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Discriminating sterile neutrinos and unitarity violation with CP invariants

Philipp Sicking[*]
Fakultät für Physik, TU Dortmund,
Otto-Hahn-Str. 4, D-44221 Dortmund, Germany
E-mail: philipp.sicking@udo.edu

We present a new method to analyze upcoming results in the search for CP violating neutrino oscillations. The CP violating amplitudes $\mathscr{A}^{kj}_{\alpha\beta}$ provide parametrization independent observables, which will be accessible by experiments soon. The strong prediction of a unique $\mathscr{A}^{kj}_{\alpha\beta}$ (the Jarlskog invariant) in case of the standard three neutrino model does not hold in models with new physics beyond the Standard Model. Nevertheless there are still correlations among the amplitudes depending on the specific model. Due to these correlations it is possible to reject specific new physics models by determining only 3 of the CP violating amplitudes.

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[*] Speaker.

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1. Introduction

A first hint for a maximal CP violation in neutrino oscillations has been reported by T2K [1, 2]. This situation cannot be understood as a proof of the minimal three neutrino picture, though. As has been shown by several authors, new physics models can fake a signal at current experiments which look like satisfying the three neutrino paradigm [3, 4, 5, 6, 7].

We present a novel approach to analyse upcoming neutrino oscillation data in the light of CP violation based on reference [8].

Neutrino oscillation probabilities are described by introducing the mixing matrix U, parametrizing the transformation from neutrino mass to flavor eigenstates, $|\nu_\alpha\rangle = \sum_k U_{\alpha k} |\nu_k\rangle$:

$$P_{\nu_\alpha\to\nu_\beta}(t) = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} e^{-i\frac{\Delta m^2_{kj} L}{2E}} \tag{1.1}$$

$$= \delta_{\alpha\beta} - 4\sum_{k>j}^{N} \mathrm{Re}(U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}) \sin^2\left(\frac{\Delta m^2_{kj} L}{4E}\right)$$

$$+2\sum_{k>j}^{N} \mathrm{Im}(U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}) \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right), \tag{1.2}$$

where $\mathscr{A}^{kj}_{\alpha\beta} = \mathrm{Im}(U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j})$. For antineutrinos the last term switches its sign, so the CP violation $P_{\nu_\alpha\to\nu_\beta} - P_{\bar\nu_\alpha\to\bar\nu_\beta}$ depends only on the CP violating amplitudes $\mathscr{A}^{kj}_{\alpha\beta}$. Here, N indicates the number of light neutrinos involved in the oscillation process. If all neutrino mass eigenstates are small compared to the relevant energy scale at production and detection (for instance the pion mass) all eigenstates are involved in the oscillation process and the mixing matrix U is unitary. If, on the other hand, at least one mass eigenstate cannot be produced due to kinematics, or the heavy flavors can be integrated out, the resulting effective mixing matrix U can be non-unitary.

A common approximative parametrization used in the literature is based on a series expansion in $\alpha = \frac{\Delta m^2_{31}}{\Delta m^2_{21}} \ll 1$ and the unitarity of the 3×3 mixing matrix[9]. Here we rely on the exact expressions given in equation (1.2) instead, which is invariant under reparametrization. In particular the CP violating amplitudes $\mathscr{A}^{kj}_{\alpha\beta}$ are independent of the parametrization [10, 11] and can be determined in various extensions to the SM case.

A specific feature which had already been pointed out by Jarlskog [12],[13] is that in the case of exactly three flavors and a unitary mixing matrix U, all CP violating amplitudes $\mathscr{A}^{kj}_{\alpha\beta}$ have identical absolute values.

Inspired by previous work [10, 11] we take a closer look to sums and ratios of the CP violating amplitudes $\mathscr{A}^{kj}_{\alpha\beta}$ and find useful correlations among them. These correlations depend highly on the specific model and therefore provide a useful test for new physics in CP violating neutrino oscillations.

2. Analytic treatment of 3+1 ν

A popular extension of the three neutrino model is to add one additional light sterile neutrino [14, 15]. In this model the mixing matrix U is now a 4×4 unitary mixing matrix but the 3×3 sub matrix is not unitary anymore. Although the resulting amplitudes are no longer unique, they are related due to the unitarity of the complete mixing matrix. By exploiting these relations it has been shown for four flavors that all amplitudes can be reduced to only three independent CP violating amplitudes [16].

