Exclusive pion-induced Drell-Yan process at J-PARC for accessing the nucleon GPDs and soft nonfactorizable mechanism

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Generalized parton distributions (GPDs) encoding multidimensional information of hadron partonic structure appear as the building blocks in a factorized description of hard exclusive reactions. The nucleon GPDs have been accessed by deeply virtual Compton scattering and deeply virtual meson production with lepton beam. A complementary probe with hadron beam is shown to be the exclusive pion-induced Drell-Yan process, $\pi^- p \rightarrow \mu^+ \mu^- n$, as demonstrated by recent theoretical advances on describing this process in terms of QCD factorization as the partonic subprocess convoluted with the nucleon GPDs and the pion distribution amplitudes, and by the feasibility study for its measurement via a spectrometer at the High Momentum Beamline being constructed at J-PARC in Japan. We also discuss the possible soft partonic mechanisms beyond the QCD factorization framework, and present an estimate of the soft mechanisms at J-PARC kinematics, making use of dispersion relations and quark-hadron duality. Realization of the measurement of the exclusive pion-induced Drell-Yan process at J-PARC will provide a new test of QCD descriptions of a novel class of hard exclusive reactions, and also offer the possibility of experimentally accessing nucleon GPDs at large timelike virtuality.
1. Introduction

We consider the pion-induced dimuon production. Summing the absolute square of the corresponding amplitudes, \( \pi N \rightarrow q\bar{q}X \rightarrow \gamma^* X \rightarrow \mu^+ \mu^- X \), over the accompanying hadronifc final state \( X \), we obtain the inclusive Drell-Yan cross section. The leading contribution comes from the transversely-polarized virtual photon \( \gamma^* \), as a consequence of the helicity conservation in the annihilation of the on-shell, massless quark and antiquark associated with the relevant partonic subprocess \( q\bar{q} \rightarrow \gamma^* \). On the other hand, the dimuon angular distribution for the production in the forward region is known to obey the pattern associated with the longitudinally-polarized virtual photon, which can be produced by the annihilation of off-shell quark \( q \) or antiquark \( \bar{q} \). The relevant off-shellness may be caused by perturbative gluon exchange between the quark and antiquark originating from the pion, and this type of mechanism with the gluon exchange plays important role for the forward production. Now, the spectator quark (or antiquark) originating from the pion may be absorbed by the target nucleon, giving rise to the exclusive final state, \( \mu^+ \mu^- N \), as represented in Fig. 1. This is the exclusive Drell-Yan process and this type of diagrams gives important contributions for the dimuon production in the forward region of the exclusive Drell-Yan process [1].

We discuss a recent cross section estimate of the exclusive Drell-Yan process as a report of our recent paper [2]. With the High Momentum Beamline being constructed at J-PARC, the secondary pion beam with moderately high energies \( \sim 15-20 \) GeV is best suited to study meson-induced hard exclusive processes like exclusive Drell-Yan process. We mention the feasibility study [2] for measuring the exclusive pion-induced Drell-Yan process with the E50 spectrometer at J-PARC. We also discuss non-factorizable mechanism, beyond QCD factorization, in exclusive Drell-Yan process and give its first estimate using the light-cone QCD sum rules [3, 4].

