

One-loop neutrino mass in $SU(5)$

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I demonstrate viability of the one-loop neutrino mass mechanism within the $SU(5)$ grand unification framework when the loop particles comprise two particular pairs of scalar leptoquarks (LQs) and the down-type quarks. The LQ multiplet components that propagate in the loop should mix for the mechanism to work. I accordingly classify all $SU(5)$ invariant operators that can yield the required mixing. If the LQs mediate proton decay at the tree-level the neutrino mass mechanism can only be viable when the LQ masses reside in a very narrow range between 10^{12} GeV and 5×10^{13} GeV. I also present one realistic $SU(5)$ set-up where the LQs in the neutrino mass loop are collider accessible.

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1. Introduction

Leptoquarks (LQs) are colored states that couple quarks to leptons. They can thus yield novel physical processes that are not present in the Standard Model (SM) of elementary particle physics¹. One such process is a generation of neutrino masses of Majorana nature at the one-loop level that requires addition of at least two particular scalar LQ multiplets [2, 3] to the SM particle content. This type of the one-loop contributions towards neutrino mass has been extensively studied in the literature [2, 3, 4, 5, 6, 7, 8]. The aim of this work is to provide a viable $SU(5)$ origin [9] of the one-loop mechanism of the neutrino mass generation when the particles in the loop are the down-type quarks and two scalar LQs. For more details on this analysis the reader is referred to Ref. [10].

I first present an overview of the most salient features of the one-loop mechanism to generate neutrino masses. There are only two possible LQ pairs that can generate the one-loop mechanism I intend to incorporate in $SU(5)$ framework. These combinations are made of either S_1 – \tilde{R}_2 or S_3 – \tilde{R}_2 mixtures. The LQ states that actually propagate in the neutrino mass loop carry an electric charge of $1/3$ in units of the absolute value of the electric charge of an electron. The SM transformation properties of the relevant scalar LQs are given in Table 1, where I adopt standard symbolic notation to represent LQ multiplets [11]. To single out a particular electric charge eigenstate component from a given LQ multiplet I use superscripts [1]. For example, S_3 comprises three electric charge eigenstates that are label $S_3^{4/3}$, $S_3^{1/3}$, and $S_3^{-2/3}$ whereas \tilde{R}_2 comprises two eigenstates denoted $\tilde{R}_2^{2/3}$ and $\tilde{R}_2^{-1/3}$.

The Yukawa interactions that are relevant for the one-loop mechanism are

$$\mathcal{L}_Y \supset -\tilde{y}_2^{RL} \bar{d}_R \tilde{R}_2^a \varepsilon^{ab} L_L^b - y_1^{LL} \bar{Q}_L^C S_1 \varepsilon^{ab} L_L^b - y_3^{LL} \bar{Q}_L^C \varepsilon^{ab} (\tau^k S_3^k)^{bc} L_L^c - y_D \bar{Q}_L^a H^a d_R + \text{h.c.}, \quad (1.1)$$

where \tilde{y}_2^{RL} , y_1^{LL} , y_3^{LL} , and y_D are 3×3 matrices in flavor space. $H (\equiv (\mathbf{1}, \mathbf{2}, 1/2))$ is the Higgs boson of the SM, τ^k , $k = 1, 2, 3$, are Pauli matrices, and $a, b, c = 1, 2$ are the $SU(2)$ indices. The couplings that are crucial for the neutrino mass generation mechanism under consideration are the Yukawa couplings of $\tilde{R}_2^{-1/3}$, S_1 , and $S_3^{1/3}$ with the left-chiral neutrinos and the down-type quarks. These couplings are $\tilde{y}_2^{RL} \bar{d}_R \nu_L \tilde{R}_2^{-1/3}$, $y_1^{LL} \bar{d}_L^C \nu_L S_1$, and $y_3^{LL} \bar{d}_L^C \nu_L S_3^{1/3}$. Note that S_1 and $S_3^{1/3}$ ($\tilde{R}_2^{-1/3}$) can only couple to the left-chiral (right-chiral) down-type quarks and the left-chiral neutrinos. It is thus essential that there is a mixing between $\tilde{R}_2^{-1/3}$ and either S_1 or $S_3^{1/3}$ if one is to close the loop that effectively yields Majorana mass term for the SM neutrinos.

