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Reduction of Couplings: Applications in Finite Theories and the MSSM

S. Heinemeyer
Campus of International Excellence UAM+CSIC, Cantoblanco, E–28049 Madrid, Spain,
Instituto de Física Teórica UAM-CSIC, C/ Nicolas Cabrera 13-15, E-28049 Madrid, Spain and
Instituto de Física de Cantabria (CSIC-UC), Avda. de Los Castros s/n, E-39005 Santander, Spain
E-mail: Sven.Heinemeyer@cern.ch

M. Mondragón*
Instituto de Física, Universidad Nacional Autónoma de México, A.P. 20-364, México 01000
E-mail: myriam@fisica.unam.mx

N. Tracas
Theoretical Physics Department, CERN, Geneva, Switzerland and
Physics Department, National Technical University, 157 80 Zografou, Athens, Greece
E-mail: ntrac@central.ntua.gr

G. Zoupanos
Theoretical Physics Department, CERN, Geneva, Switzerland and
Physics Department, National Technical University, 157 80 Zografou, Athens, Greece
E-mail: George.Zoupanos@cern.ch

The method of reduction of couplings is applied to a Finite Unified Theory and in the MSSM. We search for renormalization group invariant relations among couplings of a renormalizable theory which holds to all orders in perturbation theory. The method leads to relations, at the unification scale, between gauge and Yukawa couplings (in the dimensionless sectors of the theory) and relations among the couplings of the trilinear terms and the Yukawa couplings, as well as a sum rule among the scalar masses and the gaugino mass (in the soft breaking sector). In the Finite Unified Theory model we predict, with remarkable agreement with the experiment, the masses of the top and bottom quarks while our predictions for the light Higgs mass and the rest supersymmetric spectrum masses are in comfortable agreement with the LHC bounds on Higgs and supersymmetric particles. In the case of the reduced MSSM the predictions are less successful but recent improvements in the code used to calculate the Higgs masses give promises for better results.

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1. Introduction

The discovery, in 2012 at the LHC, of the last unobserved particle of the Standard Model (SM), namely the Higgs boson [1–4], confirms the mechanism of spontaneous electroweak breaking which explains the masses of the fundamental particles. Nevertheless, the hierarchy problem, the neutrino masses, the Dark Matter, the over twenty free parameters of the model, just to name some questions, ask for a more fundamental theory to answer some, if not all, of those.

Therefore, one of the main aims of this fundamental theory is to relate these free parameters, or rephrasing it, to achieve a *reduction of these parameters* in favour of a smaller number (or ideally only one). This reduction is usually based in the introduction of a larger symmetry rendering the theory more predictive. A very good example is the Supersymmetric Grand Unified Theories (SUSY GUTs) [5–11]. The case of minimal $SU(5)$ is one example, where the number of couplings is reduced to one due to the corresponding unification. Data from LEP [12] suggested that a $N=1$ global supersymmetry [10, 11] is required in order the prediction to be viable. Relation among the Yukawa couplings is also suggested in GUTs. For example, the $SU(5)$ predicts the ratio of the tau to the bottom mass M_τ/M_b [13] in the SM. GUTs intoduce, however, new complications such as the different ways of breaking this larger symmetry as well as new degrees of freedom.

A way to relate the Yukawa and the gauge sector, in other words achieving Gauge-Yukawa Unification (GYU) [14–16] seems to be a natural extension of the GUTs. The possibility that $N=2$ supersymmetry [17] plays such a role is highly limited due to the existence of light mirror fermions. Other phenomenological drawbacks appear in composite models and superstring theories.

A complementary approach is to search for all-loop Renormalization Group Invariant (RGI) relations [18, 19] which hold below the Planck scale and are preserved down to the scale of unification [14–16, 20–25]. With this approach [18, 19] gauge-Yukawa unification is possible [14–16, 26]. A remarkable point is that, assuming finiteness at one-loop in $N=1$ gauge theories, RGI relations that guarantee finiteness to all orders in perturbation theory can be found [27–29].

The above approach seems to need supersymmetry as an essential ingredient. However the breaking of supersymmetry has to be understood too, since it provides the SM with several predictions for its free parameters. Actually, the RGI relation searches has been extended to the soft SUSY breaking (SSB) sector [20, 30–32] relating parameters of mass dimension one and two.

Finally, the RGI approach is applied here to the MSSM too, i.e. without referring of a particular GUT.

