Effective field theory of Inflation in Hořava-Lifshitz gravity

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We study the evolution of primordial power spectra during inflation in the context of Hořava Lifshitz gravity. In this context, scalar perturbations are generally divided by 2 different d.o.f.; an inflaton fluctuation and a scalar graviton due to Lorentz invariance violation. However, we reveal that only the inflaton fluctuation contributes to a primordial curvature perturbation because the scalar graviton acquires its mass much larger than Hubble scale. Besides, the curvature perturbation becomes adiabatic, which is consistent with current observations of the Cosmic Microwave Background. In addition we find that a primordial gravitational waves provide a direct evidence of Lorentz invariance violation.

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1. Inflation as a probe for LIV

Inflation is one of the most attractive phenomena in cosmology. Historically, inflation was firstly introduced to solve the cosmological problems \cite{1}, but inflation can also generate initial condition of the universe from quantum fluctuations that make seeds of cosmic structures. In this sense, we will expect that abundant imprints of high energy physics remains in cosmological observables. In this article, we would like to focus on the imprints Lorentz invariance in the early time of the universe. Lorentz invariance (LI) is one of the most fundamental symmetry in modern physics respected by a lot of fundamental theories. We would like to discuss how observables in inflationary epoch carry the information of Lorentz invariance violation (LIV) at present. In a theoretical point view, LIV is also adequate to interest us. Actually, Hořava-Lifshitz gravity \cite{2} has been developed recently as a counter candidate of quantum gravity instead of string theory. The main aim of our study is to grasp LIV features in inflationary universe and to pioneer the search of LIV phenomena in the cosmological context. More specifically, what we do in this article is that we investigate the features of primordial fluctuations on an inflation model with Hořava-Lifshitz gravity and compute two main observables; tensor to scalar ratio \( r \) and tensor tilt \( n_t \). Usually, \( r \) and \( n_t \) are given by

\[
\begin{align}
  r &= 16c_s \epsilon, \\
  n_t &= -\frac{r}{8c_s}.
\end{align}
\]

These observables can be tested by CMB observations\cite{3}. Besides, there are future proposals to realize direct detection of primordial gravitational waves such as eLISA\cite{4} and DECIGO\cite{5}. In prior to these observations, it is meaningful to provide how LIV features are imprinted in primordial fluctuations. In the next section we briefly summarize what we find.

2. EFT treatment of scalar graviton

Hořava-Lifshitz gravity has two different properties from General Relativity; the anisotropic scaling law and the foliation-preserving diffeomorphism given as follows respectively:

\[
  t \rightarrow b^z t, \quad x \rightarrow b x, \quad t \rightarrow \tilde{t}(t), \quad x \rightarrow \tilde{x} = \tilde{x}(x,t).
\]

Here \( z \) denotes a critical exponent of time in ultraviolet limit. \( z = 1 \) makes no difference from General Relativity. However in Hořava-Gravity \( z \) is generally larger than \( 1 \) to permit to add higher spatial derivatives to keep the anisotropic scaling law. This is crucial to respect renormalizability for quantum correction of graviton self interactions. Hereafter we set \( z = 3 \) for simplicity. In more detail discussion please see reviews\cite{6,7}. Due to the breakdown of general covariance, a new scalar d.o.f, scalar graviton, becomes dynamical. The previous studies shows cosmological perturbations on fixed backgrounds \cite{8,9,10}, but the consistency relation between scalar and primordial gravitational waves given by Eqs. (1.2) have yet to be fully investigated. We build an inflation model that includes interactions between the inflaton and the scalar graviton. Consequently, we...
find that the scalar graviton get decoupled from inflaton before Hubble crossing. This is because
the scalar graviton gains its mass, \( m_K \), which is given by

\[
m_K = \sqrt{\frac{M_P}{M_{LIV}} H} \gg H ,
\]

(2.1)

where \( M_{LIV} \) is a typical scale of LIV\(^1\) and thus the scalar graviton is already decoupled from the
inflaton fluctuation. As a result, the power spectrum of a gauge invariant curvature perturbation \( \zeta \)
is determined only by inflaton and we obtain

\[
\mathcal{P}_\zeta \approx \frac{1}{\epsilon} \frac{M_{LIV}}{M_P} \left( \frac{H}{M_{LIV}} \right)^{3/z-1} .
\]

(2.2)

Here are some comments for the spectrum. First of all, \( \zeta \) is adiabatically conserved due to decoupling
of the scalar graviton. This indicates that the scalar perturbations during inflation we can put
the effect of the scalar graviton describing with an effective field theory approach. Observational
points of view, the two following things are important. The one is that the latest observations of
CMB support adiabatic initial condition of inflation. Therefore, our result is still consistent with the
observations. The other is that flatness of the spectrum. Due to a scaling invariance at \( z = 3 \), \( \mathcal{P}_\zeta \)
becomes scale invariant spectrum. Note that this result we don’t investigate a running correction of
slow-roll parameters.

### 3. PGW as a smoking gun of LIV

Let’s consider the primordial gravitational waves (PGW). P. Creminelli et al.\(^{11}\) shows that
Eq. \( (1.2) \) generally preserves in a inflation model with general covariance. However in Hořava-
Lifshitz gravity, we obtain a modified consistency relation as

\[
n_t = \frac{z - 3}{z} \epsilon .
\]

(3.1)

As we mentioned, we now consider the case \( z = 3 \) so that we find that Eq. \( (1.2) \) can be violated as

\[
n_t = 0 ,
\]

(3.2)

which means PGW provides a direct evidence to indicate LIV in the inflationary epoch. Recently
it has been reported that G-inflation \(^{12}\) or k-inflation \(^{13}\) can provide a violating consistency
relation, all of which break null energy condition. What we have to stress here is that our results is
quite different from that reports because we still respect the null energy condition but the consist-
sency relation is disrespected. This is purely due to LIV in Hořava-Lifshitz gravity. Therefore the
direct detection of PGW can be a smoking gun of Lorentz symmetry.

### 4. Conclusion

We have studied the evolution of primordial power spectra during inflation in the space-time
described by Hořava-Lifshitz gravity. During inflaton not only the inflaton fluctuation but also
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the scalar graviton are dynamical. However, the scalar graviton become massive and is decoupled from inflaton. As a result, gauge invariant curvature perturbation is determined only by inflaton. Considering together that inflaton is in WKB phase before Hubble crossing, the power spectrum of the curvature perturbation becomes adiabatic, which is consistent with current observations of the CMB. In addition, we have found that a primordial gravitational waves provide a direct evidence of Lorentz symmetry violation. We will submit these results in this article soon.

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References