The relevant parts of the scalar interactions that provide the mixing between S_1 – \tilde{R}_2 and S_3 – \tilde{R}_2 pairs are

$$\mathcal{L}_{\text{scalar}} \supset -\lambda_1 \tilde{R}_2^{\dagger a} H^a S_1^\dagger - \lambda_3 \tilde{R}_2^{\dagger a} (\tau^k S_3^{\dagger k})^{ab} H^b + \text{h.c.}, \quad (1.2)$$

LEPTOQUARK	$(SU(3), SU(2), U(1))$	$SU(5)$
S_3	$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	$\bar{\mathbf{45}}$
\tilde{R}_2	$(\mathbf{3}, \mathbf{2}, 1/6)$	$\mathbf{10}, \mathbf{15}$
S_1	$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	$\bar{\mathbf{5}}, \mathbf{45}, \bar{\mathbf{50}}$

Table 1: Transformation properties of scalar LQs under the SM gauge group $SU(3) \times SU(2) \times U(1)$ and the list of the most relevant $SU(5)$ representations that accommodate them.

¹See, for example, Ref. [1] for an exhaustive list of references on the LQ phenomenology.

where λ_1 and λ_3 are dimensionful parameters. I denote the squared-masses of the two physical LQs of the $1/3$ electric charge with m_{LQ1}^2 and m_{LQ2}^2 regardless of whether these states originate from the $S_1 - \tilde{R}_2^{-1/3*}$ or $S_3^{1/3} - \tilde{R}_2^{-1/3*}$ combination in what follows. The angle that diagonalises 2×2 squared-mass matrix m_1^2 (m_3^2) when the mixing pair is $S_1 - \tilde{R}_2^{-1/3*}$ ($S_3^{1/3} - \tilde{R}_2^{-1/3*}$) is labeled θ_1 (θ_3). The squared-mass matrices m_1^2 and m_3^2 take the form

$$m_{1,3}^2 = \begin{pmatrix} m_{11}^2 & \lambda_{1,3} \langle H \rangle \\ \lambda_{1,3} \langle H \rangle & m_{22}^2 \end{pmatrix}, \quad (1.3)$$

where $\langle H \rangle$ represents a vacuum expectation value (VEV) of electrically neutral component of the SM Higgs field. Here, m_{11}^2 and m_{22}^2 are the squares of would-be masses of S_1 and $\tilde{R}_2^{-1/3*}$ or $S_3^{1/3}$ and $\tilde{R}_2^{-1/3*}$ if there was no mixing whatsoever while the angles θ_1 and θ_3 are defined through

$$\tan 2\theta_{1,3} = \frac{2\lambda_{1,3} \langle H \rangle}{m_{11}^2 - m_{22}^2}. \quad (1.4)$$

Finally, the effective neutrino mass matrix that is generated at the one-loop level in the down-type quark mass basis reads [4]

$$\begin{aligned} (m_N)_{\alpha\beta} &= \frac{3 \sin 2\theta_{1,3}}{32\pi^2} \sum_{\delta=1,2,3} m_\delta \left[\frac{\log x_{1\delta}}{1-x_{1\delta}} - \frac{\log x_{2\delta}}{1-x_{2\delta}} \right] \{ (\tilde{y}_2^{RL})_{\delta\alpha} (y_{1,3}^{LL})_{\delta\beta} + (\tilde{y}_2^{RL})_{\delta\beta} (y_{1,3}^{LL})_{\delta\alpha} \} \\ &\approx \frac{3 \sin 2\theta_{1,3}}{32\pi^2} \log \frac{m_{LQ2}^2}{m_{LQ1}^2} \sum_{\delta=1,2,3} m_\delta \{ (\tilde{y}_2^{RL})_{\delta\alpha} (y_{1,3}^{LL})_{\delta\beta} + (\tilde{y}_2^{RL})_{\delta\beta} (y_{1,3}^{LL})_{\delta\alpha} \}, \end{aligned} \quad (1.5)$$

where $(m_1, m_2, m_3) = (m_d, m_s, m_b) = \langle H \rangle ((y_D)_{11}, (y_D)_{22}, (y_D)_{33})$ are the down-type quark masses, $\alpha, \beta, \delta = 1, 2, 3$ are flavor indices, and $x_{i\delta} = m_\delta^2 / m_{LQi}^2$. The associated one-loop Feynman diagrams are presented in Fig. 1.

