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Aspects of Skyrmion Black Hole Hair

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In this short article, prepared for the proceedings of the Corfu Meeting 2016, I review three aspects of skyrmion/baryon black hole hair which have been investigated recently [1, 2] and which I presented in my talk in Corfu. First, I will argue that one can detect the classical skyrmion/baryon hair which, as it is well-known [8], a small enough black hole can carry in classical scattering experiments of waves. Second, I will show that, under certain assumptions which I will point out, a black hole can not only carry the known classical skyrmion/baryon hair but also a quantum skyrmion/baryon hair which can be detected in Aharonov-Bohm type experiments in which a probe string encloses the black hole. In contrast to the known classical skyrmion/baryon hair, the Aharonov-Bohm type quantum skyrmion/baryon hair can be carried by black holes of arbitrary size. Third, I will argue that from the point of view of a low-energy observer who can measure the Aharonov-Bohm type skyrmion/baryon hair of a black hole, black holes respect Skyrme topological charge/baryon number.

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1. Introduction

During the last decades there has been a lot of progress in the field of classical black hole hair (see e.g. [3, 4, 5, 6, 7] for some reviews). In that context one of the most interesting results, in our opinion, has been the discovery of dynamically-stable[1] and asymptotically-flat classical black hole solutions of the Einstein-Skyrme equations with geometries which are not of Kerr-Newman form [8]. These "black holes with skyrmion hair", or "skyrmion black holes", have been found in a certain domain of parameters as classical solutions of the Einstein-Skyrme equations, essentially when parameters are chosen such that the black hole event horizon is smaller than a characteristic length scale associated to the skyrmion.[2] The skyrmion black holes are interesting, in particular, because from the point of view of a low-energy observer skyrmions are nothing but baryons [10], at least in a framework with a large number of colors. Therefore, black holes with skyrmion hair are nothing but black holes with baryon hair when considered from the point of view of a low-energy observer.

After briefly reviewing in section 2 what black holes with skyrmion hair of the type [8] are, we will in this article recapulate the following three recently studied [1, 2] aspects of black holes with skyrmion/baryon hair:

First, in section 3, we review our result [1] that the classical black hole skyrmion/baryon hair of the type [8] can be detected in classical scattering experiments of waves. We point out that the scattering cross sections of massless minimally-coupled probe scalar waves scattered by a skyrmion black hole of the type [8] look different than the analogous scattering cross sections of the same waves now scattered by a Schwarzschild black hole with same ADM mass than the skyrmion black hole; in particular the characteristic glory peaks in the cross sections are located at different scattering angles. Therefore, one can find out if a black hole with given ADM mass carries a skyrmion/baryon hair of the type [8] or not by using such classical wave-scattering processes. This method is also applicable to black holes with classical hair of different kind and can therefore have interesting astrophysical consequences in the context of "testing the no-hair conjecture" (see e.g. [11, 12] for two recent reviews about "testing the no-hair conjecture").

Second, in section 4, we recall the result from [1, 2] that black holes can not only carry classical skyrmion/baryon black hole hair of the type [8] but, under certain assumptions which we will point out, also quantum skyrmion/baryon hair which can be measured by Aharonov-Bohm type experiments in processes in which a probe string encloses the hairy black hole. In contrast to the classical skyrmion/baryon hair of the type [8] we argue that this quantum Aharonov-Bohm type hair can be present for black holes with arbitrary event horizon size.

Third, in section 5, we argue, as in [1], that from the point of view of a low-energy observer who can measure the skyrmion topological charge/baryon number of a black hole via the

[1] We want to emphasize first, that dynamical stability (in Lyapunov sense) of hairy black hole solutions of the Einstein-Skyrme equations has only been established *on the linear level* [9] and second, that on top of the hairy black hole solutions of the Einstein-Skyrme equations which are dynamically stable (at least) on the linear level, there are also dynamically unstable hairy black hole solutions of the Einstein-Skyrme equations [9]. In other words, not all of the classical hairy black hole solutions of the Einstein-Skyrme equations are dynamically stable (on the linear level). For our purpose the important point however is that such stable solutions do exist.

