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Belle results for ϕ_3 and prospects from Belle II by 2021

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We report on recent Belle measurements of the $\phi_3(\gamma)$ angle of the CKM unitarity triangle as well as on an ongoing new time-dependent analysis of $B^0 \to D^*\rho$. Belle II prospects with regards to ϕ_3 measurements and their comparison to LHCb are also presented.

9th International Workshop on the CKM Unitarity Triangle
28 November - 3 December 2016
Tata Institute for Fundamental Research (TIFR), Mumbai, India

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PoS(CKM2016)114

1. Methods for Measuring ϕ_3

The ϕ_3 (also known as γ) angle of the standard CKM unitarity triangle is the least well constrained of its angles. At the same time, it is the only CP-violating parameter that can be measured solely in tree-level processes, thus making it a benchmark for CP violation within the Standard Model. However, ϕ_3 measurement precision is limited by small branching fractions of the involved processes.

The most powerful and theoretically clean way of measuring ϕ_3 is based on the interference between $b \to \bar{u}cs$ and $b \to u\bar{c}s$ tree level amplitudes [1]. What we mean by saying theoretically clean is that there is no penguin graph contribution [2] to these processes and consequently no theoretical uncertainty connected to the hadronic parameters, as they can be obtained from experiment.

An example of the mentioned type of processes is $B^\pm \to D^{(*)}K^\pm$ followed by $D \to f$, and $B^\pm \to \bar{D}^{(*)}K^\pm$ followed by $\bar{D} \to f$, where f is a *common final state*. Interference between these two paths gives rise to direct CP violation. Diagrams of the relevant decays are depicted in figure 1.

Most analyses neglect the effects of neutral D meson mixing and CP violation as these are expected to be small [3]. However, D mixing and CP violation can be included at no cost to the uncertainty by using measured values from other studies [4].

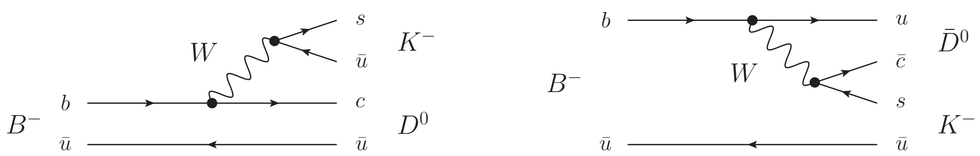


Figure 1: Diagrams of the two tree-level $B^- \to DK^-$ decays.

1.1 GLW Method

The method was first proposed by Gronau, London and Wyler [1, 5]. Their idea is that a B meson can decay weakly to a state with a D^0 or $\bar{D}^0$. But if we reconstruct the neutral D meson from a final state that is a CP-eigenstate, we actually select a superposition $(D^0 \pm \bar{D}^0)/\sqrt{2}$. We label these states D_{CP^+} and D_{CP^-} for the CP-even and CP-odd states respectively. We can then construct 4 observables that encode the CP violation parameters,

$$\mathscr{R}_{CP^\pm} = 2\frac{\Gamma(B^- \to D_{CP^\pm}K^-) + \Gamma(B^+ \to D_{CP^\pm}K^+)}{\Gamma(B^- \to D_{fav}K^-) + \Gamma(B^+ \to D_{fav}K^+)} = 1 + r_B^2 \pm 2r_B \cos(\delta_B)\cos(\phi_3), \tag{1.1}$$

$$\mathscr{A}_{CP^\pm} = \frac{\Gamma(B^- \to D_{CP^\pm}K^-) - \Gamma(B^+ \to D_{CP^\pm}K^+)}{\Gamma(B^- \to D_{CP^\pm}K^-) + \Gamma(B^+ \to D_{CP^\pm}K^+)} = \pm r_B \sin(\delta_B)\sin(\phi_3)/R_{CP^\pm}, \tag{1.2}$$

where D_{fav} signifies that the D meson is reconstructed in a favored hadronic mode such as $D^0 \to K^-\pi^+$, r_B is the magnitude of the ratio of $B \to \bar{D}^0K^-$ and $B \to D^0K^-$ amplitudes and δ_B is the strong phase difference between them. A value of $\mathscr{A}_{CP^\pm}$ different from 0 means CP is violated in these processes.