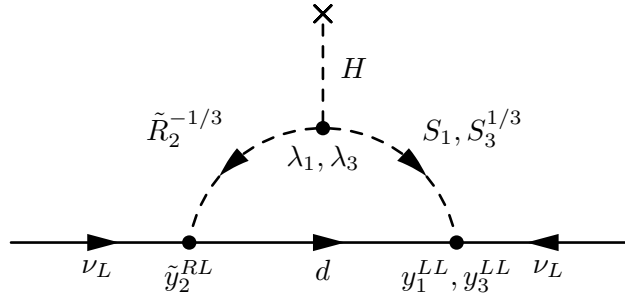


Figure 1: The neutrino mass diagrams with the down-type quarks in the loop. See text for full details.

The aim of this work is to embed this particular one-loop neutrino mass mechanism in the $SU(5)$ framework. I accordingly investigate viability of two distinct regimes in Section 2. First regime corresponds to a scenario where the LQs behind the neutrino mass generation reside at a very high energy scale. This possibility is discussed in Section 2.1. Second regime corresponds to a scenario where the neutrino masses are generated with the Large Hadron Collider (LHC) accessible scalar LQs. I demonstrate viability of that scenario in Section 2.2. The summary is presented in Section 3.

2. Grand unification vs. one-loop neutrino mass

I proceed with a realistic implementation of the one-loop neutrino mass mechanism with scalar LQs within the $SU(5)$ grand unification framework. The SM fermions reside in $\mathbf{10}_\alpha$ and $\bar{\mathbf{5}}_\alpha$ of $SU(5)$, where $\alpha(= 1, 2, 3)$ is a flavor index [9]. The exact decompositions of $\mathbf{10}_\alpha$ and $\bar{\mathbf{5}}_\alpha$ under $SU(3) \times SU(2) \times U(1)$ of the SM read $\mathbf{10}_\alpha \equiv (\mathbf{1}, \mathbf{1}, 1)_\alpha \oplus (\bar{\mathbf{3}}, \mathbf{1}, -2/3)_\alpha \oplus (\mathbf{3}, \mathbf{2}, 1/6)_\alpha = (e_\alpha^C, u_\alpha^C, Q_\alpha)$ and $\bar{\mathbf{5}}_\alpha \equiv (\mathbf{1}, \mathbf{2}, -1/2)_\alpha \oplus (\bar{\mathbf{3}}, \mathbf{1}, 1/3)_\alpha = (L_\alpha, d_\alpha^C)$, respectively. Possible embeddings of scalar LQs in the $SU(5)$ representations are presented in Table 1.

One clearly needs either one 10- or one 15-dimensional scalar representation in order to introduce one \tilde{R}_2 multiplet in any $SU(5)$ model. Relevant contraction that yields the interaction term $\tilde{y}_2^{RL} \bar{d}_R \nu_L \tilde{R}_2^{-1/3}$ of Eq. (1.1) when \tilde{R}_2 is part of 10-dimensional (15-dimensional) representation is $y_{\alpha\beta} \bar{\mathbf{5}}_\alpha \bar{\mathbf{5}}_\beta \mathbf{10}$ ($y_{\alpha\beta} \bar{\mathbf{5}}_\alpha \bar{\mathbf{5}}_\beta \mathbf{15}$). I accordingly identify \tilde{y}_2^{RL} to be $-y/\sqrt{2}$, where y is an antisymmetric (symmetric) complex matrix when \tilde{R}_2 originates from 10-dimensional (15-dimensional) representation.

The origin of the term $y_3^{LL} \bar{d}_L^C \nu_L S_3^{1/3}$ of Eq. (1.1) is unique in $SU(5)$ as can be seen from Table 1. Namely, S_3 resides in a 45-dimensional representation and the relevant contraction that generates aforementioned couplings is $y_{\alpha\beta}^{45} \mathbf{10}_\alpha \bar{\mathbf{5}}_\beta \bar{\mathbf{45}}$. One can thus identify y_3^{LL} with $y^{45}/\sqrt{2}$, where y^{45} is related to the masses of the charged fermions and down-type quarks as I show in the next paragraph.

