

New Dimensions from Gauge-Higgs Unification

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The Higgs boson is unified with gauge fields in the gauge-Higgs unification. The $SO(5) \times U(1)$ gauge-Higgs electroweak unification in the Randall-Sundrum warped space yields almost the same phenomenology at low energies as the standard model, and gives many predictions for the Higgs couplings and new W', Z' bosons around $6 \sim 8$ TeV, which can be tested at 14 TeV LHC. The gauge-Higgs grand unification is achieved in $SO(11)$ gauge theory. It suggests the existence of the sixth dimension (GUT dimension) in addition to the fifth dimension (electroweak dimension). The proton decay is naturally suppressed in the gauge-Higgs grand unification.

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1. Standard model

The Standard Model (SM) is very successful at low energies. It is gauge theory of $SU(3)_C \times SU(2)_L \times U(1)_Y$, whose Lagrangian density consists of four parts;

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{Yukawa}} , \\
\mathcal{L}_{\text{gauge}} &= -\frac{1}{2}\text{Tr} G_{\mu\nu} G^{\mu\nu} - \frac{1}{2}\text{Tr} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} , \\
\mathcal{L}_{\text{Higgs}} &= |D_\mu \Phi|^2 - V[\Phi] , \\
\mathcal{L}_{\text{fermion}} &= \sum \bar{\psi}_j i \gamma^\mu D_\mu \psi_j , \\
\mathcal{L}_{\text{Yukawa}} &= \sum \{ y_{jk}^d \bar{\psi}_j \Phi \psi_k + y_{jk}^u \bar{\psi}_j \tilde{\Phi} \psi_k \} + (h.c.) .
\end{aligned} \tag{1.1}$$

The form of the parts $\mathcal{L}_{\text{gauge}}$ and $\mathcal{L}_{\text{fermion}}$ is determined by the gauge principle, and is beautiful. The form of the Higgs potential $V[\Phi]$ in $\mathcal{L}_{\text{Higgs}}$, however, is given in ad hoc manner. The Yukawa couplings $y_{jk}^{u,d}$ in $\mathcal{L}_{\text{Yukawa}}$ are arbitrary as well. The parts $\mathcal{L}_{\text{Higgs}}$ and $\mathcal{L}_{\text{Yukawa}}$ lack a principle.

The electroweak (EW) gauge symmetry breaking in the SM is brought about by an intentional choice of $V[\Phi]$ which is assumed to have a global minimum at $\langle \Phi \rangle \neq 0$. In other words, the EW gauge symmetry breaking is enforced by hand. The Higgs boson remains mysterious in the SM.

2. Gauge-Higgs unification

In the gauge-Higgs unification one starts with gauge theory in higher dimensions.[1, 2, 3] The Higgs field becomes a part of the extra-dimensional component of gauge fields. Schematically

$$\begin{aligned}
\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} &\implies \mathcal{L}_{\text{gauge}}^{5d} , \\
\mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{Yukawa}} &\implies \mathcal{L}_{\text{fermion}}^{5d} .
\end{aligned} \tag{2.1}$$

The effective Higgs potential is generated dynamically at the quantum level from $\mathcal{L}_{\text{gauge}}^{5d} + \mathcal{L}_{\text{fermion}}^{5d}$. In short, the theory is governed by the gauge principle, and becomes concise and beautiful.[4, 5]

In the gauge-Higgs unification in five dimensions (x^μ, y)

$$A_M = \begin{cases} A_\mu \supset \gamma, W, Z \\ A_y \supset \text{Higgs boson} \sim \text{Aharonov-Bohm (AB) phase } \theta_H \end{cases} \tag{2.2}$$

When the fifth dimension is not simply connected, the Higgs field appears as an Aharonov-Bohm phase θ_H in the fifth dimension. The effective potential $V_{\text{eff}}(\theta_H)$ becomes nontrivial at the one-loop level. When $V_{\text{eff}}(\theta_H)$ is minimized at $\theta_H \neq 0$, the EW symmetry is dynamically broken. Finite Higgs boson mass is generated. The gauge-hierarchy problem is solved.