2. The Reduction of Couplings: A Brief Outline

A Renormalization Group Invariant (RGI) relation among the couplings, that is a relation which does not depend explicitly on the renormalization scale μ, can be expressed in the form $\Phi(g_1,\cdots,g_A) =$ const. This $\mu-$independence of the Φ function leads to the following differential equation

$$\frac{d\Phi}{dt}=\sum_{a=1}^{A}\frac{\partial\Phi}{\partial g_a}\frac{dg_a}{dt}=\sum_{a=1}^{A}\frac{\partial\Phi}{\partial g_a}\beta_a=0\,, \tag{2.1}$$

where $t=\ln\mu$ and β_a is the β-function of the coupling g_a. If the $\beta-$functions satisfy a certain regularity, there exist $A-1$ independent solutions of Eq.(2.1) Therefore, all the couplings can be expressed as function $g_a(g)$ of a single coupling g, the primary one. Then the solution of the above partial differential equation is equivalent to the following set of ordinary differential equations (called Reduction Equations, RE)

$$\beta_g \frac{dg_a}{dg} = \beta_a \; , \; a = 1, \cdots, A \; , \tag{2.2}$$

where β_g is the primary coupling β-function and the counting $a = 1, \cdots, A$ does not include the primary coupling [18, 19, 33]. Trying to solve these RGI relations, we demand the couplings to be expressed as power series of the primary coupling (which leads to perturbative renormalizability)

$$g_a = \sum_{n=0} \rho_a^{(n)} \, g^{2n+1} \; . \tag{2.3}$$

It should be noted that even from the one-loop order we can check on the uniqueness of the above power series as a solution of the RGI relations [18, 19, 33].

The existence of supersymmetric theories with couplings having the same asymptotic behaviour can justify the search for such power series, (2.3), as solutions of the REs (2.2). Therefore, keeping only the first terms in the power series can be regarded as good approximation.

3. Extension of the Reduction in the Soft Breaking Terms Section

The above method of reduction in the space of dimensionless couplings was extended [20, 30–32] to the dimensionful parameters of the Soft Supersymmetry Breaking (SSB) sector of a $N = 1$ supersymmetric theories. Also, in gauge-Yukawa unified models, the scalar masses of the SSB sector satisfy a universal sum rule [34, 35].

Suppose we have the (matter) fields Φ_i which transform as the irreducible R_i representation of a gauge grou G. The superpotential is written as

$$W = \frac{1}{2} \mu^{ij} \Phi_i \Phi_j + \frac{1}{6} C^{ijk} \Phi_i \Phi_j \Phi_k \; , \tag{3.1}$$

where C^{ijk} and μ^{ij} are the Yukawa couplings and the mass terms respectively. The SSB Lagrangian is expressed by

$$-\mathscr{L}_{\rm SSB} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{2} (m^2)^j_i \phi^{*i} \phi_j + \frac{1}{2} M \lambda\lambda + \text{H.c.}, \tag{3.2}$$

where λ and M are the gauginos and their unified mass, b^{ij} and h^{ijk} are the bilinear and trilinear (dimensionful) couplings, $(m^2)^j_i$ the soft scalars masses and ϕ_i the scalar components of the superfields Φ_i.

At this point, let us remind the reader that the gauge coupling β-function β_g is given by [36–40]

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2} [\sum_i T(R_i) - 3 C_2(G)] \; , \tag{3.3}$$

where $\mathrm{Tr}[T^a T^b] = T(R) \delta^{ab}$ (T^a being the generators of the group in the appropriate representation) and $C_2(G)$ is the quadratic Casimir of the adjoint representation of the group G. The β-functions of the Yukawa couplings C_{ijk}, are given by

$$\beta_C^{ijk} = \frac{dC_{ijk}}{dt} \; = \; C_{ijl} \gamma^l_k + C_{ikl} \gamma^l_j + C_{jkl} \gamma^l_i \; , \tag{3.4}$$

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where γ^i_j are the anomalous dimensions of the chiral superfields involved in the coupling which in turn are given by (at the one-loop level) [36–40]

$$\gamma^{(1)\,i}_{\;\;j} = \frac{1}{32\pi^2}\,[\,\frac{1}{2}C^{ikl}\,C_{jkl} - 2\,g^2\,C_2(R)\delta^i_j\,], \tag{3.5}$$

where $C^{ijk} = C^*_{ijk}$ and $C_2(R)$ is the quadratic Casimir of the representation R_i.

In our approach we assume that

$$C^{ijk} = g\sum_{n=0}\rho^{ijk}_{(n)}g^{2n}\ , \tag{3.6}$$

i.e. the reduction equations admit power series solutions.