To generate viable charged fermion masses the minimal $SU(5)$ scenario needs to include one 5-dimensional scalar representation beside the 45-dimensional one [12]. I denote VEVs of $\mathbf{5} \equiv \mathbf{5}^i$ and $\mathbf{45} \equiv \mathbf{45}^{ij}$ with $\langle \mathbf{5}^5 \rangle = v_5/\sqrt{2}$ and $\langle \mathbf{45}_1^{15} \rangle = \langle \mathbf{45}_2^{25} \rangle = \langle \mathbf{45}_3^{35} \rangle = v_{45}/\sqrt{2}$, where $i, j, k = 1, \dots, 5$ are the $SU(5)$ indices. The minimal set of contractions that generates mass matrices of the SM charged fermions comprises three operators: $y_{\alpha\beta}^{45} \mathbf{10}_\alpha \bar{\mathbf{5}}_\beta \bar{\mathbf{45}}$, $y_{\alpha\beta}^5 \mathbf{10}_\alpha \bar{\mathbf{5}}_\beta \bar{\mathbf{5}}$, and $\bar{y}_{\alpha\beta} \mathbf{10}_\alpha \mathbf{10}_\beta \mathbf{5}$. The 3×3 mass matrices for the down-type quarks, charged leptons, and the up-type quarks are

$$m_D = -y^{45} v_{45} - y^5 v_5/2, \quad (2.1)$$

$$m_E^T = 3y^{45} v_{45} - y^5 v_5/2, \quad (2.2)$$

$$m_U = \sqrt{2}(\bar{y} + \bar{y}^T) v_5, \quad (2.3)$$

where all the VEVs are taken to be real. The VEV normalization yields $v_5^2/2 + 12v_{45}^2 = v^2$, where $v(= 246 \text{ GeV})$ is the electroweak VEV [13]. The $SU(5)$ symmetry thus dictates that $y^{45} \equiv \sqrt{2} y_3^{LL} = -y_2^{RL} = (m_E^T - m_D)/(4v_{45})$.

One can also produce the term $y_1^{LL} \bar{d}_L^C \nu_L S_1$ of Eq. (1.1) within the $SU(5)$ framework. It originates from $y_{\alpha\beta}^5 \mathbf{10}_\alpha \bar{\mathbf{5}}_\beta \bar{\mathbf{5}}$ and $y_{\alpha\beta}^{45} \mathbf{10}_\alpha \bar{\mathbf{5}}_\beta \bar{\mathbf{45}}$ for $S_1 \in \bar{\mathbf{5}}$ and $S_1 \in \bar{\mathbf{45}}$, respectively. In the former (latter) case one can identify y_1^{LL} with $-y^5/\sqrt{2}$ ($y^{45}/2$).

Finally, there is a question of the LQ mixing in $SU(5)$. There are two very different regimes for the scalar LQ masses that can be envisaged with regard to this issue.

First option is that the LQs behind the neutrino mass generation reside at a very high energy scale. This would automatically provide compliance of the set-up with the experimental bounds on proton decay. The main issue with this regime could be associated with the size of the relevant lepton-quark-LQ couplings. Namely, the Yukawa couplings might need to be unrealistically large in order to (re)produce neutrino mass scales that are compatible with experimental observations. It turns out that this is not the case and I accordingly demonstrate in Section 2.1 why and how this particular scenario can be realised within the $SU(5)$ framework.

Second option is that the scalar LQs in the neutrino mass loop are collider accessible. The main difficulty with this particular set-up is to explain observed levels of matter stability [14]. Namely, S_1 and S_3 can both have “di-quark” couplings that, in combination with the lepton–quark–LQ couplings that are needed to generate neutrino masses, destabilise protons and bound neutrons. To avoid conflict with stringent limits on proton lifetime one would need to either forbid or substantially suppress these “di-quark” operators. This might be very difficult from the model building point of view since the unification of the matter fields dictates common origin of both types of couplings. One would also need to prevent mixing between these LQs and any other LQ in the theory that has “di-quark” couplings to insure stability of matter. This might also represent a challenge since one needs to mix specific LQ multiplets in order to generate neutrino masses in the first place. I show that both of these issues can be successfully addressed for the S_3 – \tilde{R}_2 loop scenario in Section 2.2.

