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Hadronic Higgs boson decay at order α_s^4 and α_s^5

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We compute corrections to the decay of the Standard Model Higgs boson to hadrons, to the fourth order in the strong coupling constant α_s. We use an effective theory in which the Higgs boson couples directly to bottom quarks and to gluons, via top quark–mediated effective couplings. Numerically, our results are of a comparable size to the previously-known "massless" contributions and complete the order α_s^4 corrections to the hadronic decay of the Higgs boson. In these proceedings we also provide an independent cross check of the gluonic Higgs boson decay at order α_s^5.

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1. Introduction

In the coming years, it will be an important task to precisely determine the Higgs boson couplings to other Standard Model particles. To this end, it is important to have a good theoretical understanding of the decay rate of the Higgs boson into bottom quarks. Combined with the Higgs boson's decay rate into gluons, it forms some 70% of the hadronic decay width and therefore influences all Higgs boson branching ratios.

QCD corrections to the partial width $\Gamma(H \to b\bar{b})$ have been studied for some time. At one and two loops they are known with exact bottom quark–mass dependence [1, 2, 3, 4, 5]. Beyond this order, at three and four loops, only the "massless" approximation was known [6, 7, 8]. In Ref. [9] the order α_s^4 corrections were completed. The calculation will be described in these proceedings. For a summary of further corrections, the reader is referred to [10, 11, 12].

For the calculation in the limit $M_t \gg M_H$, we consider an effective Lagrange density in which the top quark has been integrated out. It consists of QCD and effective couplings between the Higgs boson and bottom quarks and gluons,

$$\mathscr{L}_{\text{eff}} = -\frac{H^0}{v^0}\left(C_1[\mathscr{O}_1'] + C_2[\mathscr{O}_2']\right) + \mathscr{L}'_{\text{QCD}}, \tag{1.1}$$

where primed quantities refer to the five-flavour effective theory. The bare Higgs field H^0 and vacuum expectation v^0 may be identified with their renormalized quantities since we neglect electroweak effects in this calculation. The top quark–mass dependence is provided by the coefficient functions C_1 and C_2, which are known to fifth order in α_s [13, 14, 15]. The bare effective operators are given by $\mathscr{O}_1' = (G^{0\prime}_{a,\mu\nu})^2$ and $\mathscr{O}_2' = m_b^{0\prime}\bar{b}^{0\prime}b^{0\prime}$, and $[\mathscr{O}_1']$, $[\mathscr{O}_2']$ are their renormalized counterparts.

Correlators formed by the above operators can be related, via the optical theorem, to the total decay rate of the Higgs boson. With the correlator

$$\Pi_{ij}(q^2) = i\int \mathrm{d}x e^{iqx}\langle 0|T[\mathscr{O}_i', \mathscr{O}_j']|0\rangle, \tag{1.2}$$

we define the quantities

$$\Delta_{11} = \text{Im}\left[\Pi_{11}(M_H^2)\right]/(32\pi M_H^4), \qquad \Delta_{12} = \text{Im}\left[\Pi_{12}(M_H^2) + \Pi_{21}(M_H^2)\right]/(6\pi M_H^2 m_b^2),$$
$$\Delta_{22} = \text{Im}\left[\Pi_{22}(M_H^2)\right]/(6\pi M_H^2 m_b^2), \tag{1.3}$$

in terms of which,

$$\Gamma(H \to \text{hadrons}) = A_{b\bar{b}}\left[(C_2)^2(1+\Delta_{22}) + C_1C_2\Delta_{12}\right] + A_{gg}(C_1)^2\Delta_{11} \tag{1.4}$$

where $A_{b\bar{b}} = 3G_F M_H m_b^2(\mu)/(4\pi\sqrt{2})$ and $A_{gg} = 4G_F M_H^3/(\pi\sqrt{2})$.

Π_{11} and Π_{22} are already known at four [16] and five [8] loops, providing α_s^5 and α_s^4 corrections to the decay rate, respectively. The mixed contribution $\Pi_{12}(=\Pi_{21})$, however, was known only at three loops [7]. It has now been computed at four loops [9], completing the α_s^4 QCD corrections to the decay rate. Additionally, we compute new two-, three- and four-loop corrections to Π_{11} which are suppressed by one power of (m_b^2/M_H^2); they are numerically small, but are of the same order in (m_b^2/M_H^2) as the leading contributions from Π_{12} and Π_{22}. The results presented in these proceedings (Eq. (3.7)) provide an independent cross check of the four-loop $(m_b^2/M_H^2)^0$ terms of Π_{11}.

Corrections suppressed by powers of (M_H^2/M_t^2) are small, and not considered here.

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2. Calculation

The calculation was performed by making use of a well-tested collection of software. We use `qgraf` [17] to generate the diagrams. `q2e` and `exp` [18, 19, 20] are used to map the diagrams into a set of two-point integral topologies. `FORM` [21] is used to perform the Dirac traces and to re-write all integrals as linear combinations of scalar integrals. The resulting scalar integrals are reduced to master integrals by `FIRE 5.2` [22, 23]; we find 28 four-loop master integrals, which have been computed in Refs. [24, 25, 26]. We do not discuss the details of the renormalization of the bare expressions here, and refer the reader to Ref. [9].