Notice that the system is not over-constrained by the four observables even though there are three parameters, as $\mathscr{R}_{CP^+}\mathscr{A}_{CP^+} = -\mathscr{R}_{CP^-}\mathscr{A}_{CP^-}$.

1.2 ADS Method

Atwood, Dunietz and Soni [6] realized that CKM-suppressed decays can also be used to measure ϕ_3. Let us consider the process $B^- \to [K^+\pi^-]_D K^-$, where the brackets represent a final state that was produced from an intermediate D resonance. The full final state can be reached in two ways — either CKM-favored $B^- \to D^0 K^-$ followed by CKM-suppressed $D^0 \to K^+\pi^-$, or CKM-suppressed $B^- \to \bar{D}^0 K^-$ followed by CKM-favored $D^0 \to K^-\pi^+$.

In contrast to the GLW method, when the D meson decays to a non-CP-eigenstate, one has to factor in the magnitude of the ratio of the suppressed and favored D decays r_D as well as their relative strong phase δ_D, much like we did for the B decay. Fortunately, these hadronic parameters can be obtained from mixing measurements [7].

The ADS observables are similar to the GLW ones, however there are only two of them per B decay channel as they are charge-averaged,

$$\begin{aligned}\mathscr{R}_{ADS} &= \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^-\pi^+]_D K^-) + \Gamma(B^+ \to [K^+\pi^-]_D K^+)} \\ &= r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D)\cos(\phi_3),\end{aligned} \tag{1.3}$$

$$\begin{aligned}\mathscr{A}_{ADS} &= \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) - \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)} \\ &= 2 r_B r_D \sin(\delta_B + \delta_D)\sin(\phi_3)/\mathscr{R}_{ADS}.\end{aligned} \tag{1.4}$$

1.3 GGSZ Method

The Dalitz plot analysis method for ϕ_3 measurements was proposed by Giri, Grossman, Soffer and Zupan [8] and independently by Bondar [9]. The idea behind it is to use (usually self-conjugate) multi-body final states accessible to both D^0 and $\bar{D}^0$ mesons. One then measures relative phases and magnitudes of their amplitudes for D mesons coming from B decays such as $B^\pm \to DK^\pm$.

Let us consider a $B^\pm \to [K_S^0\pi^+\pi^-]_D K^\pm$ process. Its amplitude can be written as

$$A_{B^+}(s_+, s_-) = \bar{A}_D(s_+, s_-) + r_B e^{i(\delta_B + \phi_3)} A_D(s_+, s_-), \tag{1.5}$$

$$A_{B^-}(s_+, s_-) = A_D(s_+, s_-) + r_B e^{i(\delta_B + \phi_3)} \bar{A}_D(s_+, s_-), \tag{1.6}$$

where we have introduced the Dalitz variables $s_\pm = m^2_{K_S^0\pi^\pm}$ and A_D is the amplitude of the D decay. The strong phase $\delta_B \equiv \delta_B(s_+, s_-)$ has to have a large variation over the Dalitz plot; if it were constant, there would be no ϕ_3 sensitivity.

Two approaches are possible — model independent, binned analysis (as proposed in the original paper) and model dependent, unbinned analysis. The former divides the Dalitz plot into bins across which there is a small strong phase variation. The events in one bin are then treated equally. Extra input in the form of strong phase measurements from charm factories is required for this method [10].

The second method employs a certain model of the strong phase distribution function across the Dalitz phase-space. While introducing an obvious model uncertainty, it has a higher statistical power, which can be very desirable in low yield analyses. The above-mentioned models can be determined from samples of flavor tagged $D^{*\pm} \to D^0\pi^\pm$.

2. Recent Belle Analyses

All the analyses presented use the entire $711\,\mathrm{fb}^{-1}$ Belle data sample containing $772 \times 10^6 B\bar{B}$ events. Listed uncertainties are statistical first, systematic second.