Following the spurion technique [41–45] we are led to all-loop relations among SSB β-functions [46–51]. Following [47], we assume that the following relation is RGI

$$h^{ijk} = -M(C^{ijk})' \equiv -M\frac{dC^{ijk}(g)}{d\ln g}\ . \tag{3.7}$$

Taking into account the all-loop gauge β-function of Novikov *et al.* [52, 53]

$$\beta_g^{\rm NSVZ} = \frac{g^3}{16\pi^2}\left[\frac{\sum_l T(R_l)(1-\gamma_l/2) - 3C_2(G)}{1-g^2C_2(G)/8\pi^2}\right]\ , \tag{3.8}$$

we are led to the all-loop RGI sum rule [54]

$$\begin{aligned} m_i^2+m_j^2+m_k^2 = |M|^2 &\left\{\ \frac{1}{1-g^2C_2(G)/(8\pi^2)}\frac{d\ln C^{ijk}}{d\ln g} + \frac{1}{2}\frac{d^2\ln C^{ijk}}{d(\ln g)^2}\ \right\} \\ &+\sum_l \frac{m_l^2 T(R_l)}{C_2(G)-8\pi^2/g^2}\frac{d\ln C^{ijk}}{d\ln g}\ , \end{aligned} \tag{3.9}$$

where we have assumed that $(m^2)^i{}_j = m^2_j\delta^i_j$.

The all-loop relations among the β-functions of the SSB sectors lead also to all-loop RGI relations (see e.g. [32]). Assuming that the Yukawa couplings C^{ijk} are reduced, i.e.

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g} \tag{3.10}$$

and also that the following relations for the trilinear SSB couplings hold at all-orders

$$h^{ijk} = -M\frac{dC(g)^{ijk}}{d\ln g}\,, \tag{3.11}$$

then, the following RGI relations hold to all-loops [55, 56]

$$M = M_0\,\frac{\beta_g}{g}, \tag{3.12}$$

$$h^{ijk} = -M_0\,\beta_C^{ijk}, \tag{3.13}$$

$$b^{ij} = -M_0\,\beta_\mu^{ij}, \tag{3.14}$$

$$(m^2)^i{}_j = \frac{1}{2}\,|M_0|^2\,\mu\frac{d\gamma^i{}_j}{d\mu}, \tag{3.15}$$

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where M_0 is an arbitrary mass scale which will be specified later. Note that the two assumptions leading to the above relations do not depend on the specific type of solutions of these two relations.

In ref [55] it was emphasized that the RGI relations (3.12)-(3.15) are the ones that appeared in the *Anomaly Mediated SB Scenario* [57, 58], by identifying the M_0 scale to be the gravitino mass $m_{3/2}$, which is the natural scale in the supergravity framework. A final remark is in order. It concerns the resolution of the fatal problem appearing in the anomaly induced scenario in the supergravity framework, which is here solved thanks to the sum rule (3.9). Other solutions have been provided by introducing Fayet-Iliopoulos terms [59].

4. Reduction of Couplings in the MSSM

The superpotential of the MSSM is defined by

$$W = Y_t H_2 Q t^c + Y_b H_1 Q b^c + Y_\tau H_1 L \tau^c + \mu H_1 H_2 \,, \tag{4.1}$$

while the SSB Lagrangian is given by

$$\begin{aligned} -\mathscr{L}_{SSB} = \sum_\phi m_\phi^2 \phi^* \phi + \left[m_3^2 H_1 H_2 + \sum_{i=1}^{3} \frac{1}{2} M_i \lambda_i \lambda_i + \text{h.c} \right] \\ + \left[h_t H_2 Q t^c + h_b H_1 Q b^c + h_\tau H_1 L \tau^c + \text{h.c.} \right] , \end{aligned} \tag{4.2}$$

where in the last four terms we refer to the scalar components of the corresponding superfield. The Yukawa $Y_{t,b,\tau}$ and the trilinear $h_{t,b,\tau}$ couplings refer to the third generator only, neglecting the first two generations.

Following the procedure of reduction, at the first stage we keep only the g_3 coupling and treat the two other gauge coupling g_2 and g_1 (which cannot be reduced in favour of g_3) as corrections. The same happens with the tau Yukawa, since assuming that Y_τ is reduced in favour of g_3 leads to an imaginary coefficient at one-loop. This "reduced" system, holding at any scale, can serve as boundary condition of the RGE of MSSM at the unification scale [32].