2. QCD factorization formula at the leading order (LO)

We consider the exclusive Drell-Yan production, \( \pi^+ p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n \), in particular, with the production of \( \gamma^* \) in the forward region corresponding to the small invariant-momentum-transfer, \( t = \Delta^2 \) with \( \Delta \equiv q - q' \), where \( q \) and \( q' \) are the momenta of the initial pion and the produced \( \gamma^* \). In this case, as mentioned in Sec. 1, the complete annihilation of quark as well as antiquark from the pion, as in Fig. 1, plays important role. Here, the relevant amplitude is expressed as the convolution of the corresponding partonic (short distance) annihilation processes with the two separate parts of long-distance nature, associated with the pion and the nucleon, respectively: the upper long-distance part denotes the pion distribution amplitude (DA), whose information can be obtained from, e.g., \( \gamma \gamma^* \rightarrow \pi^0 \) process at Belle and Babar, while the lower long-distance part denotes the generalized parton distribution functions (GPDs) as an off-forward nucleon matrix element, whose forward limit reduces to the usual helicity distribution, \( \Delta q(x) \). Here, the GPDs are defined as the off-forward matrix element of the bilocal light-cone operators of the type, \( \langle n(p')|\bar{q}(-y^-/2)\cdots q(y^-/2)|p(p')\rangle \).
with $y^+ = \bar{y}_+ = 0$ ($j^\mu y_\mu = 0$), and are the functions of the relevant invariants, the (average) light-cone momentum fraction $x$ and the skewness $\xi$ ($= (p - p')^+ / (p + p')^+$), as well as $t$ (see Sec. II in [2]). Decomposing \( (n(p')) |q^-(\gamma^+ / 2)| q^+(\gamma^- / 2) |p(p)| \) into the independent Lorentz structures, we obtain the familiar proton GPDs, $H^q, E^q$, relevant to the Ji sum rule for quark’s angular momentum contribution to the proton spin, \( J^q = \int_1^1 dx x [H^q(x, \xi, 0) + E^q(x, \xi, 0)] / 2 \), and also $\tilde{H}^q, \tilde{E}^q$ for the case with the additional $\gamma_5$, as $(P \equiv (p + p')/2)$,

$$
2P^+ \int \frac{dy}{4\pi} e^{ixP^+ y^-} (p') |\bar{q}(-\frac{y^-}{2})\rangle \gamma^+ \gamma_5 q(\frac{y^-}{2}) |p\rangle = \bar{u}(p') [\tilde{H}^q(x, \xi, t) t^\gamma \gamma_5 + E^q(x, \xi, t) \frac{\gamma_5 \Delta^+}{2 m_N}] u(p),
$$

(2.1)

for each quark flavor $q$; here, $|p^{(l)}\rangle \equiv |p(p^{(l)})\rangle$, $u(p^{(l)})$ denotes the proton spinor with momentum $p^{(l)}$ and mass $m_N$, and we do not show the gauge-link operator between two quark fields. Those GPDs, $H, E, \tilde{H}$, and $\tilde{E}$, are measured by the deeply virtual Compton scattering (DVCS) corresponding to $\gamma^* p \rightarrow q q'$ process in the experiments at JLab, HERMES, COMPASS, etc., and also by the deeply virtual meson production (DVMP). For the case of the deeply virtual pion production, $\gamma^* p \rightarrow \pi N$, the pseudoscalar nature of the pion allows us to probe the GPDs $\tilde{H}^q$ and $\tilde{E}^q$ solely, associated with $\gamma_5$ as in (2.1). Interchanging the initial $\gamma^*$ and the final pion in the deeply virtual pion production and making the $\gamma^*$ timelike, we obtain the exclusive Drell-Yan process. This demonstrates that the exclusive Drell-Yan process at the J-PARC allows us to probe the GPDs $\tilde{H}^q$ and $\tilde{E}^q$ solely and plays a complementary role compared with the deeply virtual pion production at, e.g., JLab. The kinematical region accessible by the exclusive Drell-Yan process at the J-PARC is also complementary to those accessible by the GPD measurements by the various other experiments [2].

In Fig. 1, the “hard” gluon exchange ensures that the vertices in the partonic subprocess are separated by short distances; thus, the diagrams associated with this type of gluon exchange obey the QCD factorization into the corresponding short-distance partonic subprocess and the long-distance parts, the pion DA and the nucleon GPD. Indeed, Fig. 1 and similar diagrams of order $\alpha_s$ give the leading order (LO) in the factorization formula for exclusive Drell-Yan process. The first estimate using this LO factorization formula was performed by Berger, Diehl and Pire [1]. The corresponding cross section at the large $Q^2 \equiv q^2$ scaling limit with the fixed $\tau \equiv Q^2 / (2 p \cdot q)$ is

$$
\frac{d\sigma_L}{dt dQ^2} = \frac{4 \pi \alpha_s^2 m_N^2 \tau^2}{27 Q^4 f_\pi^2} \left[ (1 - \xi^2) |\tilde{H}^{du}|^2 - 2 \xi^2 \text{Re}(\tilde{H}^{du} \tilde{E}^{du}) - \frac{\xi t}{4 m_N} |\tilde{E}^{du}|^2 \right],
$$