In total there exist $4 \times 4 \times 4 \times 4 = 256$ ($\alpha, \beta \in \{e, \mu, \tau, s\}$ and $k, j \in \{1,2,3,4\}$) different CP violating amplitudes $\mathscr{A}^{kj}_{\alpha\beta} = \mathrm{Im}(U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j})$, whereas the number is strongly reduced by the fact that $\mathscr{A}^{kj}_{\alpha\beta} = 0$ for $\alpha = \beta$ or $k = j$ and due to symmetry, $\mathscr{A}^{kj}_{\alpha\beta} = \mathscr{A}^{kj}_{\beta\alpha}$ and $\mathscr{A}^{kj}_{\alpha\beta} = \mathscr{A}^{jk}_{\alpha\beta}$. Therefore it is sufficient to only consider $\mathscr{A}^{kj}_{\alpha\beta}$ where $\alpha < \beta$ and $k > j$. Note that the previous relations hold due to the definition of $\mathscr{A}^{kj}_{\alpha\beta}$ regardless of the underlying U and are not specific for the 3+1ν model. This reduces the number of CP violating amplitudes to 36. These 36 amplitudes are not independent of each other and can be expressed via only nine amplitudes (see Appendix A in [8]). Again, these nine amplitudes can be expressed by three remaining amplitudes via analytical expressions (see [8] for all resulting relations).

To emphasize the differences between 3ν and $3+1\nu$ we want to highlight following relations:

$$\mathscr{A}^{31}_{e\mu} = -\mathscr{A}^{32}_{e\mu} + \mathscr{A}^{43}_{e\mu} \tag{2.1}$$

$$\mathscr{A}^{21}_{e\tau} = -\mathscr{A}^{32}_{\mu\tau} + \mathscr{A}^{43}_{\tau s} \tag{2.2}$$

$$\mathscr{A}^{31}_{e\tau} = -\mathscr{A}^{32}_{e\tau} - \mathscr{A}^{32}_{\tau s} + \mathscr{A}^{43}_{\tau s} \tag{2.3}$$

The relations reduce to the 3ν case, if no mixing with the light neutrino takes place.

3. Numeric Analysis of sterile neutrinos and non-unitary scenarios

The relations in the previous section rely on the unitarity of the resulting $3+1\nu$ model. In general these relations are, if possible, harder to find and more complicated. An easier approach is to use a numeric analysis of the correlations of the different amplitudes for different models. Therefore we pick random numbers for all parameters in the specific model. Since the elements of U are independent of the parametrization we are free to choose the standard parametrization from [17].

To check if the generated combination of parameters satisfy current experimental bounds, we compare the entries of the 3×3 sub matrix of U with the bounds presented in [18], where a global fit is performed without implying a unitarity of $U^{3\times 3}$. For a viable combination of parameters all accessible amplitudes $\mathscr{A}^{kj}_{\alpha\beta}$ are calculated and extracted. For each model we extracted 100,000 viable combinations. To show the correlation we performed a kernel density estimation for different combination of amplitudes, i.e. estimating the underlying probability density function by summing up Gaussian kernels placed on every data point.

We compare 4 different approaches of neutrino physics beyond the three neutrino paradigm:

(i) a model of one additional light sterile neutrino ($3+1\nu$),

(ii) a model of two additional light sterile neutrinos ($3+2\nu$),

(iii) a scenario of non-unitarity without additional constraints (NU) realized by modifying the unitary matrix with a lower triangular matrix α[19, 20, 21]

$$U_{NU} = (I-\alpha)U^{3\times3} = \begin{pmatrix} 1-\alpha_{ee} & 0 & 0 \\ \alpha_{e\mu} & 1-\alpha_{\mu\mu} & 0 \\ \alpha_{\tau e} & \alpha_{\mu\tau} & 1-\alpha_{\tau\tau} \end{pmatrix} U^{3\times3}, \tag{3.1}$$