[2] See, for example [1] for a more detailed review on the parameter space of solutions.

Aharonov-Bohm type experiment described in section 4, Skyrme topological charge/baryon number is respected by black holes.[3]

The presentation of the points in the following sections will be far from complete. For details we refer the reader to [1, 2].

2. Skyrmion black holes of the type [8]

In this section we very briefly review what asymptotically-flat classical skyrmion black holes of the type [8] are. We restrict our review to the spherically-symmetric cases. For details we refer to [8] and [1, 2].

We consider the "Skyrme Lagrangian"[4] [13, 14],

$$\mathscr{L}^{(S)} = -\frac{F_\pi^2}{4}\mathrm{Tr}\left(U^+\partial_\mu U U^+ \partial^\mu U\right) + \frac{1}{32e^2}\mathrm{Tr}\left([\partial_\mu U U^+, \partial_\nu U U^+]^2\right) \ , \tag{2.1}$$

where F_π is the pion decay constant, e is the Skyrme coupling constant and U is a $SU(2)$ matrix which, in terms of pion fields $\pi_a(x)$ is defined as

$$U = e^{\frac{i}{F_\pi}\sigma_a \pi_a(x)} \ , \tag{2.2}$$

with σ_a the Pauli matrices.

This Lagrangian allows for topological lowest-energy configurations which assume a hedgehog ansatz,

$$\frac{\pi_a(x)}{F_\pi} = F(r) n_a \ , \tag{2.3}$$

where n_a is a unit vector in radial direction and $F(r)$ is a profile-function with boundary conditions $F(0) = B\pi$ and $F(\infty) = 0$ (B is a natural number which sets the topological charge of the configuration). The form of the solution profile-function these configurations take is not important for our purposes but can be found for example in [14, 15]. The configurations are called "skyrmions". In the case of a large number of colors they are known to be low-energy descriptions for baryons [10].

The Skyrme Lagrangian (2.1) can be coupled to gravity and the resulting Einstein field equations ("Einstein-Skyrme equations"),

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}^{(S)} \ , \tag{2.4}$$

with $G_{\mu\nu}$ the Einstein tensor and $T_{\mu\nu}^{(S)} = \frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathscr{L}^{(S)}\right)}{\delta g^{\mu\nu}}$ can be solved numerically when making a spherically-symmetric ansatz for the metric $g_{\mu\nu}$,

$$ds^2 = N^2(r)\left(1 - \frac{2M(r)G_N}{r}\right)dt^2 - \left(1 - \frac{2M(r)G_N}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \ , \tag{2.5}$$

[3] We want to emphasize that conservation of baryon number in systems with black holes on the level of the low-energy effective theory which we consider does not imply that baryon number must be conserved on the level of a theory working at higher energy scales. In fact, such high-energy baryon number violating theories are well-known, one example is baryon number violation in grand unified $SU(5)$ theory.

[4] Here we restrict to two quark flavors and to the case of the effective Lagrangian for massless pions truncated after four derivatives. Adding a mass term $\mathscr{L}_m = \frac{1}{2}m_\pi^2 F_\pi^2\,(\mathrm{Tr}U - 2)$ [15] does not change the following arguments. Effects of terms with more than four derivatives are discussed in [16].

where $N(r)$ and $M(r)$ are two ansatz-functions. Numerical solutions for the three ansatz functions $F(r)$, $N(r)$ and $M(r)$ for different choices of boundary conditions can be found in [8]. There are solutions without event horizon, "gravitating skyrmions", and solutions with event horizon, "skyrmion black holes" or "black holes with skyrmion hair". Solutions with event horizon have only been found in a certain domain of parameters (only for a certain range of boundary conditions for $F(r)$, $M(r)$ and $N(r)$ and values for F_π and e), essentially only when parameters are chosen such that the black hole event horizon is smaller than a typical size associated to the skyrmion.[5] In what follows we focus on these black holes with skyrmion hair which - as a consequence of the baryon/skyrmion correspondence - can be seen as black holes with baryon hair when considered from the point of view of a low-energy observer.

