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A proposal of a New Charged Lepton Flavor Violation Experiment: $\mu^- e^- \to e^- e^-$ in muonic atom

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We propose a new process of $\mu^- e^- \to e^- e^-$in a muonic atom for a quest of charged lepton flavor violation. The Coulomb attraction from the nucleus in a heavy muonic atom leads to significant enhancement in its rate, compared to $\mu^- e^- \to e^- e^-$. The search for this process could be complementary with search for other relevant processes and would help shed light upon the nature of charged lepton flavor violation. The wave functions of bound and scattering state leptons are properly treated by solving Dirac equations with Coulomb interaction of the finite nuclear charge distributions. This new effect contributes significantly in particular for heavier atoms, where the obtained decay rate is about one order of magnitude larger than the previous estimation for 208 Pb in particular for contact interactions. We also discuss how to observe the differences among interaction types.

This talk is based on the works[1, 2, 3].

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1. introduction

It has been well recognized that charged lepton flavor violation (CLFV) is important to search for new physics beyond the Standard Model (SM). Rare processes of muons, such as $\mu^+ \to e^+\gamma$ [4], $\mu^+ \to e^+e^-e^+$ [5], and $\mu^- \to e^-$ conversion [6], have given the strongest constraints on new physics models of CLFV interactions [7, 8]. Furthermore, it is expected that experimental sensitivity can be significantly improved in near future measurements.

As a new promising process to search for CLFV interaction, $\mu^- e^- \to e^- e^-$ in a muonic atom was proposed by Koike *et al* [1]. For heavy atoms with large atomic numbers (Z), large enhancement of the $\mu^- e^- \to e^- e^-$ rate due to the Coulomb attraction of the lepton wave functions to a nucleus is expected. Another advantage for $\mu^- e^- \to e^- e^-$ is that it can probe both the four Fermi contact and the photonic interactions, as in the $\mu^+ \to e^+e^-e^+$ decay and $\mu^- \to e^-$ conversion. In $\mu^- e^- \to e^- e^-$, a sum of the energies of two electrons in the final state would be $m_\mu + m_e - B_\mu - B_e$, where m_μ and m_e are the masses of a muon and an electron respectively, and B_μ and B_e are binding energies of the muon and electron in a muonic atom, respectively. The energy of each electron in the final state is about $m_\mu/2$, and they are emitted almost back-to-back. The search for $\mu^- e^- \to e^- e^-$ is proposed in the COMET Phase-I experiment at J-PARC, Japan [9]. This new process could be essential to identify, at near future experiments [10], a scenario for new physics via the addition of sterile neutrinos

The initial work [1] showed that the atomic number (Z) dependence of the $\mu^- e^- \to e^- e^-$ transition rate is expected to be of Z^3, owing to the probability density of the wave functions of the Coulomb-bound electrons at origin. This result was obtained by a plane wave approximation of the outgoing electrons and non-relativistic approximation of the bound states.

In this talk, I show improved analyses for the $\mu^- e^- \to e^- e^-$ process. In Sec. 2, we start from the effective CLFV interaction for the $\mu^- e^- \to e^- e^-$ process. The multipole expansion formula on the $\mu^- e^- \to e^- e^-$ rate is extended to the photonic interaction process. In Sec. 3, the improved treatments of lepton wave functions for the photonic interaction, in particular the atomic number (Z) dependence of the rate, are discussed. Then, we propose a possibility to distinguish the photonic interaction from the four Fermi interaction, using the atomic number (Z) dependenceand its angular-energy distribution of the emitted electrons. Our analysis is summarized in Sec. 4.

2. Formulation

The effective Lagrangian for $\mu^- e^- \to e^- e^-$ consists of the photonic interaction $\mathscr{L}_{\text{photo}}$ and the four Fermi interaction $\mathscr{L}_{\text{contact}}$, as follows:

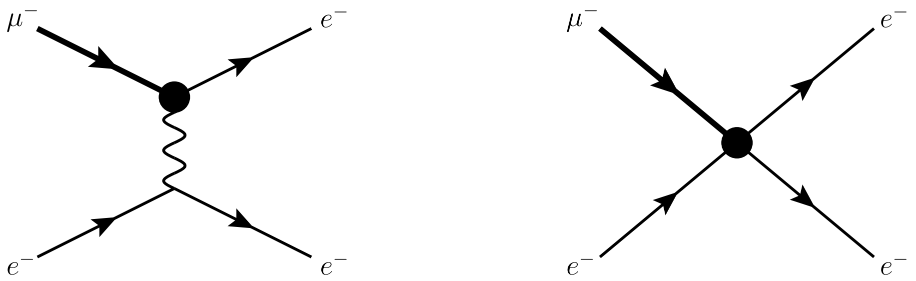
$$\mathscr{L}_{CLFV} = \mathscr{L}_{\text{photo}} + \mathscr{L}_{\text{contact}}, \tag{2.1}$$

where

$$\mathscr{L}_{\rm photo} = -\frac{4G_F}{\sqrt{2}} m_\mu \left[A_R \overline{e_L} \sigma^{\mu\nu} \mu_R + A_L \overline{e_R} \sigma^{\mu\nu} \mu_L\right] F_{\mu\nu} + [h.c.], \tag{2.2}$$

$$\begin{aligned}\mathscr{L}_{\rm contact} = -\frac{4G_F}{\sqrt{2}} &[g_1 (\overline{e_L} \mu_R)(\overline{e_L} e_R) + g_2 (\overline{e_R} \mu_L)(\overline{e_R} e_L) \\ &+ g_3 (\overline{e_R} \gamma_\mu \mu_R)(\overline{e_R} \gamma^\mu e_R) + g_4 (\overline{e_L} \gamma_\mu \mu_L)(\overline{e_L} \gamma^\mu e_L) \\ &+ g_5 (\overline{e_R} \gamma_\mu \mu_R)(\overline{e_L} \gamma^\mu e_L) + g_6 (\overline{e_L} \gamma_\mu \mu_L)(\overline{e_R} \gamma^\mu e_R)] + [h.c.].\end{aligned} \tag{2.3}$$

Here, $G_F = 1.166 \times 10^{-5}\mathrm{GeV}^{-2}$ is the Fermi coupling constant, and $A_{L/R}$ and g_i $(i = 1, \cdots, 6)$ are the coupling constants which are determined by new physics models. The left- and right-handed fields $\psi_{L/R}$ are defined as $\psi_{L/R} = P_{L/R}\psi$, using the projection operators $P_{L/R} = (1 \mp \gamma_5)/2$.



(a) (b)

Figure 1: The diagrams representing $\mu^- e^- \to e^- e^-$: the one-photon-exchange photonic interaction (a) and the four Fermi contact interaction (b). The black closed circle shows the CLFV interaction.

The one-photon-exchange photonic interaction shown in Fig. 1 (a) is given by the photonic interaction in Eq. (2.2) together with the electromagnetic interaction of

$$\mathscr{L}_{em} = -q_e \bar{e} \gamma^\lambda e A_\lambda . \tag{2.4}$$

Here $q_e = -e$ is a charge of an electron. The four Fermi interaction shown in Eq. (2.3) and Fig. 1 (b) has been studied [2]. The transition amplitude M of $\mu^- e^- \to e^- e^-$ is given by,

$$2\pi i \delta(E_{\rm f} - E_{\rm i}) M(\boldsymbol{p}_1 s_1 \boldsymbol{p}_2 s_2; \alpha_\mu s_\mu \alpha_e s_e) = \langle e^{s_1}_{\boldsymbol{p}_1} e^{s_2}_{\boldsymbol{p}_2} | T[\exp\left\{ i \int d^4x (\mathscr{L}_{CLFV} + \mathscr{L}_{em}) \right\}] | \mu^{s_\mu}_{1S} e^{s_e}_{\alpha_e} \rangle \tag{2.5}$$

$$M(\boldsymbol{p}_1 s_1 \boldsymbol{p}_2 s_2; \alpha_\mu s_\mu \alpha_e s_e) = M_{\rm photo}(\boldsymbol{p}_1 s_1 \boldsymbol{p}_2 s_2; \alpha_\mu s_\mu \alpha_e s_e) + M_{\rm contact}(\boldsymbol{p}_1 s_1 \boldsymbol{p}_2 s_2; \alpha_\mu s_\mu \alpha_e s_e). \tag{2.6}$$