2.1 Heavy LQ regime

I assume in this section that all the LQ masses need to be at or exceed 10^{12} GeV to insure proton stability. This is a very conservative estimate since it is certainly above a lower bound that can be extracted from the latest data on proton stability within the $SU(5)$ framework [15].

The mixing angle between either S_1 and $\tilde{R}_2^{-1/3*}$ or $S_3^{1/3}$ and $\tilde{R}_2^{-1/3*}$ will be rather small if the LQs are heavy. I will for definiteness assume that \tilde{R}_2 originates from 15-dimensional representation. The $S_3^{1/3}$ – $\tilde{R}_2^{-1/3*}$ mixing, in particular, originates in $SU(5)$ from three operators for $\tilde{R}_2 \in \mathbf{15}$. These operators are $45_k^{ij} \bar{\mathbf{15}}_{jl} 45_i^{lk}$, $45_k^{ij} \bar{\mathbf{15}}_{jl} 45_m^{lk} 24_i^m$, and $5^i \bar{\mathbf{15}}_{lj} 45_i^{jk} 24_k^l$, where 24-dimensional representation breaks $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$ through a very large VEV of the order of 10^{16} GeV. I list all possible $SU(5)$ operators that generate mixing between the 1/3 electric charge scalar LQs that are relevant for the loop generated neutrino masses in Table 2. For example, the operator $5^i \bar{\mathbf{15}}_{lj} 45_i^{jk} 24_k^l$ produces a mixing coefficient for the $S_3^{1/3}$ – $\tilde{R}_2^{-1/3*}$ pair that is equal to $-5v_5 v_{24} / (2\sqrt{2})$, where the VEV of $(\mathbf{1}, \mathbf{1}, 0)$ in $\mathbf{24} \equiv \mathbf{24}_j^i$ is $\langle (\mathbf{1}, \mathbf{1}, 0) \rangle = v_{24} \text{diag}(2, 2, 2, -3, -3)$. The angle θ_3 of Eq. (1.4) can thus be approximated to be at most $\theta_3 \sim (v_5 v_{24}) / m_{LQ}^2 \approx 10^{18} / 10^{24} = 10^{-6}$, where $v_5 \sim \langle H \rangle \approx 10^2$ GeV, $v_{24} \sim \lambda_3 \approx 10^{16}$ GeV, and $m_{11}^2 - m_{22}^2 \sim m_{LQ}^2 \approx 10^{24}$ GeV². The necessary mixing between $S_1 (\in \mathbf{5})$ and $\tilde{R}_2 (\in \mathbf{15})$ can be generated through the contractions of the form $\bar{\mathbf{5}}_i \bar{\mathbf{5}}_j \mathbf{15}^{ij}$ and $\bar{\mathbf{5}}_i \bar{\mathbf{5}}_j \mathbf{15}^{jk} 24_k^i$. These, again, yield an angle θ_1 that is comparable in strength to the estimate for θ_3 . One can furthermore safely assume that $m_b (\approx 1$ GeV) contribution dominates the sum in Eq. (1.5). Putting all this together implies that

$$m_N \sim \frac{3\theta_{1,3}}{32\pi^2} m_b \log \frac{m_{LQ2}^2}{m_{LQ1}^2} (\tilde{y}_2^{RL} y_{1,3}^{LL}) \approx \frac{10^{-6}}{10^2} 10^9 \text{ eV} (\tilde{y}_2^{RL} y_{1,3}^{LL}) = 10 \text{ eV} (\tilde{y}_2^{RL} y_{1,3}^{LL}), \quad (2.4)$$

where I suppress flavor indices and assume that the mass splitting between LQs is not substantial and accordingly take that $\log(m_{LQ2}^2/m_{LQ1}^2) \approx 1$. The approximation of Eq. (2.4) shows that the entries in the product $(\tilde{y}_2^{RL} y_{1,3}^{LL})$ do not have to be very large to correctly describe the neutrino mass scale. For example, in the non-degenerate normal hierarchy case for the neutrino masses² the largest entry on the left side of Eq. (2.4) needs to be at the level of 5×10^{-2} eV which would imply that $(\tilde{y}_2^{RL} y_{1,3}^{LL}) \sim 5 \times 10^{-3}$. This back-of-the-envelope estimate clearly demonstrates viability of the heavy LQ option. Note that there is an upper bound on the heavier of the two LQs in this set-up