The reduction of the top and bottom Yukawa couplings in favour of g_3, together with the corrections of g_1, g_2 and Y_τ, lead, at the unification scale M_U ($g_1 = g_2 = g_3 = g_U$), to the relations

$$Y_t^2 = c_1 g_U^2 + c_2 g_U^4/(4\pi), \qquad Y_b^2 = p_1 g_U^2 + p_2 g_U^4/(4\pi) \,, \tag{4.3}$$

where $g_U = g_3(M_U)$ and

$$\begin{aligned} c_1 &= \frac{157}{175} + \frac{1}{35} K_\tau = 0.897 + 0.029 K_\tau \,, \\ p_1 &= \frac{143}{175} - \frac{6}{35} K_\tau = 0.817 - 0.171 K_\tau \,, \\ c_2 &= \frac{1}{4\pi} \frac{1457.55 - 84.491 K_\tau - 9.66181 K_\tau^2 - 0.174927 K_\tau^3}{818.943 - 89.2143 K_\tau - 2.14286 K_\tau^2} \,, \\ p_2 &= \frac{1}{4\pi} \frac{1402.52 - 223.777 K_\tau - 13.9475 K_\tau^2 - 0.174927 K_\tau^3}{818.943 - 89.2143 K_\tau - 2.14286 K_\tau^2} \,, \\ K_\tau &= Y_\tau^2 / g_3^2 \,. \end{aligned} \tag{4.4}$$

In the SSSB sector, keeping only the first term of the perturbative expansion of the Yukawas in favour of g_3 we get also

$$h_{t,b} = -M(M_U)Y_{t,b}, \qquad m_3^2 = -M(M_U)\mu \tag{4.5}$$

and finally a set of equations resulting from the application of the sum rule

$$m_{H_2}^2 + m_Q^2 + m_{t^c}^2 = M^2(M_U), \quad m_{H_1}^2 + m_Q^2 + m_{b^c}^2 = M^2(M_U), \tag{4.6}$$

where $M(M_U)$ is the gluino mass at the GUT scale (equal of course to the mass of all gauginos).

Let us proceed now to our predictions on the reduced MSSM. Starting at the unification scale M_U with the boundary conditions described above, we run the MSSM RGEs down to the SUSY scale and then the SM ones down to the M_Z scale. At that scale we compare our calculated third generation quark masses values with the corresponding experimental ones. The gaugino mass $M(M_U)$ and $|\mu|$ at M_U are varied in the range $\sim 1-11$ TeV for both possible signs of μ. As SUSY scale we take the geometrical averages of the stop masses. For the evaluation of the bottom and tau masses the one-loop radiative corrections from the SUSY breaking are incorporated [60, 61] which can provide sizeable corrections to the bottom mass for large $\tan\beta$.

The experimental value of the top quark pole mass is taken as

$$m_t^{\exp} = (173.2 \pm 0.9) \text{ GeV} . \tag{4.7}$$

We calculate the bottom mass at M_Z in order to avoid running down to the pole mass which induces uncertainties, while we take into account the tau and bottom quark mass SUSY radiative corrections [62]

$$m_b(M_Z) = (2.83 \pm 0.10) \text{ GeV}. \tag{4.8}$$

The value of the parameter $K_\tau = Y_\tau^2/g_3^2$ (see Eq. (4.4)) is now constrained in order to get both the mass of the top and bottom quarks within 1σ and 2σ from the central experimental values simultaneously. This requirement is not fulfilled in the case that $\text{sign}(\mu) = +$ and therefore in what follows we consider only the case where $\text{sign}(\mu) = -$. In that case, the variation of the value of K_τ, demanding 2σ agreement with the top and bottom mass experimental values, is in the range $\sim 0.38 \sim 0.5$.

We proceed now to additional constraints (keeping only the case where $\mu < 0$), considering $\text{BR}(b \to s\gamma)$ and $\text{BR}(B_s \to \mu^+\mu^-)$.

We are using the value

$$\text{BR}(b \to s\gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4} \tag{4.9}$$

from the Heavy Flavour Averaging Group (HFAG) [85].

The SM prediction for $\text{BR}(B_s \to \mu^+\mu^-)$ is at the level of 10^{-9}. We consider an upper limit

$$\text{BR}(B_s \to \mu^+\mu^-) \lesssim 4.5 \times 10^{-9} \tag{4.10}$$

at the 95% C.L. [86], which is in good agreement with the measurements of CMS and LHCb [87]. We feel comfortable with the above upper limit since no sizeable impact are expected on our results.

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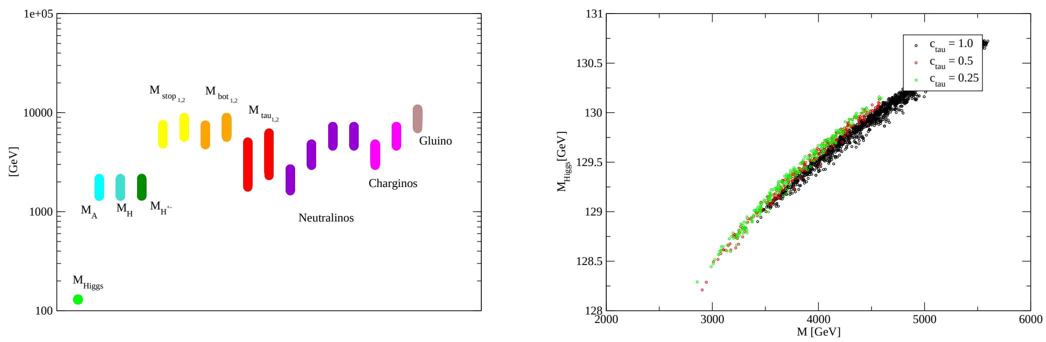