Here E_i and E_f are the energy of the initial and final state given as $E_i = m_\mu - B^{1S}_\mu + m_e - B^{\alpha_e}_e$ and $E_f = E_{p_1} + E_{p_2}$ respectively. And E_{p_i} is an energy of the electron with its momentum p_i and B^α_l is a binding energy of the lepton l in the state α. The principle quantum number n and κ [11, 12] of the bound muon and electron are collectively denoted by α_μ and α_e, respectively. We assume the initial muon is in its $1S_{1/2}$ ($n = 1$ and $\kappa = -1$) state, while we have included contribution of all bound electrons. The expression of $M_{\rm contact}$ is given as [2],

$$\begin{aligned} M_{\rm contact}(\boldsymbol{p}_1, s1, \boldsymbol{p}_2, s2; \alpha_\mu, s_\mu, \alpha_e, s_e) &\equiv \int d^3r \langle e^{s_1}_{\boldsymbol{p}_1} e^{s_2}_{\boldsymbol{p}_2} | \mathscr{L}_{\rm contact} | \mu^{s_\mu}_{\alpha_\mu} e^{s_e}_{\alpha_e} \rangle \\ = -\frac{4G_F}{\sqrt{2}} \sum_{i=1}^{6} g_i &\left[\int d^3r \overline{\psi}^{e(-)}_{\boldsymbol{p}_1,s_1}(\boldsymbol{r}) O^A_i \psi^\mu_{\alpha_\mu,s_\mu}(\boldsymbol{r}) \overline{\psi}^{e(-)}_{\boldsymbol{p}_2,s_2}(\boldsymbol{r}) O^B_i \psi^e_{\alpha_e,s_e}(\boldsymbol{r}) - (1 \leftrightarrow 2) \right], \end{aligned} \tag{2.7}$$

and the amplitude of the photonic interaction M_{photo} is given as [3],

$$M_{\text{photo}}(\boldsymbol{p}_1, s_1, \boldsymbol{p}_2, s_2; 1S, s_\mu, \alpha_e, s_e) = \left[\frac{8G_F}{\sqrt{2}} m_\mu q_e \int d^3x_1 d^3x_2 G_\nu\left(\boldsymbol{x}_1, \boldsymbol{x}_2; m_\mu - B_\mu^{1S} - E_{p_1}\right)\right.$$
$$\left. \times \overline{\psi}^e_{\boldsymbol{p}_1,s_1}(\boldsymbol{x}_1)\sigma^{\mu\nu}\left(A_L P_L + A_R P_R\right)\psi^\mu_{1S,s_\mu}(\boldsymbol{x}_1)\overline{\psi}^e_{\boldsymbol{p}_2,s_2}(\boldsymbol{x}_2)\gamma_\mu\psi^e_{\alpha_e,s_e}(\boldsymbol{x}_2)\right] - \left[\{\boldsymbol{p}_1, s_1\} \leftrightarrow \{\boldsymbol{p}_2, s_2\}\right]. \quad (2.8)$$

The second term $\{\boldsymbol{p}_1, s_1\} \leftrightarrow \{\boldsymbol{p}_2, s_2\}$ is obtained by exchanging the quantum numbers of the final electrons in the first term. The photonic interaction is a finite range interaction between the two leptons and $G_\nu(\boldsymbol{x}_1, \boldsymbol{x}_2; q_0)$ is defined as

$$G_\nu\left(\boldsymbol{x}_1, \boldsymbol{x}_2; q_0\right) = \int \frac{d^3q}{(2\pi)^3}\frac{iq_\nu e^{-i\boldsymbol{q}\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)}}{|\boldsymbol{q}|^2 - q_0^2 - i\varepsilon}. \quad (2.9)$$

