Probing the Low $x$ Gluon with Exclusive $J/\psi$ Production

S. P. Jones$^a$, A. D. Martin$^b$, M. G. Ryskin$^{b,c}$ and T. Teubner$^d$

$^a$ Max Planck Institute for Physics, Föhringer Ring 6, 80805 Munich, Germany
$^b$ Institute for Particle Physics Phenomenology, Durham University, Durham DH1 3LE, U.K.
$^c$ Petersburg Nuclear Physics Institute, NRC Kurchatov Institute, Gatchina, St. Petersburg, 188300, Russia
$^d$ Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, U.K.

E-mail: sjonessmpp.de, a.d.martindurham.ac.uk, ryskinthd.pnpi.spb.ru, thomasteubnerliverpool.ac.uk

We present an updated and improved extraction of the low-$x$ gluon distribution from the HERA and LHCb exclusive $J/\psi$ production data in the context of $k_T$ factorisation. Critically, we use a more precise expression for the photon flux and updated rapidity gap survival factors.

In collinear factorisation, exclusive $J/\psi$ photoproduction receives large perturbative corrections at next-to-leading order which depend sensitively on the factorisation scale. This prevents the use of the available data to extract a gluon PDF within this framework. Attempts to understand and overcome the problems caused by the large perturbative corrections are discussed.
1. Introduction

The first preliminary data on $J/\psi$ ultra-peripheral production $pp \rightarrow p + J/\psi + p$ at LHCb have been presented last year [1]. In principle, these new data allow the gluon distribution to be probed down to $x \sim 10^{-6}$ at a low scale $Q \sim 1.5$ GeV where it is only poorly constrained in global fits of the parton distribution functions (PDFs).

In order to be consistent with the treatment of other processes included in the global fits, one would like to study the process using the collinear factorisation framework. Here the process has been calculated at next-to-leading order (NLO) accuracy in QCD [2]. However, due to the appearance of logarithms which can become large at high energy (small $x$), it is not completely straightforward to include exclusive heavy vector meson production data into the global fits. Furthermore, for low scale processes, such as $J/\psi$ production, contributions of $\mathcal{O}(Q_0^2/M_J^2)$, where $Q_0$ is the PDF input scale, could be important.

In Section 3 we describe a partial scale fixing procedure that aims to minimize the impact of the high energy large logarithms which appear multiplied by a logarithm of the factorisation scale. We also show the results of applying a “$Q_0$ cut”, which attempts to correct for a double counting of contributions of $\mathcal{O}(Q_0^2/M_J^2)$. In Section 4 we instead study the process within the $k_T$ factorisation framework. We present a combined fit to the $J/\psi$ HERA photo/electroproduction data [3, 4] and the LHCb ultra-peripheral production data [5, 1]. The fit indicates the potential of these data to constrain the gluon distribution.

2. Framework

The photoproduction $\gamma p \rightarrow V + p, V = J/\psi, \Upsilon$ of a heavy vector meson (HVM) is described in terms of the convolution of a perturbatively calculable coefficient function, a function describing the distribution of partons in the proton and a function describing the formation of the meson. Throughout we assume that the formation of the HVM is proportional to the branching fraction to electrons $\Gamma(V \rightarrow e^+e^-)$, for which we take the experimentally measured value. In the language of

![Figure 1: Framework used in (left panel) the $k_T$ factorisation approach and (right panel) the collinear factorisation approach. The quarks connected to the photon are massive ($c$ or $b$) quarks which form the vector meson $V = J/\psi, \Upsilon$. The dashed box in the right panel encloses a (5-leg) diagram that contributes to the coefficient function $C_{k_T}^{LO}$.](attachment:image.png)
non-relativistic QCD [6] this assumption corresponds to taking only the leading non-perturbative matrix element.

In the collinear factorisation framework the photoproduction amplitude can be written as [2]

$$A \propto \int_{-1}^{1} dx \left[ C_g(x, \xi) F_g(x, \xi) + \sum_{q=u,d,s} C_q(x, \xi) F_q(x, \xi) \right], \quad (2.1)$$

where $F_g$ and $F_q$ are related to the Generalized Parton Distributions (GPDs), $C_g$ and $C_q$ are coefficient functions and $x, \xi$ are parton momentum fractions, see Figure 1 (right panel). At LO only the gluon initiated channel contributes whilst at NLO and beyond there is also a contribution from the quark channel.

In the $k_T$ factorisation framework the incoming partons are allowed also to carry some momentum $k_T$ transverse to the emitting hadron. In this work we additionally make the high energy “maximal skew” approximation in which the rightmost gluon in Figure 1 carries no momentum in the $+$-direction, see Figure 1 (left panel). We describe the parton distribution in terms of the so-called “skewed unintegrated” PDFs [7, 8].