Figure 1: The left plot shows the SUSY spectrum in the reduced MSSM. From left to right are shown: The lightest Higgs mass, the pseudoscalar one M_A, the heavy neutral one M_H, the two charged Higgses $M_{H^\pm}$; then come the two stops, two sbottoms and two staus, the four neutralinos, the two charginos, and at the end the gluino. The right plot shows the lightest Higgs mass as a function of the unified gaugino mass for three values of the uncostraint parameter c_τ.

In Fig. (1) we present the Higgs mass along with the whole sparticle and Higgs mass spectrum calculated according to Eqs. (4.3), (4.5) and (4.6), assuming the Eq.(3.11) is valid. The "mixed-scale" 1-loop approach was used in order to calculate the Higgs mass. This approach approximates the leading 2-loop corrections given by the full diagrammatic calculations [63, 64]. However, results as the ones in [65] (with more refined calculations of the Higgs mass) are not yet included.

In Fig.(1), the left plot presents the mass spectrum of the model. The heavier Higgses mass are above the TeV scale while we note a heavy SUSY spectrum in general, in agreement with the non-observation of colored SUSY particles put by the LHC bounds [66–68]. As it was mentioned above, we are considering only the case where $\mu < 0$, which is known not to be compatible with the muon anomalous magnetic moment, but our heavy spectrum provides very small corrections to the predictions of the SM.

Going to the right plot of Fig. (1) we present the mass of the light Higgs as a function of the unified gaugino mass M taking into account the variation of K_τ itself (mentioned before) and constraints on the unified gaugino mass M put by the B-physics observables. The different coloured points correspond to different values of c_τ, the constant between h_τ and Y_τ, $h_\tau = c_\tau M Y_\tau$, which is the only unconstrained parameter. The m_3^2 and μ parameters are constrained by the requirement of electroweak symmetry breaking. The value of the mass varies in the range $128 \sim 130$ GeV but we expect using the new version of the code `FeynHiggs` [69, 88–90] this value will slightly come down.

5. Finiteness

Consider a GUT with superpotential Eq. (3.1) along with SSB terms Eq. (3.2) describing a $N = 1$ globally supersymmetric, anomaly free theory based on a group G with gauge coupling g. If the β-function of g as well as the anomalous dimensions $\gamma_i^{j(1)}$ of the Yukawas vanish, then all one-loop β-functions of the theory vanish (see Eqs.(3.3),(3.5))

$$\sum_i T(R_i) = 3C_2(G)\,,\ \frac{1}{2}C_{ipq}C^{jpq} = 2\delta_i^j g^2 C_2(R)\,. \tag{5.1}$$

The above conditions are enough to guarantee two-loop finiteness [70]. At this point we should mention a theorem [27–29], that guarantees the all-loop vanishing of the β-functions. The extra requirement is that the Yukawas are reduced, to all-orders, in favour of the gauge couplings (see [71]). Similar results were obtained [72–74] with the use of the all-loop gauge β-function of NSVZ [52,75].

Considering here finite theories, we start by assuming that our group G is simple and that the gauge coupling β-function vanishes at one-loop level.[1] We further assume that the power series Eq. (3.6) can solve the reduction equations and, according to the finiteness theorem [27–29,78], if the one-loop anomalous dimensions $\gamma_i^{j(1)}$ vanish, then the theory is finite. The relation [79]

$$h^{ijk} \ = \ -MC^{ijk} + \cdots = -M\rho^{ijk}_{(0)}\,g + O(g^5) \tag{5.2}$$

can establish the finiteness of h^{ijk}, at one- and two-loops (... stands for higher orders).

Finally, in Gauge-Yukawa unified models, as we have seen, a sum rule is satisfied by the SSB scalar masses at one-loop level [34]. From the results of generalizing to two-loop [34] and to all-loops [54] for finite theories, the following sum-rule is found [35]

$$\frac{(\ m_i^2 + m_j^2 + m_k^2\ )}{MM^{\dagger}} = 1 + \frac{g^2}{16\pi^2}\Delta^{(2)} + O(g^4)\,, \tag{5.3}$$

where $m^2_{i,j,k}$ are the scalar masses, $\rho^{ijk}_{(0)} \neq 0$ and $\Delta^{(2)}$ the two-loop correction, vanishing for the case where all scalar masses are equal at the unification point and also for the model that we are considering.