2.1 $B^- \to D^0 K^-, D^0 \to K_S^0 \pi^+ \pi^-$ (GGSZ)

This analysis is the first model-independent measurement of ϕ_3 in the relevant channel. The reported result is [11]

$$\phi_3 = (77^{+15.1}_{-14.9} \pm 4.1 \pm 4.3)^\circ, \tag{2.1}$$

where the third uncertainty comes from the precision of the strong-phase parameters obtained by CLEO, which are an external input to this analysis.

This uncertainty is comparable to the model uncertainties from the latest Belle and BaBar measurements: $3^\circ - 9^\circ$. For future experiments, the model uncertainty is expected to dominate as there will be more statistics and, possibly, better systematics control. On the other hand, the precision of the strong-phase parameters measurement will improve as BES-III results [12] supersede CLEO's.

2.2 $B^- \to D^{*0} K^-, D^{*0} \to D^0 \pi^0, D^0 \gamma$ (GLW and ADS)

These two analyses are currently exclusive to B-factories as they involve low energy π^0 or γ, making them very difficult for hadron collider experiments.

Combining GLW results for $D^* \to D\pi^0, D\gamma$ yields

$$\begin{aligned} R_{CP+} &= +1.19 \pm 0.13 \pm 0.03, \\ R_{CP-} &= +1.03 \pm 0.13 \pm 0.03, \\ A_{CP+} &= -0.14 \pm 0.10 \pm 0.01, \\ A_{CP-} &= +0.22 \pm 0.11 \pm 0.01. \end{aligned}$$

The ADS analysis results are

$$\begin{aligned} R_{D\pi^0} &= [1.0^{+0.8+0.1}_{-0.7-0.2}] \times 10^{-2}, \\ R_{D\gamma} &= [3.6^{+1.4}_{-1.2} \pm 0.2] \times 10^{-2}, \\ A_{D\pi^0} &= 0.4^{+1.1+0.2}_{-0.7-0.1}, \\ A_{D\gamma} &= -0.51^{+0.33}_{-0.29} \pm 0.08. \end{aligned}$$

Here the signal of the $D^* \to D\gamma$ mode is seen with a 3.5σ significance. An interesting feature of this ADS analysis is that the values of R are different for $D^* \to D\gamma$ and $D^* \to D\pi^0$ modes, because of the π strong phase difference between the two amplitudes.

The analyses are to be published soon [13].

2.3 $B^- \to D^0 K^-, D^0 \to K^+ \pi^- \pi^0$ (ADS)

The fact that there is a three-body final-state in this ADS analysis means that the strong phase difference between the interfering processes can vary over the phase-space. This can "dilute" direct CP violation effects. To quantify this dilution, correlated $D\bar{D}$ production is used [14, 15]. Conveniently, the effect is small in this channel, which allows for a more precise CP violation measurement [16]:

$$R_{DK} = (1.98 \pm 0.62 \pm 0.24) \times 10^{-2},$$
$$A_{DK} = 0.41 \pm 0.30 \pm 0.05.$$

This is a first evidence for the suppressed decay, with a 3.2σ significance. It is noteworthy that even when the dilution is rather large, an analysis can be feasible as evidenced by [17].

2.4 $B^0 \to D^0 K^{*0}, K^{*0} \to K^+\pi^-, D^0 \to K^-\pi^+$ (ADS)

A major advantage of this analysis is that it uses a self-tagging channel, because of the K^* decay. This in turn boosts efficiency. While the study uses the ADS method, the fact that K^* has a natural width larger than the experimental resolution, leads to some complications, which require a modified definition of the ADS observables. Further details can be found in [18]. The obtained result is [19]

$$R_{DK^{*0}} = (4.1^{+5.6+2.8}_{-5.0-1.8}) \times 10^{-2}. \tag{2.2}$$

As the $R_{DK^{*0}}$ value is not significant, an upper limit is established

$$R_{DK^{*0}} < 0.16 \quad (95\% \text{ C.L.}). \tag{2.3}$$

2.5 $B^0 \to D^0 K^{*0}, K^{*0} \to K^+\pi^-, D^0 \to K^0_S\pi^+\pi^-$ (GGSZ)

This analysis also uses the model-independent approach and resulted in establishing an upper limit on the suppressed vs. favored ratio $r_S < 0.87$ at the 68% confidence level [20]. This value is closely related to ϕ_3 sensitivity because the statistical uncertainty of ϕ_3 measurements is proportional to $1/r_S$.