3. Testing The No-Hair Conjecture via Scattering of Waves

In [1] we studied a massless minimally-coupled probe scalar field Φ scattered by two particular spherically-symmetric skyrmion black holes of the type [8] (characterized by two particular sets of parameters and solution functions $M(r)$, $N(r)$ and $F(r)$) and compared the differential scattering cross sections for different monochromatic scalar waves with the analogous (known) scattering cross sections of the same scalar waves now scattered by Schwarzschild black holes which have the same ADM masses than the skyrmion black holes. In our analysis we used the same techniques which have been used frequently for studying scattering of waves by different kind of black holes (see e.g. [17] and the references therein and [18] for some more recent works); to our knowledge these well-known techniques so far have however not been applied to black holes with classical hair. We focused on the effects caused by pure gravitational interactions of the probe scalar field Φ with the black holes. Therefore we neglected possible non-gravitational interactions, in particular between Φ and the skyrmion.

The method we used is also applicable to different kind of probe waves (for example to waves of higher spin) and can easily be generalized to take into account also non-gravitational interactions. We used the most simple case of a massless scalar wave as a proof of principle.

The starting point is to consider the classical motion of Φ in the background of a given skyrmion black hole which is described by the Klein Gordon equation

$$\Box_g \Phi = 0 \,, \tag{3.1}$$

where $\Box_g$ is the d'Alambert operator in the particular skyrmion black hole spacetime with a metric g parametrized as in (2.5). Using for the scalar field Φ the expansion

$$\Phi(t,r,\theta,\phi) = \sum_{lm} \frac{A_{Wl}(r)}{r} Y_{lm}(\theta,\phi) e^{-iWt} \,, \tag{3.2}$$

where $Y_{lm}(\theta,\phi)$ are the standard spherical harmonics with ϕ and θ spherical coordinates, one can, after some straightforward algebra, separate the Klein-Gordon equation into a radial part and an angular part. The radial part can be written as

$$\partial_{x^*}^2 A(x) + \left(w^2 - V_{eff}(x)\right) A(x) = 0 \,. \tag{3.3}$$

[5] For a detailed review of this domain of parameters we refer to [1].

Here we introduced ∂_{x^*} defined as $\partial_{x^*} = N(x)h(x)\partial_x$ and the effective potential V_{eff} which is defined as

$$V_{eff}(x) = N^2(x)h(x)\frac{l(l+1)}{x^2} + \frac{N(x)}{x}h(x)\partial_x\left(N(x)h(x)\right) \ , \tag{3.4}$$

with $h(r) = \left(1 - \frac{2M(r)G_N}{r}\right)$. For convenience we introduced the dimensionless variable x defined as $x = eF_\pi r$ and the dimensionless frequency $w = W\left(eF_\pi\right)^{-1}$. Equation (3.3) has the form of a Schrödinger equation. Therefore, one can study scattering processes by using methods from standard quantum mechanical scattering theory.

For two particular examples of skyrmion black holes, we did a detailed partial wave analysis to determine differential scattering cross sections of a monochromatic scalar wave scatterd by one of the skyrmion black holes and showed how the differential scattering cross sections differ from the analogous scattering cross sections of the same wave now scattered by a Schwarzschild black hole with same ADM mass than the skyrmion black hole. One particular measure to parameterize the difference is the position of the characteristic glory peaks in the scattering cross sections. We found that the peaks are located at different scattering angles: For the skyrmion black holes the peaks are "shifted" to smaller scattering angles when compared to the analogous peaks for the Schwarzschild black holes with same ADM mass. Detailed numerical results obtained by partial wave analysis are visualized in the second part of [1].

Classical scattering of waves can also be used to distinguish black holes with different kind of classical hair from non-hairy black holes with same asymptotic characteristics (see e.g. [19]). It is therefore, in our opinion, worth investigating how this method can be used in astrophysical scenarios to detect a hair of a potential astrophysical black hole in nature.