(2.2)

$$
\tilde{H}^{du} = \frac{8 \alpha_s}{3} \int_{-1}^1 dz \frac{1}{1 - z^2} \int dx \left( \frac{e_u}{\xi - x + i\epsilon} - \frac{e_d}{\xi + x + i\epsilon} \right) \left[ \tilde{H}^d(x, \xi, t) - \tilde{H}^u(x, \xi, t) \right],
$$

(2.3)

where $f_\pi$ is the pion decay constant, $e_q$ is the quark’s electric charge, and $\tilde{H}^{du}$ is a function of $\xi$ and $t$ as the convolution of the hard part, the pion DA $\phi_\pi(z)$ of leading twist, and the GPD $\tilde{H}^q$, while $\tilde{E}^{du}$ denotes the similar convolution with $\tilde{H}^q$ replaced by the GPD $\tilde{E}^q$. In (2.3), the $p \rightarrow n$ transition GPDs arising in Fig. 1 is expressed by the proton GPDs of (2.1) using isospin invariance relations [1, 2]. The subscript “$L$” in (2.2) indicates that this cross section is obtained at the leading twist, associated with the production of the longitudinally-polarized $\gamma^*$. (The production of the transversely-polarized $\gamma^*$ requires higher twist effects, see [5].) With $Q^2 = 5 \text{ GeV}^2$ for the mass of the produced dimuon, the cross section (2.2) is plotted in Fig. 2. The results of [1] are labeled as “BMP2001”, where a model of the nucleon GPDs $\tilde{H}$ and $\tilde{E}$, based on the double distributions, and the asymptotic pion DA for $\phi_\pi$ are used. The cross section is of the pb level.
Recently [2], we have updated the estimate of the corresponding cross section and obtained the results labeled as “GK2013” in Fig. 2: we have used a recent parameterization for the GPDs $\tilde{H}$ and $\tilde{E}$, determined by comparing with the HERMES data for $\pi^+\gamma$ electroproduction, as well as to the pion DA with the pre-asymptotic corrections for $\phi_\pi$. The updated cross section is enhanced compared with the previous result. For more detail, we refer the readers to [2].

Using this updated estimate of (2.2) as an input, we have performed the Monte Carlo simulation and the feasibility study [2] for measuring $\pi^-\gamma$ at J-PARC. We are able to obtain Monte Carlo simulation signals of the dimuon mass spectra for the secondary pion beam with the J-PARC high-momentum beam line, assuming a minimal extension of the E50 spectrometer at J-PARC (see Fig. 9 in [2]). Our results of the Monte Carlo simulated missing-mass $M_X$ spectra demonstrate that the exclusive Drell-Yan signal is well-separated from the inclusive as well as other signals, see Fig. 14 in [2]. Also, the accuracy expected for the corresponding J-PARC data will allow us to distinguish the typical parameterizations for the GPDs. For further detail, see [2].