The authors also report the "raw" observables $x_\pm$, $y_\pm$ which are defined as

$$x_\pm = r_S \cos(\delta_S \pm \phi_3),$$
$$y_\pm = r_S \sin(\delta_S \pm \phi_3)$$

and are measured to be

$$x_- = +0.4^{+1.0+0.0}_{-0.6-0.1} \pm 0.0,$$
$$y_- = -0.6^{+0.8+0.1}_{-1.0-0.0} \pm 0.1,$$
$$x_+ = +0.1^{+0.7+0.0}_{-0.4-0.1} \pm 0.1,$$
$$y_+ = +0.3^{+0.5+0.0}_{-0.8-0.1} \pm 0.1,$$

where the third uncertainty again comes from the precision of the strong-phase parameters, as in Sec. 2.1.

These observables have the benefit, that they can be readily merged with similar results from other studies for a combined measurement.

3. Time-dependent Measurements

All the measurements mentioned in the previous Section were time-independent. However, time-dependent analyses are also possible. Namely channels

- $B^0 \to D^{(*)}\pi$ [21],
- $B^0 \to D^{(*)}\rho$,

can be used to constrain ϕ_3, by giving us access to $2\phi_1 + \phi_3$. All of these processes are subject to *mixing induced CP violation*, which arises from interference between different decay paths that lead to the same final state, i.e., $B^0 \to f$ and $B^0 \to \bar{B}^0 \to f$.

In order to extract the weak angle from $B \to$ (scalar-scalar) or (scalar-vector) decays, some model parameters must be supplied externally, because they cannot be extracted by the analyses. Concretely, the ratio of amplitude magnitudes $r = |A_{\rm DCS}/A_{\rm CF}|$, where DCS and CF stand for *doubly Cabibbo suppressed* and *Cabibbo favored* respectively, must come from other measurements. To this end, one can use decays such as $B_s^0 \to D_s^{(*)}\pi$ with the added assumption of $SU(3)$ symmetry. This approach, however, is burdened by a theoretical uncertainty on the assumption.

3.1 $D^*\rho$ Introduction

From the listed channels, $B^0 \to D^*\rho$ is unique, because it is a $B \to$ (vector-vector) decay. Therefore, there are three possible helicity/transversity configurations of the $D^*\rho$ state, its amplitude being the superposition

$$A = \sum_{\lambda \in \{+,0,-\}} H_\lambda = \sum_{\lambda \in \{\parallel,0,\perp\}} A_\lambda .$$

This form gives rise to interference terms in the decay rate $\Gamma = |A|^2$, which means more information about the A_λ terms is preserved. We can take advantage of this and in principle extract all the parameters, including r_λ, from data. This is very valuable, not only because we don't include a hard to quantify theoretical uncertainty, but also combined with, e.g., $B_s^0 \to D_s^*\rho$, this measurement can be used to probe the $SU(3)$ assumption.

Let us mention a rather important technical caveat, which forces us to make a change of variables. r_λ appears in the decay rate formula in the following expressions

$$\rho_\lambda = r_\lambda e^{i(-2\phi_1 - \phi_3 + \delta_\lambda)}, \qquad \bar{\rho}_\lambda = r_\lambda e^{i(+2\phi_1 + \phi_3 + \delta_\lambda)},$$