3. $SO(5) \times U(1)$ gauge-Higgs EW unification

The Randall-Sundrum (RS) warped space is specified with the metric

$$ds^2 = e^{-2\sigma(y)} dx^\mu dx_\mu + dy^2 ,$$

$$\begin{aligned}\sigma(y) &= \sigma(-y) = \sigma(y+2L), \\ \sigma(y) &= k|y| \quad \text{for } |y| \leq L.\end{aligned}\tag{3.1}$$

The RS space has topology of $M^4 \times (S^1/Z_2)$, in which $(x^\mu, y), (x^\mu, -y)$ and $(x^\mu, y+2L)$ are identified. Its fundamental region is 5d AdS space sandwiched by UV and IR branes, at $y_0 = 0$ and $y_1 = L$. The 5d cosmological constant is given by $\Lambda = -6k^2$. The $SO(5)$ and $U(1)_X$ gauge fields, A_M and B_M , satisfy

$$\begin{aligned}\begin{pmatrix} A_\mu \\ A_y \end{pmatrix}(x, y_j - y) &= P_j \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}(x, y_j + y) P_j^{-1}, \quad P_j \in SO(5), \quad P_j^2 = 1, \\ \begin{pmatrix} B_\mu \\ B_y \end{pmatrix}(x, y_j - y) &= \begin{pmatrix} B_\mu \\ -B_y \end{pmatrix}(x, y_j + y),\end{aligned}\tag{3.2}$$

Although gauge potentials themselves are not single-valued, physical gauge-invariant quantities are single-valued.[6]-[10]

The set of the matrices P_0, P_1 is called the orbifold boundary condition. We take

$$P_0 = P_1 = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & +1 \end{pmatrix},\tag{3.3}$$

by which gauge symmetry $\mathcal{G} = SO(5) \times U(1)_X$ is reduced to $\mathcal{H} = SO(4) \times U(1)_X$. Zero modes (parity even-even modes) appear in the \mathcal{H} part of A_μ, B_μ , and in the \mathcal{G}/\mathcal{H} part of A_y . The latter is an $SO(4) \simeq SU(2)_L \times SU(2)_R$ vector, or an $SU(2)_L$ doublet, corresponding to the 4d Higgs field in the SM.

Quark-lepton multiplets are introduced in the vector representation of $SO(5)$ in the bulk. In addition, one introduces dark fermions in the spinor representation in the bulk. On the UV brane at $y = 0$ brane fermions in $SU(2)_L$ doublet and a brane scalar Φ in $SU(2)_R$ doublet are introduced. The brane scalar Φ spontaneously breaks $SU(2)_R \times U(1)_X$ to $U(1)_Y$, and at the same time gives rise to additional mass terms for fermions. The resultant spectrum at low energies (< 1 TeV) is that of the SM. The effective potential $V_{\text{eff}}(\theta_H)$ is evaluated at the one loop. Contributions from the top quark multiplet and dark fermions triggers the EW gauge symmetry breaking with a Higgs boson mass $m_H = 125$ GeV.

4. Success

The $SO(5) \times U(1)_X$ gauge-Higgs unification is successful. The gauge principle governs the theory, including dynamics of the 4d Higgs boson.[10]-[24]

(1) The 4d Higgs boson, the four-dimensional fluctuation mode of the AB phase θ_H in the fifth dimension, is massless at the tree level but acquires a nonvanishing mass at the one loop level

which is free from divergence and independent of regularization methods employed. The gauge hierarchy problem, a cumbersome problem in many theories, is naturally solved.

(2) The phenomenology at low energies (≤ 1 TeV) for $\theta_H < 0.1$ is almost the same as in the SM.

(3) There is no vacuum instability problem associated with the 4d Higgs scalar field.[25] The effective potential for the 4d Higgs field $H(x)$ is given by $V_{\text{eff}}(\theta_H + H/f_H)$. The large gauge invariance guarantees the periodicity $V_{\text{eff}}(\theta_H + 2\pi) = V_{\text{eff}}(\theta_H)$, which in turn implies that there never occurs the instability. It has been explicitly shown that $V_{\text{eff}}(\theta_H)$ is finite at the one loop level.