4. Aharonov-Bohm Skyrmion Black Hole Hair

Following [1, 2], we argue that the Skyrme topological charge/baryon number of a skyrmion/baryon in flat spacetime can, under certain assumptions which we will point out, be measured by an asymptotic low-energy observer in processes in which a probe string encloses the skyrmion/baryon. We argued in [1, 2] that in this way we can also measure the same Skyrme topological charge/baryon number if the skyrmion/baryon was swallowed by a black hole. In this sense the black hole which swallowed a skyrmion/baryon carries a "skyrmion/baryon hair". This hair has a priori not to be of the kind [8] which was considered in the previous two sections. In fact, in contrast to the hair of type [8], it can be assigned to black holes of arbitrary size and therefore can for large enough black holes, not be of the type [8].

The Skyrme topological charge B of the chiral theory with Lagrangian (2.1) can be defined as volume integral over the zeroth component of the Skyrme topological current,

$$J_\mu = -\frac{1}{24\pi^2}\varepsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\left(U^+\partial^\nu UU^+\partial^\alpha UU^+\partial^\beta U\right) \ . \tag{4.1}$$

This Skyrme topological charge,

$$B = \int d^3x J_0 \ , \tag{4.2}$$

corresponds to the baryon number a high energy observer would measure [10].

In [1, 2] we pointed out that there is a differential two-form $S_{\mu\nu}$ such that the topological current can be expressed as exterior derivative of that two-form. When parametrizing U in terms of the three angles α, β and γ as

$$U = \begin{pmatrix} \cos\gamma e^{i\alpha} & \sin\gamma e^{i\beta} \\ -\sin\gamma e^{-i\beta} & \cos\gamma e^{-i\alpha} \end{pmatrix} , \tag{4.3}$$

this two-form takes the form

$$S_{\mu\nu} = -\frac{1}{4\pi^2}\cos^2\gamma\partial_{[\mu}\alpha\partial_{\nu]}\beta \ . \tag{4.4}$$

Evaluated on the hedgehog (2.3) the two-form can be written as[6]

$$S_{\mu\nu} = -\frac{1}{4\pi^2}\left(F(r) - \frac{1}{2}\sin(2F(r)) - B\pi\right)\partial_{[\mu}\cos(\theta)\partial_{\nu]}\phi \ , \tag{4.5}$$

where ϕ and θ are the standard spherical coordinates. We introduced $B\pi$ in (4.5) to make the two-form well-defined everywhere. Using the well-defined two-form we can, after using Stokes theorem, write the Skyrme topological charge as a boundary integral over a two-sphere S_2 at infinity,

$$\int_{S_2} dX^\mu \wedge dX^\nu S_{\mu\nu} = B \ , \tag{4.6}$$

where X^μ are the world-volume coordinates of the two-sphere. This boundary integral defines the Skyrme topological charge/baryon number at infinity. We argued in [1, 2] that we can therefore define the Skyrme topological charge at infinity in this way also after a black hole (with arbitrary event horizon size) is formed and the skyrmion which carries the charge is swallowed by the black hole. (This defines the Skyrme topological charge/baryon number of the black hole.)

If an asymptotic observer can measure this charge at infinity, he can thus measure the Skyrme topological charge/baryon number of a black hole. If measurable, in this sense a black hole which swallowed a skyrmion/baryon carries a skyrmion/baryon hair. And, indeed, we argued in [1, 2] that a low-energy observer can measure this surface integral in case he has a probe string at hand which couples to the two form via[7]

$$g\int d^2\sigma\partial_a X^\mu\partial_b X^\nu \varepsilon^{ab}\delta^{(4)}(x-X) \ , \tag{4.7}$$

with σ_a the world-sheet coordinates, $X^\mu(\sigma)$ the embedding coordinates of the string and g a coupling constant which is not an integer-multiple of 2π: A process in which the probe string encloses the skyrmion induces a change in the action,

$$\Delta\mathscr{S} = gB \ , \tag{4.8}$$

which can be measured as Aharonov-Bohm phase shift provided gB is not an integer-multiple of 2π. The probe string can be a fundamental string or any other string like object such as the cosmic

[6]Plugging the hedgehog ansatz (2.3) into (4.4), namely $\gamma = \arcsin(\sin F\sin\theta)$, $\beta = \phi$ and $\alpha = \arctan(\tan F\cos\theta)$, gives (4.5) modulo the exterior derivative of a one-form.