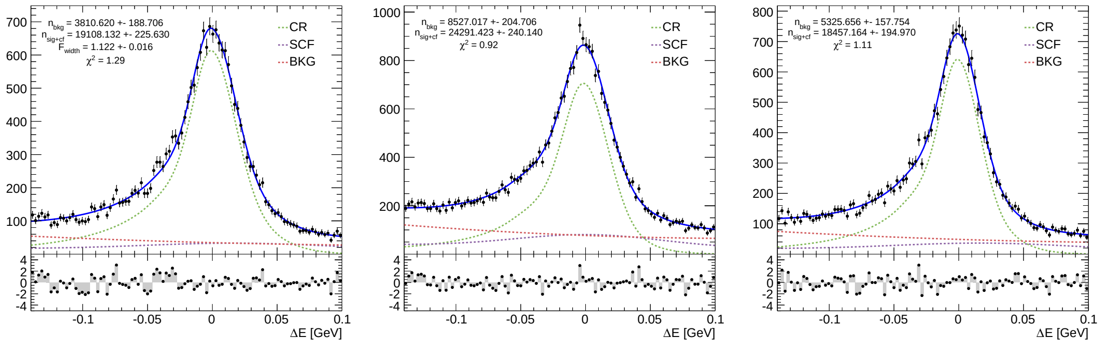
where $2\phi_1 + \phi_3$ is the weak phase and δ_λ are the strong phases. It is apparent that $r_\lambda = 0$ is a pole in the *sensitivity* of other variables. When the r_λ are small, fitting and error estimation can fail. To avoid this problem we introduce Cartesian variables defined as follows

$$\rho_\lambda = x_\lambda + i y_\lambda, \qquad \bar{\rho}_\lambda = \bar{x}_\lambda + i \bar{y}_\lambda .$$

The change $\{r_\lambda, \delta_\lambda, \phi_w\} \to \{x_\lambda, y_\lambda, \bar{x}_\lambda, \bar{y}_\lambda\}$ introduces five new variables and an additional step is needed to extract the physical parameters.

3.2 $D^*\rho$ Current Status

We are currently conducting an analysis of this channel and have reached several milestones. We finalized our signal selection algorithm, continuum suppression and yield extraction. The signal yield extraction fitter was tested on the official Belle MC simulation dataset (see figure 2). This sample was produced using the `EvtGen` [22] and `GEANT3` [23] software packages to generate the events and to simulate the detector response, respectively. The fitter was also validated using real data (with modification to account for MC–data differences). Close to 6×10^4 signal events pass our selection. This might seem like high statistics, however, $r = |A_{\mathsf{DCS}}/A_{\mathsf{CF}}|$ is expected to be very small at $\sim 1-2\%$, and the CP violation effect tiny.



(a) $D^0 \to K\pi$ **(b)** $D^0 \to K\pi\pi^0$ **(c)** $D^0 \to K\pi\pi\pi$

Figure 2: Monte Carlo yield fits for a sample equivalent to the Belle dataset

Now we want to present first results from a realistic angular time-dependent fit to a signal Monte Carlo sample. The sample was generated using these settings:

- 6×10^4 events passing selection
- helicity amplitudes taken from CLEO [24]
- conservative $r_\lambda = 1\%$
- randomly chosen values of strong phases:
  - $\delta_+ = -0.393$
  - $\delta_0 = 0.785$
  - $\delta_- = 1.571$
- weak phase $2\phi_1 + \phi_3 = 1.79371$

Results from the fit to this dataset are listed in table 1. The most interesting values we obtain are the uncertainties labeled σ. They allow us to make an estimation of the final statistical uncertainty of the analysis.

We mentioned in section 3.1 that one must make an additional step to extract the physical parameters from our observables. It is a rather involved procedure called *constrained supremum method* and we won't discuss it here. Suffice to say, it is a robust way to extract $2\phi_1+\phi_3$ from x_λ

and y_λ. Plots from a simpler toy Monte Carlo study validating the algorithm can be seen in figure 3. More information will be available in [25].

Adjusting results from the aforementioned toy study for expected yield, resolution and flavor tagging effects we get the following expected uncertainties

- $\sigma(2\phi_1 + \phi_3) \approx 80^\circ(\text{stat})$ for Belle
- $\sigma(2\phi_1 + \phi_3) \approx 11^\circ(\text{stat})$ for Belle II at 50 ab^{-1}

Let us stress that we expect these uncertainties in the case of no external input whatsoever. If we wish so, we can trade the analysis' theoretical 'purity' for decreased statistical uncertainty, if we take, e.g., r_λs from other measurements.