6. An $SU(5)$ Finite Unified Theory

We shall study an all-loop Finite Unified Theory (FUT) based on the $SU(5)$ gauge group, applying the coupling reduction to quarks and leptons of the third generation. The model consists of three ($\overline{\mathbf{5}}+\mathbf{10}$) supermultiplets (the three generations of leptons and quarks) and four ($\overline{\mathbf{5}}+\mathbf{5}$) and a **24** supermultiplets (the Higgses). By breaking the gauge group we assume that we are left with the MSSM, while our theory is not any more finite [15,21–24].

The following three properties, in addition to the requirements mentioned already, should a predictive Gauge-Yuakwa unified $SU(5)$ model poses, being finite to all orders

- The anomalous dimension, at one-loop order, should be diagonal i.e., $\gamma_i^{(1)\,j} \propto \delta_i^j$.
- The three ($\overline{\mathbf{5}}+\mathbf{10}$) representations of the fermions should not couple to the **24**.
- The MSSM two Higgs doublets should mostly be made out of a pair of Higgs **5** and $\overline{\mathbf{5}}$, which couple to the third generation.

[1] Finiteness implying three generations of matter have been studied for realistic finite unified theories with product gauge groups [76,77].

Reducing the couplings, the superpotential of the enhances symmetry theory is

$$
\begin{aligned}
W = & \sum_{i=1}^{3} [\, \frac{1}{2} g_i^u \, \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \, \mathbf{10}_i \overline{\mathbf{5}}_i \overline{H}_i \,] + g_{23}^u \, \mathbf{10}_2 \mathbf{10}_3 H_4 \\
& + g_{23}^d \, \mathbf{10}_2 \overline{\mathbf{5}}_3 \overline{H}_4 + g_{32}^d \, \mathbf{10}_3 \overline{\mathbf{5}}_2 \overline{H}_4 + g_2^f \, H_2 \, \mathbf{24} \overline{H}_2 + g_3^f \, H_3 \, \mathbf{24} \overline{H}_3 + \frac{g^\lambda}{3} (\mathbf{24})^3 \, .
\end{aligned} \tag{6.1}
$$

The solutions to $\gamma_i^{(1)} = 0$ (isolated and non-degenerate) are

$$
\begin{aligned}
& (g_1^u)^2 = \frac{8}{5} \, g^2 \, , \; (g_1^d)^2 = \frac{6}{5} \, g^2 \, , \; (g_2^u)^2 = (g_3^u)^2 = \frac{4}{5} \, g^2 \, , \\
& (g_2^d)^2 = (g_3^d)^2 = \frac{3}{5} \, g^2 \, , \; (g_{23}^u)^2 = \frac{4}{5} \, g^2 \, , \; (g_{23}^d)^2 = (g_{32}^d)^2 = \frac{3}{5} \, g^2 \, , \\
& (g^\lambda)^2 = \frac{15}{7} g^2 \, , \; (g_2^f)^2 = (g_3^f)^2 = \frac{1}{2} \, g^2 \, , \; (g_1^f)^2 = 0 \, , \; (g_4^f)^2 = 0 \, ,
\end{aligned} \tag{6.2}
$$

and the sum rule gives:

$$
m_{H_u}^2 + 2 m_{\mathbf{10}}^2 = M^2 \, , \; m_{H_d}^2 - 2 m_{\mathbf{10}}^2 = -\frac{M^2}{3} \, , m_{\overline{\mathbf{5}}}^2 + 3 m_{\mathbf{10}}^2 = \frac{4M^2}{3} \, . \tag{6.3}
$$

Allowing a rotation of the Higgs sector, through the introduction of appropriate mass terns, we can end up with two Higgs doublets as is expected, since we assume that after the $SU(5)$ breaking we are left with the MSSM. [21–25, 80–82]. This procedure allows only one Higgs pair, coupled mainly to the third family, to remain light and acquire a vev. The problem of fast proton decay is treated with the double-triplet splitting as usual, with some delicate differences from the $SU(5)$ case because of the extended Higgs sector in the present case.

7. Predictions of the Finite Model

Having spontaneously broken the gauge symmetry, only boundary conditions remain, at the M_{GUT} scale, from the finiteness conditions on the gauge and Yukawa couplings (6.2), as well as the relation $h = -MC$ (5.2), along with the sum rule for the soft scalar masses at M_{GUT}.

The **FUT** predictions are shown in Fig.2, for the top mass m_t and the bottom mass $m_b(M_Z)$ as a function of the gaugino mass M, distinguishing the two cases $\mu < 0$ and $\mu > 0$. The bounds on the two quark masses lead to the $\mu < 0$ case [83, 84].

