing the statistical uncertainty on  $\gamma$ . Further, the measurement of strong phase differences of this mode in eight different phase space regions, allows a model-independent GGSZ estimation of  $\gamma$  from this mode alone. It is estimated that a single-mode uncertainty on  $\gamma$  of  $\sigma_\gamma = 3.5^\circ$  is achievable with a  $50 \text{ ab}^{-1}$  sample of data at Belle II. This could be improved with optimized  $c_i$  and  $s_i$  values, provided a proper amplitude model is available and a finer binning using a larger sample of quantum correlated data from the BESIII experiment.

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