[7]On the level of low-energy effective theory this coupling is fully legitimate because it respects all the symmetries.

Nielsen-Olesen string which we considered in [2]. Therefore, under the assumptions that a low-energy observer has such probe strings at hand and that a coupling of the form (4.7) exists with gB not an integer-multiple of 2π, the observer can measure the baryon/skyrmion number of a black hole. In this sense the black hole carries a (quantum) Aharonov-Bohm type baryon/skyrmion hair. In [2] we argued that this hair is similar to the well-known Aharonov-Bohm hair of Krauss and Wilczek [20] (see alse [21, 22]) as well as to other known Aharonov-Bohm type black hole hair [23, 24, 25].

5. Conservation of Skyrmion Charge/Baryon Number by Black Holes

Given that a low-energy observer can measure the baryon number/Skyrme topological charge of a black hole at infinity, one can perform the following Gedankenexperiment as we did in [1]: Assume that there is an (asymptotic) observer who can measure a non-vanishing Skyrme topological charge/baryon number of a large evaporating black hole via the above-described Aharonov-Bohm type experiment. Then, as we argued in [1], this observer can monitor the skyrmion charge/baryon number of the black hole when the black hole shrinks down. In other words, since such a low-energy observer can continuously measure the Skyrme topological charge/baryon number of an evaporating black hole, from his point of view black holes respect baryon number/Skyrme topological charge.

One interesting question is if this conserved charge is revealed in terms of a classical object after the black hole shrinks down to a some critical size. And indeed, we argued in [1], that there is a self-consistent possiblity for such a charge-revealing process. In fact, since for event horizon sizes smaller than a typical length scale associated to the Skyrmion (typically much bigger than L_P) classical black hole objects of the type [8] which carry a skyrmion/baryon hair exist, these are natural candidates which might carry the baryon/skyrmion hair after the black hole shrinks down to that size. In other words, one self-consistent possibility is that, after evaporating down to a certain critical size, a black hole which initially swallowed a skyrmion/baryon and which carries an Aharonov-Bohm type skyrmion/baryon hair undergoes a phase transition during which it develops a classical baryon/skyrmion hair of the type [8]. This option is useful to consider in particular because, given that skyrmion charge/baryon number is conserved, it avoids the necessity to assume the existence of exotic objects such as “Planck size black hole remnants” carrying the baryon number. However, whether this option is realized in nature or some other potentially more exotic option, in case a low-energy observer can measure the skyrmion topological charge/baryon number of a black hole via the above-described Aharonov-Bohm type experiment, this already implies that from the point of view of that low-energy observer the Skyrme topological charge/baryon number is respected by black holes.

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References

[1] G. Dvali and A. Gußmann, “Skyrmion Black Hole Hair: Conservation of Baryon Number by Black Holes and Observable Manifestations,” Nucl. Phys. B **913** (2016) 1001 doi:10.1016/j.nuclphysb.2016.10.017 [arXiv:1605.00543 [hep-th]].

[2] G. Dvali and A. Gußmann, “Aharonov-Bohm protection of black hole’s baryon/skyrmion hair,” arXiv:1611.09370 [hep-th].

[3] M. S. Volkov and D. V. Gal’tsov, “Gravitating nonAbelian solitons and black holes with Yang-Mills fields,” Phys. Rept. **319** (1999) 1 doi:10.1016/S0370-1573(99)00010-1 [hep-th/9810070].

[4] P. Bizon, “Gravitating solitons and hairy black holes,” Acta Phys. Polon. B **25** (1994) 877 [gr-qc/9402016].

[5] J. D. Bekenstein, “Black hole hair: 25 - years after,” In *Moscow 1996, 2nd International A.D. Sakharov Conference on physics* 216-219 [gr-qc/9605059].