(4) Dynamical EW symmetry breaking takes place in the RS space. The existence of a heavy quark, the top quark $m_t > m_W$, is crucial. $V_{\text{eff}}(\theta_H)$ is controlled by the W and Z bosons, the top quark multiplet, and the dark fermions. Light quarks and leptons multiplets are irrelevant for the EW symmetry breaking in the RS space.

5. Predictions

The gauge-Higgs unification gives many predictions to be confirmed by the forthcoming and future experiments. Although the model contains several parameters, most of physical quantities are determined by the AB phase θ_H .

(a) The Yukawa couplings of quarks and leptons, Y_α , the three-point couplings of the Higgs boson to W, Z bosons, g_{HWW}, g_{HZZ} , are given, in good approximation, by[8, 11]

$$Y_\alpha, g_{HWW}, g_{HZZ} \simeq (\text{SM values}) \times \cos \theta_H. \quad (5.1)$$

The deviation from the SM is less than 1% for $\theta_H < 0.1$.

(b) Decay of the Higgs boson to $\gamma\gamma, Z\gamma$, and two gluons take place through one-loop diagrams. In the gauge-Higgs unification an infinite number of various Kaluza-Klein (KK) modes run inside the loop. (Fig. 1) Each of their contributions gives $O(1/n)$ correction to the decay width where n is the KK number. There appears miraculous cancellation among them so that the sum of all contributions turns out finite and small. It gives less than 1% correction to those in which SM particles run inside the loop for $\theta_H < 0.1$. The cancellation in the process $H \rightarrow Z\gamma$ is highly nontrivial, as the KK number can change inside the loop.[10, 19]



Figure 1: Diagrams contributing to $H \rightarrow \gamma\gamma, Z\gamma$. The infinite sums converge and give small corrections to the SM. There are diagrams in which the top quark and its KK tower run inside the loops.

(c) An immediate consequence of (a) and (b) is that both the production rate of the Higgs boson at LHC and decay widths of the Higgs boson to various modes are all suppressed in good approximation by a factor $\cos^2 \theta_H$ compared to those in the SM. Branching fractions of various decay modes

are nearly the same as in the SM. The signal strengths of the various decay modes are suppressed by a factor $\cos^2 \theta_H$. For $\theta_H < 0.1$ the deviation from the SM is less than 1%.

(d) The Higgs cubic and quartic self-couplings, λ_3^H, λ_4^H , deviate from those in the SM, which can be tested in future. Although the model has several parameters to be fixed, many of physical quantities such as λ_3^H, λ_4^H , the KK mass scale m_{KK} , and the masses of the first KK modes $\gamma^{(1)}, Z^{(1)}, W^{(1)}$ depend only on θ_H in very good approximation. This property is called the universality. (Fig. 2)

The universality leads to strong prediction power in the gauge-Higgs unification. Suppose that the first KK mode $Z^{(1)}$ is found at m_0 . From the relation $m_{Z^{(1)}}(\theta_H) = m_0$, the value θ_H is determined. Then other quantities $\lambda_3^H(\theta_H), \lambda_4^H(\theta_H), m_{W^{(1)}}(\theta_H)$ etc. are determined, and can be checked experimentally.

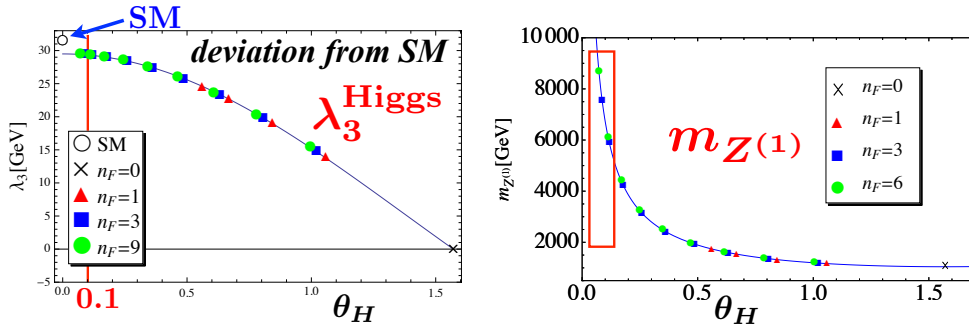


Figure 2: Universality. $\lambda_3^H, \lambda_4^H, m_{\text{KK}}, m_{Z^{(1)}}, m_{\gamma^{(1)}}, m_{W^{(1)}}$ etc. are determined by θ_H , almost independent of the details of the model. In particular, they do not depend on the number n_F of dark fermions.