We use the the code `FeynHiggs` [69,88–90] for our prediction on the lightest Higgs mass M_h which is shown in Fig. 3 (for the **FUT** with $\mu < 0$). The constraints of the B physics observables have been taken into account. The lightest Higgs mass is in the range

$$
M_h \sim 121 - 126 \, \mathrm{GeV} \, . \tag{7.1}
$$

The uncertainty is due to the variation of the soft scalar masses. A value of ± 2 GeV should be added from unknown corrections of higher orders [69]. A small variation of up to 5% of the FUT boundary conditions, due to threshold corrections at the GUT scale, is also included. The heavier Higgs masses are larger comparing with our previous analyses [83, 91–93]. The reason is that the bound on $\mathrm{BR}(B_s \to \mu^+ \mu^-)$ pushes beyond ~ 1 TeV these masses, thus excluding any discovery at the LHC.

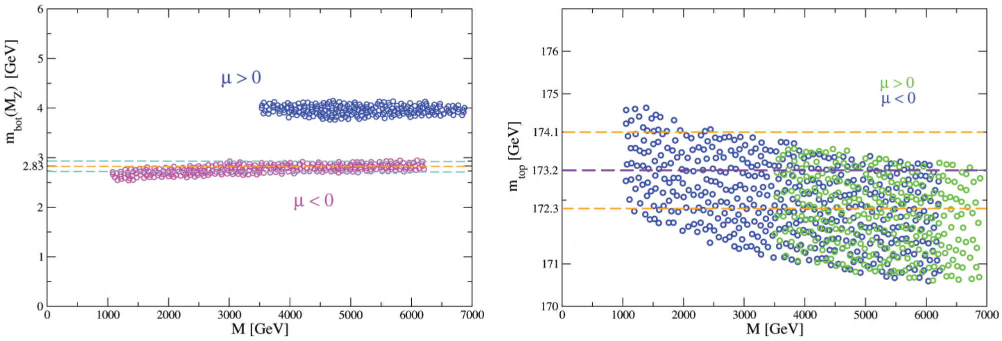

Figure 2: The masses $m_b(M_Z)$ (left) and m_t (right) as function of the unified gaugino mass M.

We now impose the constraint of the lightest Higgs boson mass on our results, which is the value of the Higgs mass

$$M_h \sim 125.1 \pm 3.1 \pm 2.1 \text{ GeV} , \tag{7.2}$$

where ± 3.1 GeV corresponds to the current theory and experimental uncertainty, and ± 2.1 GeV to a reduced theory uncertainty in the future. We find that constraining the allowed values of the Higgs mass puts a limit on the allowed values of the unified gaugino mass, as can be seen from Fig.(3). The dashed-dotted lines indicate the current uncertainty, placing an upper bound of $M \lesssim 3.8$ TeV. The anticipated future uncertainty (keeping the current central value) would lower this bound to $M \lesssim 2.6$ TeV. These upper bounds yield restrictions to the corresponding SUSY spectrum.

In Fig.4 we show the full **FUT** model for the case $\mu < 0$, respecting the constraints from the quark masses and the B-physics observables. The light scalar tau appears to be the lightest observable SUSY particle. The right (left) plot corresponds to $M_h = 126 \pm 1(3)$ GeV. Having no restriction on M_h, the SUSY mass spectrum stays above ~ 1.8 TeV which agrees with the non-observation of those particles at the LHC [66–68]. The lower part of the SUSY particle mass spectra is favoured if we include the constraints from the Higgs mass, but at the same time the very low values are excluded [94–97]. As far as the anticipated uncertainty of M_h in a future theory (see

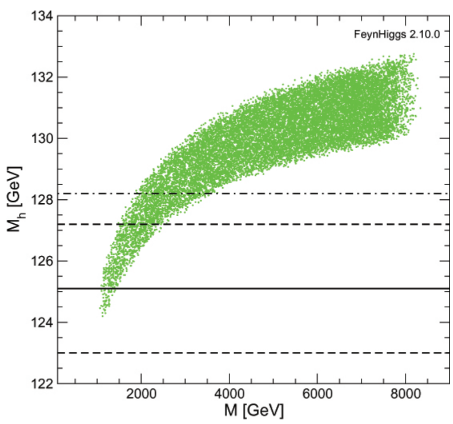

Figure 3: The lightest Higgs mass, M_h, as function of M for the model **FUT** with $\mu < 0$.