To proceed, we derive a multipole expansion of the transition amplitude. Based on a standard partial wave expansion of the scattering wave functions and the bound state wave functions of Dirac particles given in Eqs. (11), (12) and (13) of Ref. [2], the transition amplitude is expressed as

$$\begin{aligned} M(\boldsymbol{p}_1, s_1, \boldsymbol{p}_2, s_2; 1S, s_\mu, \alpha_e, s_e) = & 2\sqrt{2}G_F \sum_{\kappa_1,\kappa_2,\nu_1,\nu_2,m_1,m_2} (4\pi)^2 (-i)^{l_{\kappa_1}+l_{\kappa_2}} e^{i\left(\delta_{\kappa_1}+\delta_{\kappa_2}\right)} \\ & \times Y_{l_{\kappa_1},m_1}(\hat{p}_1) Y_{l_{\kappa_2},m_2}(\hat{p}_2)\left(l_{\kappa_1}, m_1, 1/2, s_1 | j_{\kappa_1}, \nu_1\right)\left(l_{\kappa_2}, m_2, 1/2, s_2 | j_{\kappa_2}, \nu_2\right) \\ & \times \sum_{J,M}\left(j_{\kappa_1}, \nu_1, j_{\kappa_2}, \nu_2 | J, M\right)\left(j_{-1}, s_\mu, j_{\kappa_e}, s_e | J, M\right) \\ & \times \frac{\sqrt{2\left(2j_{\kappa_1}+1\right)\left(2j_{\kappa_2}+1\right)\left(2j_{\kappa_e}+1\right)}}{4\pi} N\left(J, \kappa_1, \kappa_2, E_{p_1}, \alpha_e\right), \end{aligned} \quad (2.10)$$

where $(l_\kappa, m, 1/2, s | j_\kappa, \nu)$ and $Y_{l_\kappa,m}(\hat{p})$ are the Clebsch-Gordan coefficients and the spherical harmonics, respectively. Here l_κ, j_κ are the orbital and the total angular momentum of the state with κ. δ_κ is a phase shift of the scattering state. The partial wave amplitude, $N(J, \kappa_1, \kappa_2, E_{p_1}, \alpha_e)$ for the contact and the photonic interactions is given in [2] and [3] respectively.

Finally, the angular and energy distributions of the emitted electron are expressed in terms of the partial wave amplitude by

$$\begin{aligned} \frac{d^2\Gamma_{\alpha_e}}{dE_{p_1} d\cos\theta} = & \frac{G_F^2}{2\pi^3}|\boldsymbol{p}_1||\boldsymbol{p}_2| \sum_{\kappa_1,\kappa_2,\kappa_1',\kappa_2',J,l} (2J+1)(2j_{\kappa_e}+1)\left(2j_{\kappa_1}+1\right)\left(2j_{\kappa_2}+1\right)\left(2j_{\kappa_1'}+1\right)\left(2j_{\kappa_2'}+1\right) \\ & \times \frac{1+(-1)^{l_{\kappa_1}+l_{\kappa_1'}+l}}{2}\frac{1+(-1)^{l_{\kappa_2}+l_{\kappa_2'}+l}}{2} i^{-l_{\kappa_1}-l_{\kappa_2}+l_{\kappa_1'}+l_{\kappa_2'}} e^{i\left(\delta_{\kappa_1}+\delta_{\kappa_2}-\delta_{\kappa_1'}-\delta_{\kappa_2'}\right)} \\ & \times (j_{\kappa_1}, 1/2, j_{\kappa_1'}, -1/2 | l, 0)(j_{\kappa_2}, 1/2, j_{\kappa_2'}, -1/2 | l, 0) W(j_{\kappa_1} j_{\kappa_2} j_{\kappa_1'} j_{\kappa_2'}; Jl) \\ & \times (-1)^{J-j_{\kappa_2}-j_{\kappa_2'}} N(J, \kappa_1, \kappa_2, E_{p_1}, \alpha_e) N^*(J, \kappa_1', \kappa_2', E_{p_1}, \alpha_e) P_l(\cos\theta), \end{aligned} \quad (2.11)$$

where $P_l(x)$ is Legendre polynomials.

3. Results

The wave functions of the bound muon and electron and the emitted electrons in the final state are obtained by numerically solving Dirac equations with the Coulomb potential. We use the uniform nuclear charge distribution, $\rho_C(r)$, for the Coulomb potential, which is given as

$$\rho_C(r) = \frac{3Ze}{4\pi R^3}\theta(R-r), \tag{3.1}$$

with $R = 1.2A^{1/3} fm$. We have also examined a realistic charge distribution of the Woods-Saxon form. However the rate changes by less than 1% from that of the uniform distribution. Therefore we decided to use the uniform charge distribution is decided to use in our calculation from now on. A sufficiently large number of partial waves of the scattering electron state has to be included. The convergence property is almost the same as the contact interaction. For ^{40}Ca, we have to sum the partial waves up to $|\kappa| \leq 20$. For larger Z nuclei, the rate converges faster due to a smaller radius of the bound muon.