Var	Fit	σ
$\lvert a_\parallel \rvert$	0.2861	0.0022
$(a_\parallel)$	0.5919	0.0130
$\lvert a_0 \rvert$	0.9306	0.0008
$(a_\perp)$	3.1159	0.0110
$x_\parallel$	0.0530	0.0222
x_0	0.0640	0.0137
$x_\perp$	0.0700	0.0228
$y_\parallel$	0.0109	0.0143
y_0	-0.0131	0.0046
$y_\perp$	-0.0369	0.0225
$\bar{x}_\parallel$	0.0669	0.0240
$\bar{x}_0$	0.0829	0.0114
$\bar{x}_\perp$	0.0530	0.0249
$\bar{y}_\parallel$	0.0123	0.0168
$\bar{y}_0$	0.0055	0.0046
$\bar{y}_\perp$	0.0297	0.0235

Table 1: Results from a realistic toy fit

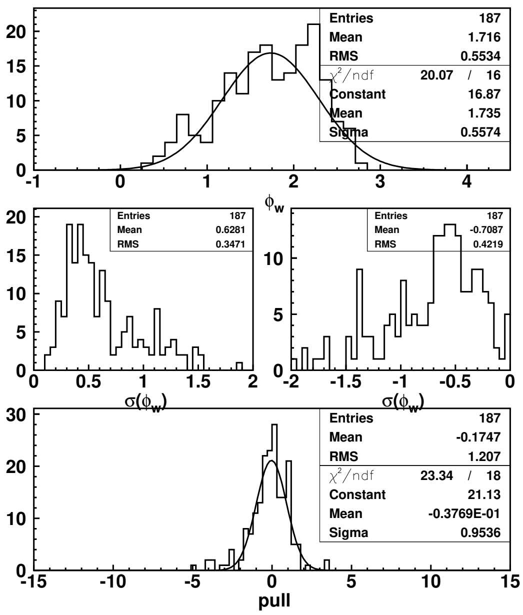


Figure 3: Constrained supremum toy study

4. Belle II Prospects

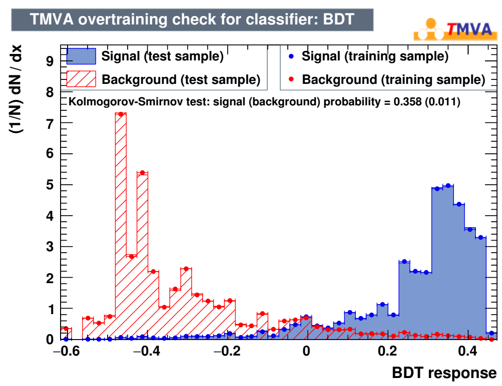
4.1 Sensitivity Simulation

Recently, a model-independent GGSZ sensitivity simulation study was conducted on the $B \to [K_S h^+ h^-]_D K$ decay. Its goal was to establish a realistic prediction of the mode's sensitivity to ϕ_3. It incorporates the full Belle II simulation with particle reconstruction, signal yield fit (see figure 4b), etc. However, the study was not finalized and several aspects such as continuum suppression (see figure 4a) and particle identification were not fully tuned. We, therefore, expect better performance from the actual analysis.

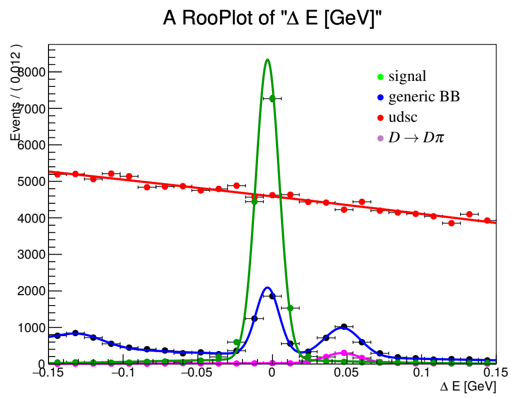
The resulting expected statistical ϕ_3 uncertainty for a measurement of this decay with Belle II can be seen in figure 4c. The values for two milestone data sample sizes are

- $\sigma(\phi_3) = 9.5^\circ$ with 10 ab^{-1}
- $\sigma(\phi_3) = 2.9^\circ$ with 50 ab^{-1}

