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Non-linear dynamics in DIS at NLO

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The growth of the hadronic and nuclear scattering cross sections with energy is described by the linear BFKL evolution equation. At very high energies, when the parton density becomes high, the hadronic cross section reaches the unitarity limit, and the transition to high parton density is described by the non-linear BK and JIMWLK evolution equations. These equations are currently the starting point of discussion for future experiments like the proposed Electron Ion Collider (EIC) or the Large Hadron electron Collider (LHeC) and also play an important role in current RHIC and LHC experiments involving nucleus-nucleus and proton-nucleus collisions. Most of the current phenomenology of high-energy and high-density QCD is based on the leading-order evolution equations with only running coupling corrections. I will present a systematic procedure to include higher-order corrections into the BFKL/BK/JIMWLK evolution equations and the corresponding scattering cross sections using the high-energy Operator Product Expansion in terms of composite Wilson line operators.

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1. Introduction

At high energy the fields of the colliding particles are ordered according to their rapidity, thus, the scattering amplitude can be factorized in rapidity space. Similarly to the usual collinear factorization where one introduces a factorization scale that separates the perturbative contributions from the non-perturbative ones, at high energy the fields, now ordered in rapidity, are separated in slow fields and fast fields by a rapidity factorization scale. In this way, the high energy scattering amplitude may be organized into coefficient functions as result of the integration over the slow fields, and into matrix elements of the relevant operators given by the integration over the fast fields. This is the logic of the Operator Product Expansion (OPE) at high-energy. The main difference from the collinear factorization is that at high-energy the coefficient functions and the matrix elements both contains perturbative and non-perturbative contributions. The latter, however, are screened by the saturation scale, which is much larger then the scale of the confinement region justifying the applicability of pertubative methods. The saturation scale is the scale at which the non-linear dynamics of the process takes place. This is one of the features of high-energy scattering processes.

Suitable operators for the description of amplitudes at high-energy are Wilson lines: infinite gauge link ordered along the straight line collinear to the particle's velocity. The energy dependence of the amplitude is then encoded in evolution equation of the Wilson lines with respect to rapidity parameter. Each step of rapidity evolution generates a new Wilson line at a different point in the impact parameter space. This feature makes the evolution equation of Wilson lines a non-linear one. Known evolution equations at high-energy are the Balitsky-Kovchegov (BK) equation, the Balitsky-hierarchy of coupled evolution equations, the Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov and Kovner (JIMWLK) equation and the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation (see Ref. [1] for a review).

2. From linear to non-linear dynamics in DIS at NLO

BFKL equation is obtained in the leading log approximation (resummation of terms proportional to the strong coupling constant and the logarithm of the center-of-mass energy) and it includes the energy dependence of the scattering processes at high-energy. Because of its linearity BFKL violates unitarity since it describes only the proliferation of partons. On the other hand, evolution equations obtained in a semi-classical (shock-wave) formalism with the use of Wilson lines operators, are non-linear and thus respect unitarity: the non linear term in the equation describes the recombination process that tames the growth of the parton density beyond the unitarity limit.

DIS in the Bjorken limit is described by an incoherent interaction of the virtual photon emitted by the lepton with the asymptotically free partons inside the proton. The diagrammatic description of DIS process in the Bjorken limit is represented in the right panel of Fig. 1. At high-energy (Regge limit) the density of partons that may interact with the virtual photon increases, consequently the DIS cross-section is better described by coherent interactions in the dipole model: the virtual photon split in to a quark-anti-quark pair long before scattering with the target. In the Regge limit the diagrammatic description of DIS cross-section is represented in the left panel of Fig. 1.

In the BFKL approximation one resums contribution proportional to $\alpha_s \ln s$ where s is the center-of-mass energy of the system virtual photon-target. The typical diagrams in the LL approx-

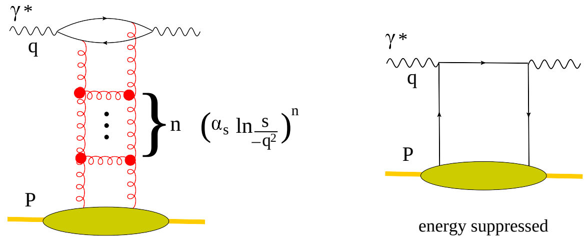

Figure 1: Left panel shows the diagrammatic description of DIS amplitude in the Regge limit. The right panel shows the DIS amplitude in the Bjorken limit which get energy suppressed in the Regge limit.