(e) The prediction of Z' events gives the cleanest test of the model. (Fig. 3) The first KK modes of the photon, Z boson, and Z_R boson appear as Z' events. (Z_R is associated with $SU(2)_R$, and has no zero mode.) For $\theta_H = 0.114$, their masses are $(m_{Z_R^{(1)}}, m_{Z^{(1)}}, m_{\gamma^{(1)}}) = (5.73, 6.07, 6.08)$ TeV and the widths are $(\Gamma_{Z_R^{(1)}}, \Gamma_{Z^{(1)}}, \Gamma_{\gamma^{(1)}}) = (482, 342, 886)$ GeV. For $\theta_H = 0.073$, their masses are $(m_{Z_R^{(1)}}, m_{Z^{(1)}}, m_{\gamma^{(1)}}) = (8.00, 8.61, 8.61)$ TeV and the widths are $(\Gamma_{Z_R^{(1)}}, \Gamma_{Z^{(1)}}, \Gamma_{\gamma^{(1)}}) = (553, 494, 1040)$ GeV.[24]

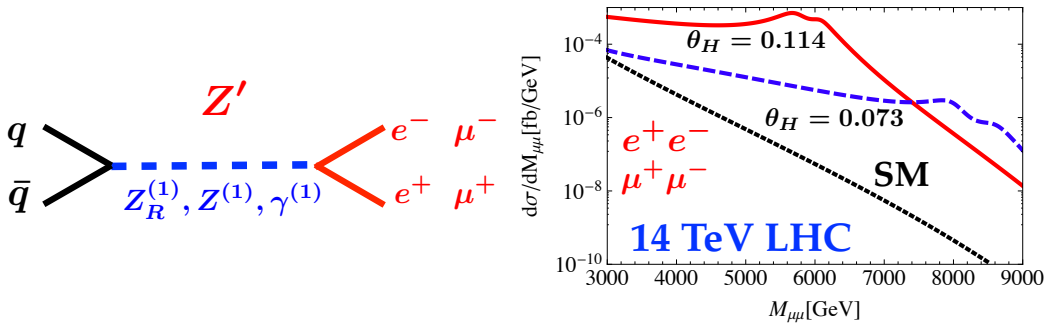


Figure 3: Z' production at LHC.

6. $SO(11)$ gauge-Higgs grand unification

It is necessary to incorporate strong interactions in the framework of gauge-Higgs unification. This leads to gauge-Higgs grand unification.[26]-[37] We look for a scenario in which the EW Higgs boson appears as the extra-dimensional component of gauge potentials, and electromagnetic, weak, and strong interactions are unified in a single group, and no exotic particles appears at low energies.

One might think that the gauge group should contain $SO(5) \times U(1)_X \times SU(3)_C$ as a subgroup. This turns out not to be the case. It is seen that $SO(11)$ gauge theory does a job, keeping good features of the $SO(5) \times U(1)_X$ gauge-Higgs EW unification.[34, 36]

One starts with $\mathcal{G} = SO(11)$ gauge theory in the Randall-Sundrum warped space (3.1). The orbifold boundary condition is given by

$$\begin{aligned} P_0^{\text{vec}} &= \text{diag}(I_{10}, -I_1), & P_1^{\text{vec}} &= \text{diag}(I_4, -I_7), \\ P_0^{\text{sp}} &= I_{16} \otimes \sigma^3, & P_1^{\text{sp}} &= I_2 \otimes \sigma^3 \otimes I_8 \end{aligned} \quad (6.1)$$