[6] C. A. R. Herdeiro and E. Radu, “Asymptotically flat black holes with scalar hair: a review,” Int. J. Mod. Phys. D **24** (2015) no.09, 1542014 doi:10.1142/S0218271815420146 [arXiv:1504.08209 [gr-qc]].

[7] M. S. Volkov, “Hairy black holes in the XX-th and XXI-st centuries,” arXiv:1601.08230 [gr-qc].

[8] H. Luckock and I. Moss, “Black Holes Have Skyrmion Hair,” Phys. Lett. B **176** (1986) 341. doi:10.1016/0370-2693(86)90175-9

S. Droz, M. Heusler and N. Straumann, “New black hole solutions with hair,” Phys. Lett. B **268** (1991) 371. doi:10.1016/0370-2693(91)91592-J

P. Bizon and T. Chmaj, “Gravitating skyrmions,” Phys. Lett. B **297** (1992) 55. doi:10.1016/0370-2693(92)91069-L

T. Tamaki, K. i. Maeda and T. Torii, “Internal structure of Skyrme black hole,” Phys. Rev. D **64** (2001) 084019 doi:10.1103/PhysRevD.64.084019 [gr-qc/0106008].

N. Shiiki and N. Sawado, “Black holes with skyrme hair,” gr-qc/0501025.

N. Sawado, N. Shiiki, K. i. Maeda and T. Torii, “Regular and black hole Skyrmions with axisymmetry,” Gen. Rel. Grav. **36** (2004) 1361 doi:10.1023/B:GERG.0000022392.89396.4b [gr-qc/0401020].

[9] M. Heusler, S. Droz and N. Straumann, “Linear stability of Einstein Skyrme black holes,” Phys. Lett. B **285** (1992) 21. doi:10.1016/0370-2693(92)91294-J

[10] E. Witten, “Global Aspects of Current Algebra,” Nucl. Phys. B **223** (1983) 422. doi:10.1016/0550-3213(83)90063-9

E. Witten, “Current Algebra, Baryons, and Quark Confinement,” Nucl. Phys. B **223** (1983) 433. doi:10.1016/0550-3213(83)90064-0

E. Witten, “Skyrmions And Qcd,” In *Treiman, S.b. (Ed.) Et Al.: Current Algebra and Anomalies*, 529-537 and Preprint - WITTEN, E. (84,REC.OCT.) 7p

[11] T. Johannsen, “Testing the No-Hair Theorem with Observations of Black Holes in the Electromagnetic Spectrum,” Class. Quant. Grav. **33** (2016) no.12, 124001 doi:10.1088/0264-9381/33/12/124001 [arXiv:1602.07694 [astro-ph.HE]].

[12] V. Cardoso and L. Gualtieri, “Testing the black hole ’no-hair’ hypothesis,” Class. Quant. Grav. **33** (2016) no.17, 174001 doi:10.1088/0264-9381/33/17/174001 [arXiv:1607.03133 [gr-qc]].

[13] T. H. R. Skyrme, "A Nonlinear field theory," Proc. Roy. Soc. Lond. A **260** (1961) 127. doi:10.1098/rspa.1961.0018

T. H. R. Skyrme, "A Unified Field Theory of Mesons and Baryons," Nucl. Phys. **31** (1962) 556. doi:10.1016/0029-5582(62)90775-7

[14] G. S. Adkins, C. R. Nappi and E. Witten, "Static Properties of Nucleons in the Skyrme Model," Nucl. Phys. B **228** (1983) 552. doi:10.1016/0550-3213(83)90559-X

[15] G. S. Adkins and C. R. Nappi, "The Skyrme Model with Pion Masses," Nucl. Phys. B **233** (1984) 109. doi:10.1016/0550-3213(84)90172-X

[16] C. Adam, O. Kichakova, Y. Shnir and A. Wereszczynski, Phys. Rev. D **94** (2016) no.2, 024060 doi:10.1103/PhysRevD.94.024060 [arXiv:1605.07625 [hep-th]].