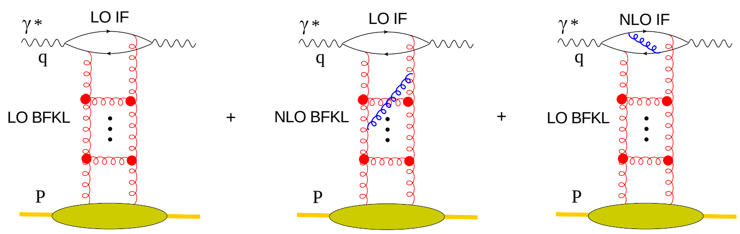

Figure 2: DIS cross section at NOL with Leading Log and Nexto-to-leading Log resummation.

imation are ladder type of diagrams. In Fig. 2 the diagrammatic representation of DIS amplitude is shown up the NLO: in the left panel is shown the DIS amplitude at LO and the LL approximation is represented by the ladder-gluon diagram; the interaction of the quark-anti-quark pair with the gluon ladder is the photon impact factor (IF). At NLO accuracy, both the photon IF and the LL ladder diagrams receive an α_s correction. This is diagrammatically represented in the center panel and right panel of Fig. 2. The NLO BFKL has been calculated in [3]. In order to obtain the full DIS amplitude one needs a model for the hadronic target.

At NLO accuracy, the IF has two contributions: one which can be considered pure NLO correction to the IF and another one that has a Log of the energy which is, in other words, a contribution already present in the LL BFKL ladder diagram. This means that when we consider DIS at NLO we have an ambiguity in the definition of what is NLO IF and what is instead part of BFKL resummation. This ambiguity is absent if we calculate DIS amplitude in the high-energy operator product expansion in terms of Wilson lines. As we will see, requiring that the NLO IF respect conformal invariance in the two-dimensional transverse space represents a guiding line for the systematics of the high-energy (Regge limit) perturbative expansion.

The Operator Product Expansion at high energies is obtained using the background field technique: the T-product of the two electromagnetic currents is evaluated in the background of gluon field generated by the hadronic target. In the spectator frame the background field reduces to a shock wave. As explained before, in the dipole model, the virtual photon which mediate the interactions between the lepton and the nucleon (or nucleus), splits into a quark-anti-quark pair long before the interaction with the target. The propagation of the quark-anti-quark pair in the background of a shock wave, reduces to two Wilson lines. If the quark fluctuates perturbatively in a quark and a gluon before interacting with the target, then the number of Wilson lines increases.

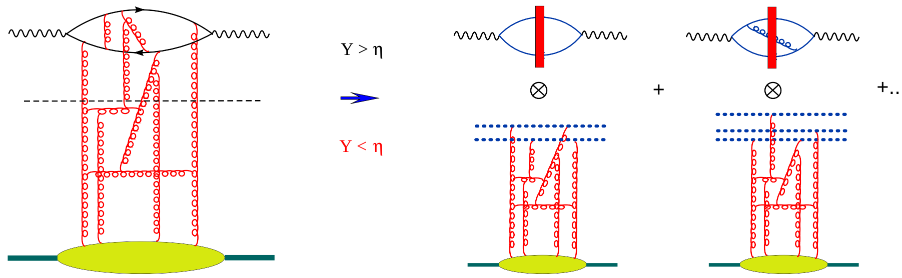

Figure 3: Expansion of the T-product of two electromagnetic currents in terms of Wilson-line operators. The blue dotted lines represent the Wilson line operators.