3. Scale Fixing and the $Q_0$ Cut

![Figure 2: The imaginary part of the tree-level, $A^{(0)}_g$, and the 1-loop, $A^{(1)}_g + A^{(1)}_q$, $J/\psi$ photoproduction amplitude as a function of the $\gamma p$ centre-of-mass energy $W$ (left panel) without the scale fixing procedure and (right panel) with the scale fixing procedure for $\mu^2 = 2.4$ GeV$^2$. The dot-dashed, solid and dashed lines correspond to the low, central and high values of the scale $\mu^2 \equiv \mu^2 = 1.7, 2.4, 4.8$ GeV$^2$ respectively.](image)

In Figure 2 (left panel) and Figure 3 (left panel) we show the tree-level and 1-loop amplitude for $J/\psi$ and $\Upsilon$ photoproduction computed using Equation 2.1. The GPDs are obtained by applying the Shuvaev transform [9, 10] to the CTEQ6.6 partons [11]. For both processes there is a large scale dependence both at the tree-level and at the 1-loop level. Typically, one expects that the scale dependence of the coefficient function at NLO partly compensates that of the strong coupling and the GPD, leading to a reduced dependence. However, for the photoproduction process this seems not to be the case.

The origin of the large scale dependence becomes clear by examining the amplitude at high-energy $W^2 \gg M_V^2$ (where $W$ is the centre-of-mass energy of the incoming $\gamma p$ system and $M_V$ is
the mass of the vector meson). The imaginary part of the amplitude dominates and the leading contribution to the NLO correction comes from the region $\xi \ll |x| \ll 1$. At high energy the 1-loop contribution to the amplitude may be written as \[2, 12]\[
A^{(1)} \approx -i\alpha_s(\mu_R)C_R^{(0)} \ln \left(\frac{m_F^2}{\mu_F^2}\right) \left[ C_A \int_{\xi}^{1} \frac{dx}{x} F_g(x, \xi, \mu_F) + C_F \int_{\xi}^{1} dx (F_S(x, \xi, \mu_F) - F_S(-x, \xi, \mu_F)) \right],
\]
where $\mu_R, \mu_F$ are the renormalization and factorisation scales respectively, $m$ is the mass of the $c$ quark (for $J/\psi$) or $b$ quark (for $\Upsilon$) and $C_R^{(0)}$ is the tree-level coefficient function. Inserting $F_g(x, \xi, \mu_F) \sim \text{const}$ and $F_S(x, \xi, \mu_F) \sim 1/x$ (at small $x$) we see that the NLO amplitude contains terms $\sim \alpha_s(\mu_R)^2 \ln(m^2/\mu_F^2) \ln(1/\xi)$ which induce a large factorisation scale dependence for small $\xi$. For the photoproduction process the logarithmically enhanced scale dependence of the coefficient function for small $\xi$ is not compensated by the behaviour of the GPD, leading to the large scale dependence.

One way to reduce the scale dependence of the NLO result is to fix the factorisation scale in part of the calculation \cite{12}. The amplitude at some factorisation scale $\mu_f$ may be written as

$$A^{(0+1)} \sim C^{(0)} \otimes F(\mu_f) + \alpha_s(\mu_R)C^{(1)}(\mu_f) \otimes F(\mu_f),$$

here the tree-level coefficient function $C^{(0)}$ does not depend on the factorisation scale. We are free to evaluate the LO contribution at a different scale $\mu_F$ and to compensate the change in the NLO coefficient function, which also becomes dependent on $\mu_F$. Introducing the new scale the amplitude may then be written as

$$A^{(0+1)} \sim C^{(0)} \otimes F(\mu_f) + \alpha_s(\mu_R)C^{(1)}(\mu_F) \otimes F(\mu_f),$$

here the first and second terms on the right hand side individually depend on $\mu_F$ but their sum does not (to $O(\alpha_s^0)$). The new scale $\mu_F$ can be used to move part of the NLO corrections into the LO amplitude. The residual scale dependence can still be accessed by varying the factorisation scale.
\( \mu_f \) which appears in the GPD multiplying the 1-loop coefficient function. The scale choice \( \mu_F = m \) zeros the potentially large \( \ln(m^2/\mu_F^2)\ln(1/\xi) \) terms. In Figure 2 (right panel) and Figure 3 (right panel) we show the \( J/\psi \) and \( \Upsilon \) amplitudes resulting from Eq. 3.3 with \( \mu_F = m \), the remaining dependence on \( \mu_f \) is greatly reduced.

Of course, by fixing the scale in this way we may miss terms containing a large \( \ln(1/\xi) \) but which do not depend on the factorisation scale. A method to compute these missing terms at a given order in \( \alpha_s \) has been devised and used by other authors \[13, 14\] to compute contributions that would appear beyond NLO, with the eventual goal of resumming all such terms. The first few terms are given by

\[
A \sim 1 + z \ln \left( \frac{m^2}{\mu_F^2} \right) + z^2 \left[ \frac{\pi^2}{6} + \frac{1}{2} \ln^2 \left( \frac{m^2}{\mu_F^2} \right) \right] + \ldots, \quad z^n \sim \alpha_s^n \ln^n(1/\xi). \tag{3.4}
\]

If the scale fixing procedure \[12\] is used without resummation then the potentially large scale independent terms appearing in Equation 3.4 would be absorbed into the GPDs.