in vectorial and spinorial representations. At the UV brane $SO(11)$ is broken to $SO(10)$ by P_0 , whereas at the IR brane it is broken to $SO(4) \times SO(7)$. As a whole $\mathcal{G} = SO(11)$ is broken to $\mathcal{H} = SO(4) \times SO(6)$. Note that $SO(4) \simeq SU(2)_L \times SU(2)_R$, and $SO(6) \simeq SU(4)$. At this stage A_μ has zero modes in the block \mathcal{H} . On the other hand A_y has zero modes in the block $[\mathcal{G}/SO(10)] \cap [\mathcal{G}/SO(4) \times SO(7)]$. In the vectorial representation A_y has zero modes in the components A_y^{a11} ($a = 1 \sim 4$), which correspond to the 4d Higgs field in the SM. (Fig. 4)

$$A_\mu : \begin{pmatrix} \boxed{(+,+)} & (+,-) & (-,-) \\ & & \\ & \boxed{(+,+)} & (-,+) \\ & & \end{pmatrix} \quad A_y : \begin{pmatrix} (-,-) & (-,+) & \boxed{(+,+)} \\ & & \\ & (-,-) & (+,-) \\ & & \end{pmatrix}$$

Figure 4: $SO(11)$ gauge-Higgs grand unification. Parity $(P_0, P_1) = (+, +)$ modes appear in the $SO(4) \times SO(6)$ block of A_μ and in the $[SO(11)/SO(10)] \cap [SO(11)/SO(4) \times SO(7)]$ block of A_y, A_y^{a11} ($a = 1 \sim 4$).

On the UV brane a brane scalar Φ_{16} is introduced. Φ_{16} spontaneously breaks $SO(10)$ to $SU(5)$. As a result $\mathcal{G} = SO(11)$ is reduced to $\mathcal{G}_{\text{SM}} = SU(2)_L \times U(1)_Y \times SU(3)_C$. Note that $SU(3)_C \subset SO(6)$, and that $U(1)_Y$ is a combination of $SU(2)_R$ and $SO(6)$. \mathcal{G}_{SM} is dynamically broken to $U(1)_{\text{EM}} \times SU(3)_C$ through the Hosotani mechanism. The Weinberg angle at the GUT scale becomes $\sin^2 \theta_W = \frac{3}{8}$, the same value as in the $SU(5)$ or $SO(10)$ GUT in four dimensions. See the comparison of gauge-Higgs EW and grand unification in Fig. 5.

Fermions are introduced in the spinor (Ψ_{32}) and vector (Ψ_{11}) representations of $SO(11)$. Ψ_{32} ,

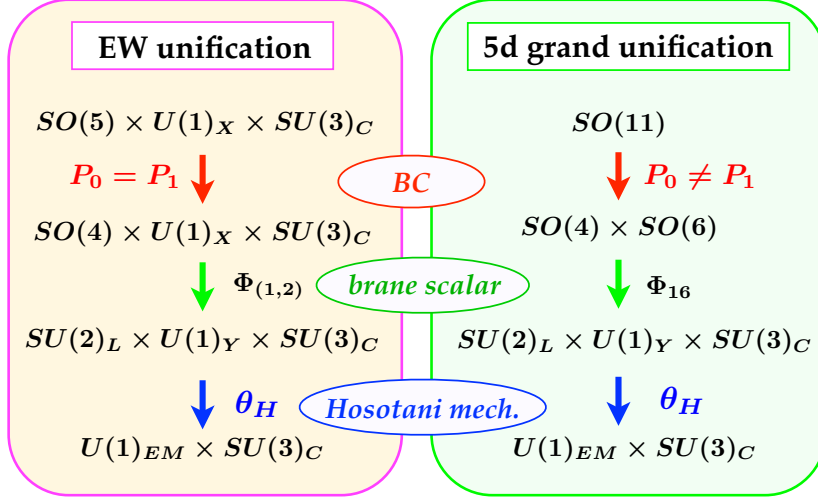


Figure 5: Comparison of gauge-Higgs EW and grand unification

for instance, satisfies $\Psi_{32}(x, y_j - y) = -\gamma^5 P_j^{\text{sp}} \Psi_{32}(x, y_j + y)$. The content of Ψ_{32} is given by