Formally, we can write down the expansion of the T-product of two electromagnetic currents in the following way

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\, I^{\rm LO}_{\mu\nu}(z_1,z_2,x,y)[{\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}] \tag{2.1}$$
$$+\int d^2z_1 d^2z_2 d^2z_3\, I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}] + \cdots$$

The infinite gauge link ordered along the straight line collinear to particle's velocity n^μ is defined as:

$$U^\eta(x_\perp) = {\rm Pexp}\Big\{ig\int_{-\infty}^{\infty} du\, n_\mu\, A^\mu(un+x_\perp)\Big\} \tag{2.2}$$

where A_μ is the gluon field of the target, $x_\perp$ is the transverse position of the particle which remains unchanged throughout the collision, and the index η is the rapidity of the particle. The energy dependence of the DIS cross section is translated into the dependence of the color dipole on the rapidity η. The evolution equation of color-dipole is [4, 5, 6]:

$$\begin{aligned}\frac{d}{d\eta}\,\hat{\mathcal{U}}^\eta(z_1,z_2) \;=\; \frac{\alpha_s N_c}{2\pi^2}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2 z_{23}^2}[\hat{\mathcal{U}}^\eta(z_1,z_3)+\hat{\mathcal{U}}^\eta(z_3,z_2))\\ -\hat{\mathcal{U}}^\eta(z_1,z_3)-\hat{\mathcal{U}}^\eta(z_1,z_3)\hat{\mathcal{U}}^\eta(z_3,z_2)]\end{aligned} \tag{2.3}$$

where $\eta = \ln\frac{1}{x_B}$ and $z_{12}\equiv z_1-z_2$ etc. (we denote operators by "hat"). The operator $\hat{\mathcal{U}}$ is

$$\hat{\mathcal{U}}^\eta(x_\perp,y_\perp) = 1-\frac{1}{N_c}{\rm tr}\{\hat{U}^\eta(x_\perp)\hat{U}^{\dagger\eta}(y_\perp)\}\,. \tag{2.4}$$

If the non-linear term in (2.3) factorized in a product of two dipoles we go from the Balitsky-JIMWLK equation to the BK equation which is valid in the mean-field approximation. The first three terms in the BK equation correspond to the linear BFKL evolution [2] and describe the partons emission while the last term is responsible for the partons annihilation. For sufficiently low x_B the partons emission balances the partons annihilation so the partons reach the state of saturation with the characteristic transverse momentum Q_s growing with energy $1/x_B$.

The diagrammatic description of the OPE at high energy is represented in Fig. 2. As already mentioned before, at NLO the IF receives two types of contributions: one which is pure NLO correction and another one which has dependence on the evolution parameter. This is the reason for which the NLO IF, although made of tree diagrams, is not conformal invariant. Formally we can write

$$\left[\langle T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}\rangle_A\right]^{\rm NLO} = \int \frac{d^2z_1 d^2z_2}{z_{12}^4} d^2z_3 \left[I_1^{\mu\nu}(z_1,z_2,z_3) + I_2^{\mu\nu}(z_1,z_2,z_3)\right]$$
$$\times\,[{\rm tr}\{U_{z_1}U^\dagger_{z_3}\}{\rm tr}\{U_{z_3}U^\dagger_{z_2}\} - N_c{\rm tr}\{U_{z_1}U^\dagger_{z_2}\}]$$

Where $I_2^{\mu\nu}$ is conformal invariant in the two-dimensional transverse space ($SL(2,C)$ invariant), while $I_1^{\mu\nu}$ is rapidity dependent

$$I_1^{\mu\nu}(z_1,z_2,z_3) = \frac{\alpha_s}{2\pi^2} I^{\rm LO}_{\mu\nu}\frac{z_{12}^2}{z_{13}^2 z_{23}^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\, e^{i\frac{s\alpha}{4}\mathscr{Z}_3}\,.$$