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**Figure 4:** The imaginary part of the tree-level, \( A_g^{(0)} \), and the 1-loop, \( A_g^{(1)} + A_q^{(1)} \), \( J/\psi \) photoproduction amplitude as a function of the \( pp \) centre-of-mass energy \( W \) with both the scale fixing procedure applied with \( \mu_F^2 = 2.4 \text{ GeV}^2 \) and the “\( Q_0 \)” cut. The dot-dashed, solid and dashed lines correspond to the low, central and high values of the scale \( \mu^2 = 1, 1.7, 2.4, 4.8 \text{ GeV}^2 \) respectively. Axes kept as in Figure 2 to facilitate comparison.

After the scale fixing procedure the NLO correction \( (A_g^{(1)} + A_q^{(1)}) \) for \( J/\psi \) photoproduction is still large and dominates the LO term \( (A_g^{(0)}) \), see Figure 2 (right panel). In \[15\] it was argued that due to the low scale of the \( J/\psi \) photoproduction process relative to the PDF input scale, \( Q_0 \), terms of \( \mathcal{O}(Q_0^2/M_J^{2/3}/\psi) \), which may be included both in the PDF and in the coefficient function, give a sizeable contribution to the amplitude. The idea presented was to compute the leading part of the 1-loop contribution (ladder diagrams only) limiting the virtuality of the loop momentum to less than the scale \( Q_0 \), then to subtract this contribution from the full 1-loop coefficient functions, effectively removing the double counting. The result of this procedure is shown in Figure 4, as hoped the 1-loop contribution is greatly reduced and becomes smaller than the tree-level contribution.

4. \( k_T \) Factorisation fit update

In the \( k_T \) factorisation context we present an updated fit \[16\] to the HERA and LHCb data. The new fit includes data from the LHCb 2014 7 TeV analysis \[5\], which supersedes the LHCb
2013 analysis [17], and data from the LHCb 2016 13 TeV preliminary analysis [1]. The new fitting procedure allows for bin-to-bin correlated errors within individual data sets (our previous fit [18] assumed all errors to be uncorrelated). Additionally, the new fit implements a better description of the photon flux based on the Budnev et al. dipole approximation [19, 20]. The gap survival factors, used to model ultra-peripheral production in terms of photoproduction, have also been updated to better fit the TOTEM data [21].

To relate our fitted gluons to the PDFs we make use of an approximate form of the Shuvaev transform based on the “maximal skew” approximation and assume that at small $x$ the gluon has a power like behaviour. In Ref. [22] it was shown that the parametrisation used in our work can introduce an $\mathcal{O}(20 - 30\%)$ effect on the total cross-section as compared to using the full Shuvaev transform.

Our fitted gluons are displayed in Figure 5 (left panel) where, for comparison, we also show the global fit gluons. Note that the global fit gluons are extracted using collinear factorisation and the $\overline{\text{MS}}$-bar scheme whilst our gluons are produced using $k_T$ factorisation. Therefore, our fit should only be interpreted as indicative of the potential to constrain the gluons and should not be seen as a direct comparison.

Recently, another approach to extracting the small $x$ gluon based on forward charm data has been advocated. In Figure 5 (right panel) we display our fit alongside a gluon obtained by reweighting a global fit to include the forward charm data [23]. The forward charm data fit appears to prefer a significantly reduced and flatter gluon at small $x$. However, one should be careful when comparing the two fitted gluons. Firstly, our gluons are produced using $k_T$ factorisation. Secondly, only the “normalized” (ratio to a reference rapidity bin) forward charm data are included in the fit of Ref. [23], thus, the data are used predominantly to extract the shape of the gluon distribution at low $x$ with the normalisation provided by matching to the larger $x$ gluon PDF obtained from the global analysis. An analysis which includes the forward charm data without taking ratios currently indicates a larger gluon at small $x$ [24].

![Figure 5](image-url)

**Figure 5:** (Left panel) the gluon distribution resulting from our $k_T$ factorisation fit, “Model 2”, compared to three global fits [25, 26, 27]. The widths of the bands represent the the 1σ uncertainty propagated from the experimental error. The $x$ regions probed by the 7 TeV LHCb data are darkly shaded. The extended $x$ region probed by the 13 TeV data is indicated by the less darkly shaded region of the $\mu^2 = 2.4$ GeV$^2$ band. (Right panel) our fit, “Model 2”, compared to the central, $\mu_R = \mu_F = \sqrt{m_c^2 + p_T^2}, m_c = 1.5$ GeV, gluon extracted from forward charm production [23].
5. Conclusion

We have summarised two proposals [12, 15] that together may enable the $J/\psi$ photoproduction and ultra-peripheral production to be included consistently within a global fit. Going forward, a reasonable step towards this goal may be to re-weight existing global fits to include the $J/\psi$ data. The impact and necessity of the “$Q_0$ cut” [15] should be carefully investigated in this context.

An extraction of the gluon using the $k_T$ factorisation framework is also presented [16]. The LHCb 2016 preliminary data [1] extend the support for the fit down to $x \sim 10^{-6}$. We find that both the shape and uncertainty of the fit, compared to a fit including only HERA [3, 4] and earlier LHCb [17] data, are largely unchanged.

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