### 3. Nonfactorizable mechanism

To calculate the convolution implied by Fig. 1, corresponding to the LO factorization for exclusive Drell-Yan process, we integrate the corresponding amplitudes over the momentum associated with the gluon propagator. When the gluon momentum becomes small compared with $\Lambda_{QCD}$, such soft and nonperturbative degrees of freedom should be separated into the long-distance parts in the spirit of QCD factorization. Absorbing the soft nonperturbative gluon propagator into either the pion DA or the nucleon GPDs leads to the “tree” diagrams, which are obtained formally by removing the gluon propagator from Fig. 1. Thus, the tree diagrams correspond to the lower order in $\alpha_s$ than the LO in the QCD factorization framework and physically represent the “Feynman mechanism”: the antiquark (quark) carrying almost all pion-momentum annihilates with the quark (antiquark) carrying almost all momentum-transfer from the nucleon, to produce $\gamma^*$, while the “wee” parton is directly transferred between the pion and the nucleon. The corresponding partonic process is not ensured to be of short-distance, and thus this diagram is not factorizable into the short- and long-distance parts. Moreover, we do not have a boundary to separate the pion and the nucleon wave functions because they are directly connected by the soft parton line; thus, the nonperturbative function arising in the tree diagrams is also nonfactorizable between those hadrons.

A similar soft nonfactorizable mechanism is known to play an important role in the QCD description of the pion electromagnetic form factor. In addition to the QCD factorization formula associated with a hard-gluon exchange in the partonic subprocess, nonfactorizable mechanisms corresponding to the “tree” partonic-process without gluon exchange, where the two pions are connected by a soft pion transferred directly between them, are indispensable for reproducing empirical behaviors of the form factor, especially for moderate momentum-transfer region [6].

![Figure 2: Cross section (2.2) of $\pi^- p \to \gamma^* n$ with $Q'^2 = 5 \text{GeV}^2$ as a function of $|t|$ for $\tau = 0.2$ [2].](image)
To perform QCD calculation of the nonfactorizable mechanism in the exclusive Drell-Yan process [3, 4], we first make the external leg of the initial pion off-shell, and replace the corresponding pion wave function by the axial vector vertex, to which the pion can couple. This procedure leads to a description using the two-point correlator, \( \mathcal{H}_{\mu \nu} = i \int d^4x e^{iqx} \langle n(p')|T_{\mu \nu}^{em}(0)f_{\mu \nu}^m(x)|p(p)\rangle \), with \( q = q' + p' - p \) and \( q^2 \neq m^2_\pi \), corresponding to the off-forward virtual Compton amplitude with one of the electromagnetic currents, \( f_{\mu \nu}^m = e_u \bar{u} \gamma_\mu u + e_d \bar{d} \gamma_\mu d \), replaced by the axial vector current, \( \bar{d} \gamma_\mu u \). For deeply virtual region, \( |q^2| \gg \Lambda^2_{\text{QCD}} \), the correlator \( \mathcal{H}_{\mu \nu} \) can be systematically treated by the operator product expansion (OPE), and the corresponding long-distance contribution can be expressed by the nucleon GPDs \( \hat{H} \) and \( \hat{E} \), which are the same GPDs as appeared in (2.2).

We may also write down the dispersion relation for \( \mathcal{H}_{\mu \nu} \) with respect to its dependence on \( q^2 \), and it can be shown that the residue at the pion pole, \( q^2 = m^2_\pi \), for this dispersion relation corresponds to the exclusive Drell-Yan amplitude associated with the on-shell pion leg. The corresponding residue may be determined from the behavior of the OPE for the off-forward deeply virtual amplitude \( \mathcal{H}_{\mu \nu} \), when we perform the OPE at the tree level, the result should give the nonfactorizable mechanism due to the tree diagrams. For an efficient matching between the OPE and dispersion relation to determine the relevant pole residue, we rely on quark-hadron duality to deal with the unwanted higher resonance contributions arising in the dispersion relation. This procedure yields the soft nonfactorizable amplitude for \( q^2 = m^2_\pi \to 0 \) as \( q^2 \rightarrow 0 \) [3, 4],

\[
\langle n|f_{\mu \nu}^m|\pi^+ p\rangle = -g_{\mu \nu}^2 \int_0^{\infty} dx e^{xq^2/m_\pi^2} \left[ \bar{u}(p')\gamma_\mu p(p) - \bar{u}(p')\gamma_\nu p(p) \right] \left[ \hat{H}_{\mu \nu}(x) \right] \hat{H}_{\mu \nu}(x, \xi, t) \left[ \bar{u}(p')\gamma_\nu p(p) \right] + \cdots,
\]