S. B. Gudnason, M. Nitta and N. Sawado, JHEP **1609** (2016) 055 doi:10.1007/JHEP09(2016)055 [arXiv:1605.07954 [hep-th]].

[17] J. A. H. Futterman, F. A. Handler and R. A. Matzner "Scattering from Black Holes," Cambridge Monographs on Mathematical Physics, Cambridge University Press (1988)

[18] K. Glampedakis and N. Andersson, "Scattering of scalar waves by rotating black holes," Class. Quant. Grav. **18** (2001) 1939 doi:10.1088/0264-9381/18/10/309 [gr-qc/0102100].

S. Dolan, C. Doran and A. Lasenby, "Fermion scattering by a Schwarzschild black hole," Phys. Rev. D **74** (2006) 064005 doi:10.1103/PhysRevD.74.064005 [gr-qc/0605031].

S. R. Dolan, "Scattering and Absorption of Gravitational Plane Waves by Rotating Black Holes," Class. Quant. Grav. **25** (2008) 235002 doi:10.1088/0264-9381/25/23/235002 [arXiv:0801.3805 [gr-qc]].

L. C. B. Crispino, S. R. Dolan and E. S. Oliveira, "Electromagnetic wave scattering by Schwarzschild black holes," Phys. Rev. Lett. **102** (2009) 231103 doi:10.1103/PhysRevLett.102.231103 [arXiv:0905.3339 [gr-qc]].

L. C. B. Crispino, S. R. Dolan and E. S. Oliveira, "Scattering of massless scalar waves by Reissner-Nordstrom black holes," Phys. Rev. D **79** (2009) 064022 doi:10.1103/PhysRevD.79.064022 [arXiv:0904.0999 [gr-qc]].

C. F. B. Macedo, E. S. de Oliveira and L. C. B. Crispino, "Scattering by regular black holes: Planar massless scalar waves impinging upon a Bardeen black hole," Phys. Rev. D **92** (2015) no.2, 024012 doi:10.1103/PhysRevD.92.024012 [arXiv:1505.07014 [gr-qc]].

C. F. B. Macedo, L. C. B. Crispino and E. S. de Oliveira, "Scalar waves in regular Bardeen black holes: Scattering, absorption and quasinormal modes," Int. J. Mod. Phys. D **25** (2016) no.09, 1641008 doi:10.1142/S021827181641008X [arXiv:1605.00123 [gr-qc]].

[19] A. Gußmann, "Scattering of massless scalar waves by magnetically charged black holes in Einstein-Yang-Mills-Higgs theory," Class. Quant. Grav. **34** (2017) 065007 doi:10.1088/1361-6382/aa606c [arXiv:1608.00552 [hep-th]].

[20] L. M. Krauss and F. Wilczek, "Discrete Gauge Symmetry in Continuum Theories," Phys. Rev. Lett. **62** (1989) 1221. doi:10.1103/PhysRevLett.62.1221

[21] J. Preskill and L. M. Krauss, "Local Discrete Symmetry and Quantum Mechanical Hair," Nucl. Phys. B **341** (1990) 50. doi:10.1016/0550-3213(90)90262-C

[22] S. R. Coleman, J. Preskill and F. Wilczek, "Quantum hair on black holes," Nucl. Phys. B **378** (1992) 175 doi:10.1016/0550-3213(92)90008-Y [hep-th/9201059].

[23] M. J. Bowick, S. B. Giddings, J. A. Harvey, G. T. Horowitz and A. Strominger, "Axionic Black Holes and a Bohm-Aharonov Effect for Strings," Phys. Rev. Lett. **61** (1988) 2823. doi:10.1103/PhysRevLett.61.2823

[24] T. J. Allen, M. J. Bowick and A. Lahiri, "Axionic Black Holes From Massive Axions," Phys. Lett. B **237** (1990) 47. doi:10.1016/0370-2693(90)90459-J

[25] G. Dvali, "Black holes with quantum massive spin-2 hair," Phys. Rev. D **74** (2006) 044013 doi:10.1103/PhysRevD.74.044013 [hep-th/0605295].