(iv) a scenario of non-unitarity where additional fermions trigger rare decays like $\mu \to e\gamma$. The corresponding constraints from rare decays and electroweak precision observables are presented in [22] ("minimal unitarity violation" (MUV), the non unitarity is parametrized as in scenario (iii))

4. Results

The 95% CL of the generated kernel density estimates for oscillations of ν_μ are shown in figure 1. As can be seen clearly for the scenarios with additional light neutrinos and non unitarity without constraints, the corresponding parameter spaces allow for significant deviation from the SM prediction of uniform CP violating. The MUV scenario albeit provides only a comparatively small allowed region. The strong constraints for the unitary violating parameters α as priors strongly restrict deviations from the SM prediction. The allowed regions fulfill all current bounds and display the current uncertainties and the not yet determined CP phase(s).
The differences between the $3+1\nu$- and $3+2\nu$-model are negligible. Due to invariance under re-parametrization the amplitudes in the 3×3 sub matrix do not change by rotations in the 4-5-Plane in case of a $3+2\nu$-model. To investigate a difference between $3+1\nu$ and $3+2\nu$ scenarios, amplitudes with sterile states or additional mass squared differences have to be taken into account which are not expected to be accessible experimentally in the near future.
Comparing the models with additional light neutrinos with the scenario of unconstrained non-unitarity one can find large deviations. The scenario of non unitarity provides viable parameter sets which are far outside the 95% CL of the models with additional light neutrinos.
The MUV scenario provides only a small deviation from the SM due to the strong constraints from electroweak precision observables. The expected deviations are out of reach of current experiments. Therefore a sizable measured deviation from the SM has to have another source than the MUV scenario.

5. Conclusion

We have presented a new method to test and discriminate the standard three neutrino paradigm and several extensions based on the study of various combinations of CP violating amplitudes $\mathscr{A}^{kj}_{\alpha\beta}$. These amplitudes are easily accessible via oscillation experiments searching for CP violation. The amplitudes and the relations among them have been translated into the notation commonly used in the neutrino community. Moreover, the concept has been generalized to scenarios with five neutrinos and non-unitary mixing matrices. Powerful discriminators between different scenarios of

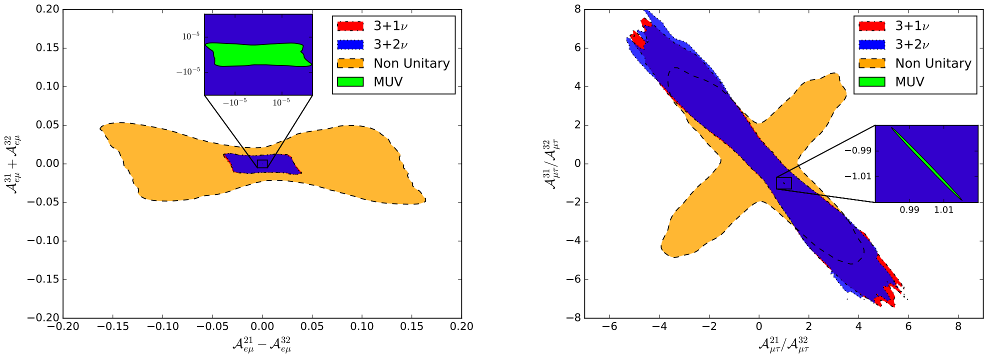

Figure 1: Kernel density estimates for the different scenarios: $3+1\nu$ in red, $3+2\nu$ in blue, Non-Unitarity in yellow and Minimal Unitarity Violation in green. In the left (right) panel it is shown the differences (ratios) of the 3 different CP violating amplitudes in the $\nu_e \to \nu_\mu$ ($\nu_\mu \to \nu_\tau$) -channel. The colored area corresponds to the 95% CL of the KDE.

physics beyond the SM can be exploited once experiments determine three different amplitudes. In this case it is possible to rule out not only the three neutrino paradigm but also models of additional sterile light neutrinos or the scenario of MUV in large regions of the respective parameter spaces. On the other hand, a determination of a unique amplitude would be in agreement with both the three neutrino model but also with specific parameter combinations of new physics models.
Note, that these calculations rely on the vacuum values of neutrino properties. They are independent of specific mass differences.

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