(3.1)
in terms of the proton GPDs, \( \hat{H}_{\mu \nu}(x) = H_{\mu \nu}(x) - \hat{H}_{\mu \nu}(x) \), and we have also the similar term associated with \( \hat{E}_{\mu \nu}(x, \xi, t) = \hat{E}_{\mu \nu}(x, \xi, t) \), as well as the terms arising from higher-twist corrections to the OPE for \( \mathcal{H}_{\mu \nu} \), in the ellipses. Here, \( x_0 \) is related to the threshold parameter \( q^2_0 \), from which the continuum contribution in the dispersion-relation integral for \( \mathcal{H}_{\mu \nu} \) starts, as the approximation for the higher resonance contributions invoking quark-hadron duality. We note that the factor \( g_{\mu \nu} \) in (3.1) indicates the longitudinal polarization of the produced \( \gamma^* \).

Compared with the QCD factorization formula (2.3), the pion DA does not appear in (3.1); instead, we have the exponential factor, \( \exp(-[(x - \xi)Q^2]/[(x + \xi)M_B^2]) \), depending on the Borel parameter \( M_B \), characteristic of the QCD sum rule approach; the relevant nonperturbative effects arising in the “sum rule” are encoded in the light-cone dominated quantities, the GPDs, and thus (3.1) corresponds to the light-cone sum rule for the nonfactorizable amplitude in the exclusive Drell-Yan process. We note that the light-cone QCD sum rules have been derived for e.g., the pion electromagnetic form factor in [6]. We present the behaviors of the soft nonfactorizable amplitude from the light-cone sum rule (3.1), using the BMP2001 parameterization for the GPDs, which was used in the estimate shown in Fig. 2. We note that the predictions using the QCD sum rules should not depend strongly on the Borel parameter, \( M_B \), introduced auxiliarily for the matching procedure. In Fig. 3, we show (3.1) as a function of \( M_B^2 \) with \( Q^2 = 5 \text{ GeV}^2 \), \( |t| = 0.2 \text{ GeV}^2 \), and \( \tau = 0.2 \). We obtain good stability in a relevant range for \( M_B^2 \). This result for (3.1) leads to the prediction to the cross section, \( d\sigma / (dt dQ^2) \), due to the soft nonfactorizable mechanism for the exclusive pion-induced Drell-Yan process, \( \pi^- p \to \gamma^* n \), as shown by the solid curve in the right figure of Fig. 3 as a function of \( t \), for the case with the same kinematics as in Fig. 2. For comparison, we also plot
the dashed curve in Fig. 3, which is same as the black curve in Fig. 2. The soft nonfactorizable mechanism gives the cross section larger by a factor of $\sim 5$ than the QCD factorization, reflecting the $O(\alpha_s^0)$ and $O(\alpha_s^2)$ cross sections using (3.1) and (2.3), respectively, and also shows the stronger dependence on $t$. The $\tau$ dependence of the soft nonfactorizable mechanism is also obtained [3]. Our results indicate that the soft nonfactorizable mechanism should be very important at the J-PARC kinematics. We note that even larger enhancement of the exclusive Drell-Yan cross section at J-PARC, caused by a different type of soft mechanisms beyond the QCD factorization, has been obtained by Goloskokov and Kroll [5] (see also the discussion in [2, 3]). Further study is needed to clarify the interplay in the soft/hard QCD mechanisms relevant for the J-PARC processes.

Figure 3: Light-cone sum rule (3.1) for the soft nonfactorizable amplitude of $\pi^- p \rightarrow \gamma^* n$ as a function of the Borel parameter $M_B^2$, with $|t| = 0.2$ GeV$^2$ (left) and the corresponding cross section as a function of $|t|$ (right), with $Q^2 = 5$ GeV$^2$ and $\tau = 0.2$ using the BMP2001 input for the nucleon GPDs and $q_\pi^2 = 0.7$ GeV$^2$ for the threshold parameter. The solid and dashed curves are obtained using (3.1) and (2.3), respectively.

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References