²For a recent analysis of neutrino oscillation data see, for example, Ref. [16].

if one demands perturbativity of Yukawa coupling entries in \tilde{y}_2^{RL} and $y_{1,3}^{LL}$ matrices. I find it to be roughly at 5×10^{13} GeV. These estimates imply that the two LQs in the neutrino mass loop must reside in relatively narrow mass window from 10^{12} GeV to 5×10^{13} GeV in order to accommodate all the constraints. One can then infer that $\log(m_{LQ2}^2/m_{LQ1}^2) \lesssim 5$ which is in agreement with the initial ansatz that $\log(m_{LQ2}^2/m_{LQ1}^2) \sim 1$.

$SU(5)$		S_1		S_3
		5	45	45
\tilde{R}_2	10	$5^i \bar{10}_{jk} 45_i^{jk}$ $\bar{5}_i \bar{5}_j 10^{jk} 24_k^i$ $5^i \bar{10}_{lj} 45_i^{jk} 24_k^l$ $5^i \bar{10}_{ij} 45_l^{jk} 24_k^l$ $5^i \bar{10}_{lm} 45_j^{lm} 24_i^j$	$5^i \bar{10}_{jk} 45_i^{jk}$ $5^i \bar{10}_{lj} 45_i^{jk} 24_k^l$ $5^i \bar{10}_{ij} 45_l^{jk} 24_k^l$ $5^i \bar{10}_{lm} 45_j^{lm} 24_i^j$	$5^i \bar{10}_{jk} 45_i^{jk}$ $5^i \bar{10}_{lj} 45_i^{jk} 24_k^l$ $5^i \bar{10}_{lm} 45_j^{lm} 24_i^j$
	15	$\bar{5}_i \bar{5}_j 15^{ij}$ $\bar{5}_i \bar{5}_j 15^{jk} 24_k^i$	$45_k^{ij} \bar{15}_{jl} 45_i^{lk}$ $5^i \bar{15}_{lj} 45_i^{jk} 24_k^l$ $5^i \bar{15}_{ij} 45_l^{jk} 24_k^l$ $45_k^{ij} \bar{15}_{jl} 45_m^{lk} 24_i^m$	$45_k^{ij} \bar{15}_{jl} 45_i^{lk}$ $5^i \bar{15}_{lj} 45_i^{jk} 24_k^l$ $45_k^{ij} \bar{15}_{jl} 45_m^{lk} 24_i^m$

Table 2: $SU(5)$ operators that generate mixing between the 1/3 electric charge scalar LQs if one assumes that the only VEVs in the theory are the ones proportional to v_{24} , v_{45} , and v_5 .

This particular possibility to generate neutrino masses has been completely overlooked in the literature on grand unification.

2.2 Light LQ regime

To demonstrate that the collider accessible LQ scenario is a viable option to generate neutrino masses one needs to study the mixing of the LQs in a given model in detail. Namely, if the genuine LQ states mix with the states that have “diquark” couplings it is hard to imagine that matter stability holds at the experimentally observed levels. I again focus exclusively on a scenario when \tilde{R}_2 originates from 15-dimensional representation in what follows. The analysis for the 10-dimensional representation case is completely analogous [10]. The $SU(5)$ scenario that I will consider comprises the following scalar representations: **5**, **15**, **24**, and **45**. Note that R_2 , \tilde{R}_2 , and S_3 do not have “diquark” couplings [17] at renormalizable level if the charged fermion mass relations are given with Eqs. (2.1), (2.2), and (2.3). The scalar LQs in the $SU(5)$ set-up under consideration are $S_1^* \in \mathbf{5}$, $(\tilde{R}_2^{2/3}, \tilde{R}_2^{-1/3}) \in \mathbf{15}$, and $(S_3^{4/3*}, S_3^{1/3*}, S_3^{-2/3*}, R_2^{5/3*}, R_2^{2/3*}, \tilde{S}_1, S_1^*) \in \mathbf{45}$. All in all, there is one LQ with the 5/3 charge, two LQs with the 4/3 charge, three LQs with the 2/3 charge, and four LQs with the 1/3 charge.