$$\Psi_{32} = \begin{pmatrix} \Psi_{16} \\ \Psi_{\overline{16}} \end{pmatrix}, \quad \Psi_{16} = \begin{pmatrix} v \\ e \\ \hat{e} \\ \hat{v} \\ u_k \\ d_k \\ \hat{d}_k \\ \hat{u}_k \end{pmatrix}, \quad \Psi_{\overline{16}} = \begin{pmatrix} v' \\ e' \\ \hat{e}' \\ \hat{v}' \\ u'_k \\ d'_k \\ \hat{d}'_k \\ \hat{u}'_k \end{pmatrix}, \quad (k = 1 \sim 3),$$

$$\text{zero modes : } \begin{pmatrix} v_L \\ e_L \end{pmatrix}, \begin{pmatrix} u_{kL} \\ d_{kL} \end{pmatrix}, \begin{pmatrix} v'_R \\ e'_R \end{pmatrix}, \begin{pmatrix} u'_{kR} \\ d'_{kR} \end{pmatrix}. \quad (6.2)$$

\hat{e} , \hat{u} , and \hat{d} fields have charges $+1$, $-\frac{2}{3}$, and $+\frac{1}{3}$, respectively. Zero modes appear only for the components of quarks and leptons. Vector multiplets Ψ_{11} are introduced to reproduce the mass spectrum of down-type quarks and leptons.

One interesting feature is that all quarks and leptons appear in Ψ_{32} as particles with the Ψ -fermion number $N_\Psi = +1$. N_Ψ is conserved even in the presence of Ψ_{11} . A proton has $N_\Psi = 3$, whereas $\pi^0 e^+$ has $N_\Psi = -1$. Thus the proton decay $p \rightarrow \pi^0 e^+$ is forbidden. This should be contrasted to the situation in the 4d GUT. In $SO(10)$ GUT in four dimensions a fermion multiplet is introduced in the spinor representation Ψ_{16} for left-handed fields. In the notation in (6.2), $(u_k, d_k) \rightarrow (u_{kL}, d_{kL})$ and $(\hat{u}_k, \hat{d}_k) \rightarrow (u^c_{kL}, d^c_{kL})$. As $u^c_{kL} \sim u_{kR}^\dagger$, gauge and Higgs interactions convert a particle to an anti-particle, which induces proton decay. In the gauge-Higgs grand unification such process is absent and the proton decay is naturally suppressed.

However, there is a problem. Careful examination reveals that in the first and second generations \hat{u} , \hat{d} , \hat{e} have light masses, which contradicts the observation. The source of this difficulty lies in the fact that the parity at $y_0 = 0$ and $y_1 = L$ is (even, odd) or (odd, even) for \hat{u} , \hat{d} , \hat{e} . In the RS

warped space it leads to light masses. In other words, $P_0 \neq P_1$ in the RS warped space gives rise to a trouble.

7. Gauge-Higgs grand unification in six dimensions

The difficulty is solved in gauge-Higgs unification in six-dimensional hybrid-warped space.[38] Consider the six-dimensional space with a metric

$$\begin{aligned} ds^2 &= e^{-2\sigma(y)}(dx^\mu dx_\mu + dv^2) + dy^2, \\ \sigma(y) &= \sigma(-y) = \sigma(y + 2L_5), \\ \sigma(y) &= k|y| \text{ for } |y| \leq L_5. \end{aligned} \quad (7.1)$$

We identify points

$$\begin{aligned} (x^\mu, y, v) &\sim (x^\mu, y + 2L_5, v) \sim (x^\mu, y, v + 2\pi R_6) \\ &\sim (x^\mu, -y, -v). \end{aligned} \quad (7.2)$$

The spacetime has topology of $M^4 \times (T^2/Z_2)$. The fundamental region can be taken as $\{0 \leq y \leq L_5, 0 \leq v < 2\pi R_6\}$. The metric (7.1) solves the Einstein equation with five-dimensional branes at $y = 0$ and $y = L_5$. Six-dimensional spacetime is an AdS space with $\Lambda = -10k^2$. The sixth dimension is curled up in a circle with a very small radius R_6 . We suppose that $z_L = e^{kL_5} \gg 1$ and

$$m_{\text{KK}_5} = \frac{\pi k}{e^{kL_5} - 1} \sim \pi k e^{-kL_5} \ll m_{\text{KK}_6} = \frac{1}{R_6}. \quad (7.3)$$