The branching ratio of $\mu^- e^- \to e^- e^-$ is given as

$$Br(\mu^- e^- \to e^- e^-) \equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \to e^- e^-), \tag{3.2}$$

where $\tilde{\tau}_\mu$ is a mean life time of the muonic atom, given in Ref. [14].

The upper limit of this branching ratio is calculated by the upper limit for effective couplings derived from $\mu^- e^- \to e^- e^-$ for g_i and from $\mu^+ \to e^+\gamma$ for A_R and A_L

Assuming the dominance of g_1 we get the data for the current upper limit for $\mu^- e^- \to e^- e^-$ as shown in Fig.2 shows the data assuming the photonic interaction, in Fig.3. We draw the figure of the upper limit of $Br(\mu^- e^- \to e^- e^-)$

The dashed (blue) line in both figures shows the result of previous work [1], whereas the results of this work with taking into account the 1S electrons and all the bound electrons are shown in a solid (red) and dotted (orange) lines, respectively. From the improved estimations using the relativistic Coulomb lepton wave functions, the branching ratio $Br(\mu^- e^- \to e^- e^-)$ is about 10^{-19} for ^{208}Pb. The non-1S bound electrons increase the branching ratio by about 20%.

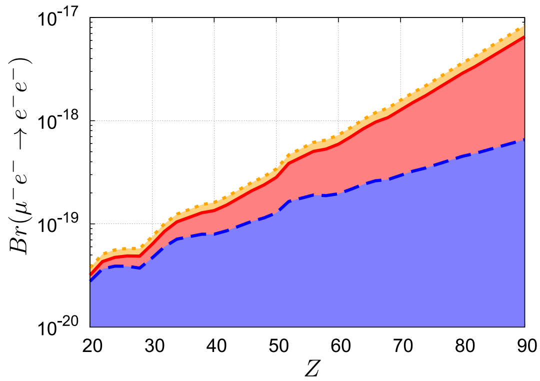


Figure 2: Upper limits on CLFV decay of the muonic atom $Br(\mu^- e^- \to e^- e^-)$, imposed by the experimental upper limits of $Br(\mu^+ \to e^+ e^+ e^-)$. The dashed(blue) curve shows the result of previous work [1]. Our results including only $1S$ bound electrons and all S-state bound electrons are shown by the solid(red) curve and the dotted(orange) curve, respectively.

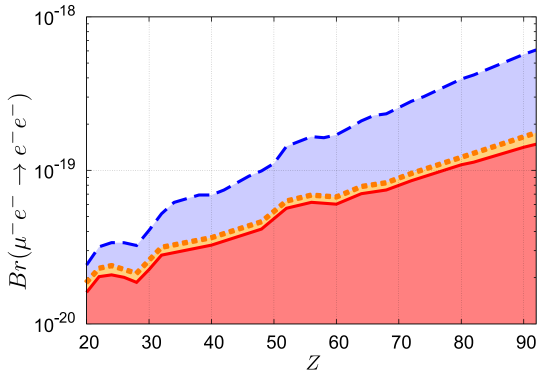


Figure 3: Upper limits on $Br(\mu^- e^- \to e^- e^-)$, constrained by the experimental upper limits of $Br(\mu^+ \to e^+\gamma) < 4.2 \times 10^{-13}$ [4]. The dashed (red) curve shows the result of previous work [1]. Our results including only the 1S bound electrons and all the S-state bound electrons are shown by the solid (red) and the dotted (orange) lines, respectively.

4. Summary

We have analyzed the $\mu^- e^- \to e^- e^-$ CLFV process in muonic atoms. We find that the relativistic treatment of the emitted electrons and bound leptons is important for the qualitative understanding of the rate, in particular the atomic number Z dependence of the rate and the angular and energy distribution of electrons.

We note the Z dependence of the $\mu^- e^- \to e^- e^-$ rate and the distributions of emitted electrons could be used to distinguish between the photonic and the four Fermi contact CLFV interactions.

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