The term $I_1^{\mu\nu}$ is proportional to the LO impact factor $I_{LO}^{\mu\nu}$ which is conformal invariant and it can be written in terms of conformal invariant vectors $\kappa = \frac{\sqrt{s}}{2x_*}(\frac{p_1}{s} - x^2 p_2 + x_\perp) - \frac{\sqrt{s}}{2y_*}(\frac{p_1}{s} - y^2 p_2 + y_\perp)$ and $\zeta_i = \left(\frac{p_1}{s} + z_{i\perp}^2 p_2 + z_{i\perp}\right)$. The explicit expression of $I_2^{\mu\nu}$ and $\mathscr{Z}_3$ can be found in Ref. [7]. At LO the DIS amplitude is

$$\langle T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}\rangle_A \;=\; \frac{s^2}{2^9\pi^6 x_*^2 y_*^2}\int d^2z_{1\perp}d^2z_{2\perp}\frac{{\rm tr}\{U_{z_1}U^\dagger_{z_2}\}}{(\kappa\cdot\zeta_1)^3(\kappa\cdot\zeta_2)^3}\times$$
$$\frac{\partial^2}{\partial x^\mu\partial y^\nu}\left[2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2) - \kappa^2(\zeta_1\cdot\zeta_2)\right] \;+\; O(\alpha_s) \tag{2.5}$$

The procedure of restoring the loss of conformal symmetry due to the regularization of the rapidity divergence by rigid cut-off, is analog to the procedure of restoring gauge invariance by adding counterterms to local operator when the rigid cut-off is used instead of dimensional regularization, that automatically preserve gauge symmetry, to regulate ultraviolet divergence at one loop order. Hence, we add to the LO operator ${\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$ a suitable counter term that restore conformal invariance

$$[{\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]^{\rm conf} \;=\; {\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\} \tag{2.6}$$
$$+\;\frac{\alpha_s}{4\pi}\int d^2z_3\,\frac{z_{12}^2}{z_{13}^2z_{23}^2}[\frac{1}{N_c}{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - {\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]\ln\frac{az_{12}^2}{z_{13}^2z_{23}^2} \;+\; O(\alpha_s^2) \tag{2.7}$$

The parameter a is the analog of μ_F in the usual OPE. Note also that at this order the operator proportional to the NLO impact factor does not need to be modified: it would get a counterterm at NNLO accuracy. Using, then, the composite operator, the NLO impact factor is conformal invariant and it can be written entirely in terms of the conformal vectors we defined above (see Ref. [7] for its explicit expression). Such result is an analytic expression of the photon impact factor in coordinate space which is relevant for DIS off a large nucleus where the non linear operator appearing at NLO level is relevant at high parton density regime [7] (see also [8] for a Light-front perturbation theory calculation of the NLO IF). The NLO IF in the BFKL approximation has been calculated in [9].

It is suggestive to rewrite the composite operator (2.7) in the following way

$$[{\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]^{\rm conf} \;=\; {\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\} + BK_{\rm LO}\int_0^{\sqrt{\frac{a\, z_{12}^2}{z_{13}^2 z_{23}^2}}}\frac{d\alpha}{\alpha} + O(\alpha_s^2) \tag{2.8}$$

where BK_{LO} is the right-hand-side of the BK evolution equation. Equation (2.8) suggests that we can choose a conformal invariant way of cutting-off the rapidity divergent in the NLO IF so that one gets automatically conformal invariance without the notion of composite operator. Note also that the conformal invariant cut-off has the BK kernel in the square root.

The main point is that, whether one uses the composite operator or the conformal invariant rigid cut-off, the requirement of restoring conformal ($SL(2,C)$ invariance) provides a guiding line to systematically factorize the DIS amplitude in rapidity space and systematically separate the energy-dependent terms encoded in the matrix elements of Wilson line operators from the coefficient functions (photon impact factors).

In order to get the full DIS amplitude at NLO one needs also the evolution equation of a color dipole Wilson line operators at NLO [10, 11].

3. Conclusions

In this talk we have discussed how Operator Product Expansion in terms of composite Wilson lines operators can provide a systematic procedure to factorize the DIS amplitude. As usual, at LO almost any type of factorization would work. Only when the NLO correction are considered one realizes whether the chosen factorization scheme is still valid or needs to be modified. At high-energy OPE the factorization is realized in terms of composite Wilson lines operators which reproduce order by order the Balitsky hierarchy of evolution equation.

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