There are ten non-trivial operators that mix the LQ states of the same electric charge if the only VEVs present are the ones proportional to v_{24} , v_{45} , and v_5 . These contractions are $5^i \bar{15}_{ij} 5^j$, $5^i \bar{45}_{ij} 24_k^j$, $45_k^{ij} \bar{15}_{jl} 45_i^{lk}$, $5^i \bar{5}_i 45_j^{jk} 45_i^i$, $5^i \bar{15}_{lj} 45_i^{jk} 24_k^l$, $5^i \bar{15}_{ij} 45_l^{jk} 24_k^l$, $\epsilon_{ijklmn} 5^k 15^{io} 45^l 45^{mn}$, $5^j \bar{5}_i 45_l^{jk} 45_j^l$, $\bar{5}_i \bar{5}_j 15^{jk} 24_k^i$, and $45_k^{ij} \bar{15}_{jl} 45_m^{lk} 24_i^m$. The LQ components affected by these contractions are the ones with the 1/3 and 2/3 electric charges. There are no contractions that mix LQs of the 4/3 electric charge through these particular VEVs.

It turns out that one can write a 4×4 squared-mass matrix for the $1/3$ electric charge LQs in a block diagonal form where the relevant two blocks are of dimension 2×2 each. The basis for this matrix is $(S_1^*(\mathbf{45}), S_1^*(\mathbf{5}), S_3^{1/3*}, \tilde{R}_2^{-1/3})$, where I explicitly denote the origin of the two LQs that both transform as S_1 under the SM gauge group. The mixing term $\lambda_3 \langle H \rangle$ of Eq. (1.2) between $S_3^{1/3*}$ and $\tilde{R}_2^{-1/3}$ is proportional to a product of v_{24} with v_5 . Since the LQs of the $4/3$ electric charge do not mix the associated 2×2 squared-mass matrix has only diagonal entries. These findings guarantee the matter stability even in the presence of the mixing that is needed to generate neutrino masses. Components of S_3 and \tilde{R}_2 can thus be very light and the resulting neutrino mass matrix is correctly described through Eq. (1.5) due to a block diagonal form of the relevant LQ squared-mass matrix. I omit discussion of the mixing between the LQ states with electric charge of $2/3$ since these originate from R_2 , \tilde{R}_2 , and S_3 multiplets that have no “diquark” couplings in this set-up and consequently do not directly affect matter stability.

Let me summarise the main features of the light LQ set-up. $\tilde{R}_2 (S_3)$ originates from $\mathbf{15} (\mathbf{45})$ of $SU(5)$. Again, \tilde{R}_2 could alternatively originate from 10-dimensional representation. The $SU(5)$ symmetry is broken by the VEV of $\mathbf{24}$ down to $SU(3) \times SU(2) \times U(1)$. The Higgs field VEVs that complete the electroweak symmetry breaking reside in both $\mathbf{5}$ and $\mathbf{45}$. The light LQ states are components of \tilde{R}_2 and S_3 and they help generate neutrino masses. Three out of six LQs of the model — $S_1(\mathbf{45})$, $S_1(\mathbf{5})$, and \tilde{S}_1 — mediate proton decay and need to be heavy. R_2 can in principle be of an arbitrary mass. Finally, the mass matrix for the up-type quarks should be symmetric in accordance with Eq. (2.3) in this set-up.

3. Conclusions

The one-loop neutrino mass mechanism with scalar LQs and the down-type quarks in the loops can be embedded within the $SU(5)$ framework of grand unification. There exist two distinct regimes for the LQ masses. One option is to have heavy LQs in the loops that generate neutrino masses. This option can be naturally realised if \tilde{R}_2 mixes with either S_1 or S_3 . One particularly nice feature of the heavy LQ limit is that the masses of the LQs in the loop can only be between 10^{12} GeV and 5×10^{13} GeV in order to simultaneously avoid experimental limits on partial proton decay lifetimes and still satisfy perturbativity constraints on the strength of the lepton-quark-LQ couplings. The other option is to have collider accessible LQs in the loop. That particular option can be realised with the mixture of S_3 and \tilde{R}_2 multiplets in a well-defined $SU(5)$ set-up that contains 5-, 15-, 24-, and 45-dimensional representations in the scalar sector when the up-type quark mass matrix is symmetric in the flavor space.

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