Under Z_2 parity $(y, v) \rightarrow (-y, -v)$, there appear four fixed points. (See Fig. 6.)

$$(y_0, v_0) = (0, 0), (y_1, v_1) = (L_5, 0), (y_2, v_2) = (0, \pi R_6), (y_3, v_3) = (L_5, \pi R_6). \quad (7.4)$$

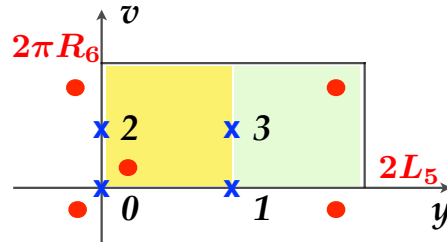


Figure 6: Four fixed points (in blue) in the 6 dimensional gauge-Higgs grand unification. The fundamental region is given by $0 \leq y \leq L_5, 0 \leq v < 2\pi R_6$. Red circle points represent a single spacetime point. Around each fixed point, parity is defined.

We consider $SO(11)$ gauge theory in the 6-dimensional hybrid-warped space (7.1). Gauge potentials A_M satisfy

$$\begin{pmatrix} A_\mu \\ A_y \\ A_v \end{pmatrix} (x, y_j - y, v_j - v) = P_j \begin{pmatrix} A_\mu \\ -A_y \\ -A_v \end{pmatrix} (x, y_j + y, v_j + v) P_j^{-1},$$

$$P_j \text{ or } -P_j \in SO(11), \quad P_j^2 = 1, \quad P_3 = P_1 P_0 P_2 = P_2 P_0 P_1. \quad (7.5)$$

Note that only three of the four P_j 's are independent, and the condition $P_1 P_0 P_2 = P_2 P_0 P_1$ must be satisfied for the consistency. We take, in place of (6.1),

$$\begin{aligned} P_0^{\text{vec}} &= P_1^{\text{vec}} = \text{diag}(I_4, -I_7), \\ P_2^{\text{vec}} &= P_3^{\text{vec}} = \text{diag}(I_{10}, -I_1). \end{aligned} \quad (7.6)$$

Fermion multiplets Ψ_{32} and Ψ_{11} are introduced in the bulk. Ψ_{32} is a 6d Weyl fermion, and satisfies $\Psi_{32}(x, y_j - y, v_j - v) = P_j^{\text{sp}} \bar{\gamma} \Psi_{32}(x, y_j + y, v_j + v)$ where $\bar{\gamma} = -i\Gamma^5 \Gamma^6$. With this boundary condition zero modes appear chiral, with the quark-lepton content given in (6.2). Furthermore, the lightest modes of hat fields $\hat{e}, \hat{d}, \hat{u}$ etc. have large masses of $O(R_6^{-1})$.

The symmetry breaking pattern is similar to the five-dimensional case. The orbifold boundary condition in the sixth dimension reduces $SO(11)$ to $SO(10)$, and the condition in the fifth dimension reduces $SO(11)$ to $SO(4) \times SO(7)$. A brane scalar $\Phi_{32}(x, v)$ is introduced on the five-dimensional UV brane at $y = 0$. It spontaneously breaks $SO(11)$ to $SU(5)$. As a result the SM symmetry $\mathcal{G}_{\text{SM}} = SU(2)_L \times U(1)_Y \times SU(3)_C$ survives. By the Hosotani mechanism the symmetry is further broken to $U(1)_{\text{EM}} \times SU(3)_C$. Zero modes of A_y correspond to the 4d Higgs doublet. There appear zero modes of A_v in the same $SO(11)$ components as A_y , which acquire masses of order gR_6^{-1} by the Hosotani mechanism.

8. Summary

The gauge-Higgs unification is promising. The $SO(5) \times U(1)$ gauge-Higgs EW unification gives definitive predictions to be tested in the forthcoming LHC experiments. The incorporation of strong interactions leads to the $SO(11)$ gauge-Higgs grand unification. It seems necessary to introduce the sixth dimension to have a spectrum consistent at low energies. The fifth dimension serves as an EW dimension, whereas the sixth dimension as a GUT dimension. We are entering into an era of ‘‘New Dimensions’’.

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