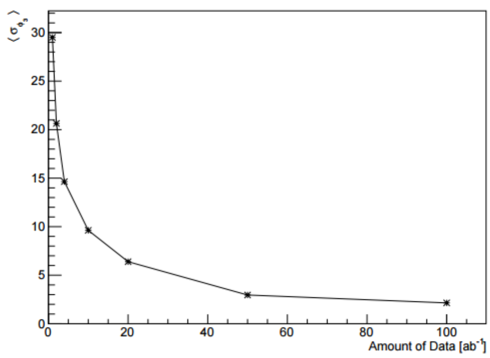
The study shows that the overall precision improvement with increasing luminosity should be as expected, even though it's estimates are incomplete and conservative. Improvement on the actual value of the uncertainty is to be expected.



(a) Continuum suppression



(b) Signal yield fit



(c) Estimated ϕ_3 uncertainty

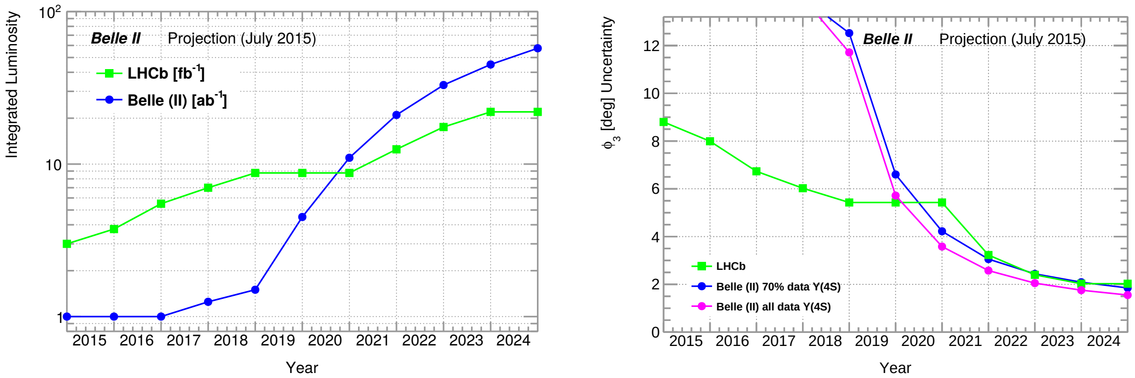
Figure 4: $B \to [K_S h^+ h^-]_D K$ sensitivity simulation

4.2 ϕ_3 Combination Projection

In this last section, we want to present estimates of the total Belle II ϕ_3 sensitivity and its comparison to LHCb's. For Belle II the projections are based on a combination of $B \to D^{(*)}K^{(*)}$ measurements that were already performed at Belle and the combined ϕ_3 value is taken from CKM-Fitter. LHCb values are based on LHCb-PAPER-2014-041. It is a combination of measurements $B^+ \to D\pi^+$, $B^+ \to DK^+$ and $B^0 \to DK^{*0}$. Combined value was taken from CKMFitter. For both experiments, the following D decay modes were considered

- $D \to KK$, $D \to \pi\pi$, $D \to K\pi$
- $D \to K\pi\pi\pi$
- $D \to K_S\pi\pi$

The expected integrated luminosity used for the comparison and the resulting ϕ_3 sensitivity in can be seen in figure 5. It is important to note that both experiments will include more channels in their ϕ_3 measurements. Work is ongoing to estimate the final sensitivity more precisely.



(a) Integrated luminosity **(b)** Estimated ϕ_3 uncertainty

Figure 5: $B \to [K_S h^+ h^-]_D K$ sensitivity simulation

5. Conclusion

Searches looking for New Physics (NP) in ϕ_3 usually compare tree-level processes with penguin dominated ones. However, recent studies show that NP contributions to tree-level C_1 and C_2 Wilson coefficients of the order of $\mathcal{O}(40\%)$ and $\mathcal{O}(20\%)$ are not excluded [26]. A rough estimate shows that deviations in ϕ_3 of $\mathcal{O}(4^\circ)$ are consistent with current experimental constraints. As you can see, there is a strong motivation for both theoretical and experimental study of ϕ_3.

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