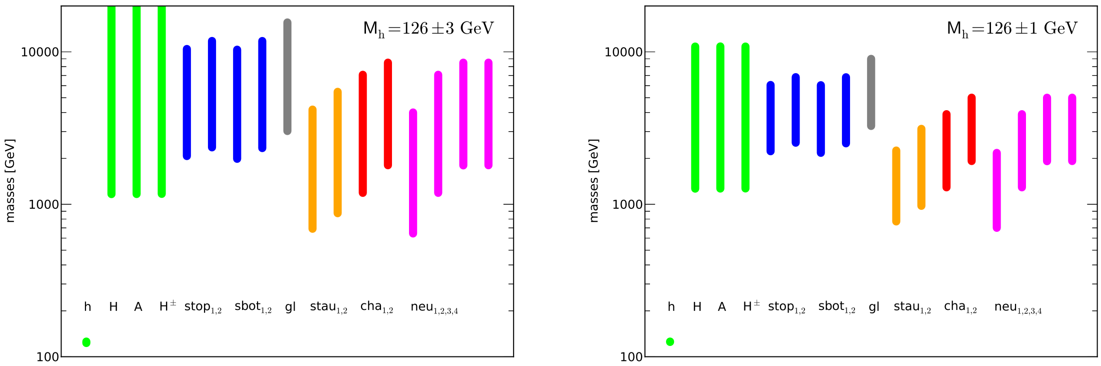

Figure 4: The left (right) plot shows the spectrum after imposing the constraint $M_h = 126 \pm 3\,(1)$ GeV. The light (green) points are the various Higgs boson masses, the dark (blue) points following are the two scalar top and bottom masses, the gray ones are the gluino masses, then come the scalar tau masses in orange (light gray), the darker (red) points to the right are the two chargino masses followed by the lighter shaded (pink) points indicating the neutralino masses.

Fig.4), unobservable SUSY masses at the LHC, as well as at the ILC or CLIC, are still permitted. But, at CLIC with $\sqrt{s} = 3$ TeV, the lighter scalar tau or the lighter neutralinos could be accessible.

8. Conclusions

The MSSM, although considered as the beyond the SM best candidate, the problem of many free parameters present in the latter is proliferated. Assuming a GUT beyond the scale of gauge coupling unification, based on the idea that a (complete) Particle Physics Theory is more symmetric at high scales, seems to fit with the MSSM. On the other hand the unification scenario seems to be unable to further reduce the number of free parameters.

Trying to reduce the free parameters, in refs. [18, 19] a new approach is proposed where RGI relations among couplings is investigated. Although this approach could uncover further symmetries, its application opens new horizons. The Finite Unified Theories and the MSSM seem to be a very promising field for applying the reduction approach. In the **FUT** case, the discovery of RGI relations among couplings above the unification scale ensures at the same time finiteness to all orders. In the MSSM case, the GUT idea is not necessary, since the search for the RGI relations is performed within the MSSM itself.

In the **FUT** case, the previous discussion shows that the results are impressive. Of course one could add several comments on **FUT**. The developments on treating the problem of divergencies include string and non-commutative theories, as well as $N = 4$ SUSY theories [98, 99], $N = 8$ supergravity [100–104] and the AdS/CFT correspondence [105]. It seems that the $N = 1$ **FUT**, discussed here, includes many ideas which survived phenomenological and theoretical tests as well as the ultraviolet divergence problem. It is actually solving that problem in a minimal way. Going to the phenomenological ground, the **FUT** case succeeded in the prediction of the top quark mass [21,22] while the SUSY spectrum agrees with the findings of LHC and its subsequent bounds.

The Higgs mass was an excellent prediction of the theory well before its discovery. The difficulty with the Higgs mass in the reduced MSSM is expected to be resolved as soon as the new version of the `FeynHiggs` code will be used.

In the forthcoming years improved calculations of the light Higgs mass are expected, among other improvements, on theory side. The corrections appearing in [65] introduces a shift in M_h, probably covered by theory uncertainties. The later ones will also be reduced by these corrections, see [65, 106], leading to a refine selection of the model points and to a sharper prediction of the spectrum. Higher order corrections can drive the M_h uncertainty below the 0.5 GeV level.

Of course important improvements are expected at the collider experiments. A large extension on the SUSY search is expected from LHC with a new record of energy $\sqrt{s} \lesssim 14$ TeV. Therefore, the lower part of our colored SUSY spectra could be tested. On the other hand, e^+e^- colliders could be a better option for EW particles. The International Linear Collider (ILC), at $\sqrt{s} \lesssim 1$ TeV, seems to have limited potential for our predicted spectra. A possible higher energy ($\sqrt{s} \lesssim 3$ TeV) at the Compact Linear Collider (CLIC) could have a better access to our spectra.

However, our spectra contains regions unaccessible by LHC, ILC or CLIC. In that case, it will still be impossible to distinguish the lightest MSSM Higgs from the SM one. Our hopes remain in improving M_h calculation.

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