Using this setup, we have computed one-, two-, three- and four-loop corrections to each of the correlators Π_{11}, Π_{12} and Π_{22}. In the case of Π_{11} we compute both the leading $(m_b^2/M_H^2)^0$ and sub-leading (m_b^2/M_H^2) contributions. In all cases we employ a generic gauge parameter ξ and expand the amplitudes to linear order in ξ before integral reduction. All dependence on ξ drops out after reduction to a minimal set of master integrals.

3. Results

To investigate the numerical size of the new corrections, we re-write Eq. (1.4) in the form

$$\Gamma(H \to \text{hadrons}) = A_{b\bar{b}} \left(1 + \Delta_{\text{light}} + \Delta_{\text{top}} + \Delta_{gg}^{m_b=0}\right), \tag{3.1}$$

with a common prefactor. Δ_{light} gives the terms for $C_1 = 0$ and $C_2 = 1$, i.e., terms with no top quark–mass dependence. $\Delta_{gg}^{m_b=0}$ collects the leading terms from Δ_{11}, which are proportional to $(m_b^2/M_H^2)^0$. The remaining terms are collected inside Δ_{top}. For completeness, these are defined in terms of the symbols of Eq. (1.4) as

$$\begin{aligned}
\Delta_{\text{light}} &= \Delta_{22}, \qquad \Delta_{gg}^{m_b=0} = \frac{16M_H^2}{3m_b^2}(C_1)^2\Delta_{11}^{m_b=0}, \\
\Delta_{\text{top}} &= \left[(C_2)^2 - 1\right](1 + \Delta_{22}) + C_1C_2\Delta_{12} + \frac{16M_H^2}{3m_b^2}(C_1)^2\Delta_{11}^{m_b^2}.
\end{aligned} \tag{3.2}$$

Numerically, we find

$$\begin{aligned}
\Delta_{\text{light}} &\approx 5.6667a_s + 29.1467a_s^2 + 41.7576a_s^3 - 825.7466a_s^4 \\
&\approx 0.2033 + 0.03752 + 0.001929 - 0.001368,
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
\Delta_{\text{top}} \approx\; & a_s^2\,[2.5556_{12} + 0.9895_{22}] \\
& + a_s^3\,[0.2222_{11} + 42.1626_{12} + 13.0855_{22}] \\
& + a_s^4\,[8.3399_{11} + 338.9021_{12} + 50.6346_{22}] \\
& + a_s^5\,[137.145_{11}] \\
\approx\; & 0.003290_{12} + 0.001274_{22} \\
& + 0.00001026_{11} + 0.001947_{12} + 0.0006043_{22} \\
& + 0.00001382_{11} + 0.0005616_{12} + 0.00008390_{22} \\
& + 0.00000815_{11},
\end{aligned} \tag{3.4}$$

$$
\begin{aligned}
\Delta_{gg}^{m_b=0} &\approx \frac{M_H^2}{27m_b^2}\left(a_s^2+17.9167a_s^3+153.0921a_s^4+392.6176a_s^5\right) \\
&\approx 0.09699+0.06235+0.01911+0.001759,
\end{aligned}
\tag{3.5}
$$

in which the expansion parameter is $a_s=\alpha_s^{(5)}(\mu)/\pi$ and we have used the values $\alpha_s^{(5)}(M_H)=0.1127$, $m_b(M_H)=2.773$ GeV, $M_H=125.09$ GeV, $M_t=173.21$ GeV. The subscripts $_{11}$, $_{12}$ and $_{22}$ in Eq. (3.4) tag the origin of each number. $\Delta_{gg}^{m_b=0}$ includes the a_s^5 term from Ref. [16] and $\Delta_{\rm top}$ the new $a_s^5(m_b^2/M_H^2)$ term computed here.

We observe that the a_s^4 term of $\Delta_{\rm light}$ is only about 30% smaller than the a_s^3 term. For this reason it is important to consider also the complete a_s^4 term of $\Delta_{\rm top}$, as we have done here. It is of the same order of magnitude, but appears with an opposite sign. Its inclusion significantly improves the convergence of the sum of all (m_b^2/M_H^2) terms,

$$
1+\Delta_{\rm light}+\Delta_{\rm top} \approx 1+0.2033+0.04208+0.004490-0.0007090,
\tag{3.6}
$$

to which the a_s^4 term now provides a correction of just 0.0567%. The (m_b^2/M_H^2) corrections to $\Delta_{\rm top}$ are very small.

We now present the a_s^3 corrections to Δ_{11}, which have not been fully presented before, in terms of their colour factors and including the scale-dependence. The (m_b^2/M_H^2) terms are new. We find

$$
\begin{aligned}
\Delta_{11}\Big|_{a_s^3} =& C_A^3\bigg(\frac{15420961}{46656}-\frac{704}{9}\zeta_2-\frac{44539}{432}\zeta_3+\frac{385}{24}\zeta_5+\Big[\frac{965285}{5184}-\frac{1331}{72}\zeta_2-\frac{605}{24}\zeta_3\Big]L_H \\
&+\frac{352}{9}L_H^2+\frac{1331}{432}L_H^3\bigg)-n_lC_A^2\bigg(\frac{667627}{3888}-\frac{2021}{48}\zeta_2-\frac{2443}{144}\zeta_3+\frac{5}{12}\zeta_5+\Big[\frac{170263}{1728} \\
&-\frac{121}{12}\zeta_2-\frac{11}{3}\zeta_3\Big]L_H+\frac{2021}{96}L_H^2+\frac{121}{72}L_H^3\bigg)-n_lC_FC_A\bigg(\frac{23221}{576}-\frac{143}{48}\zeta_2-\frac{341}{16}\zeta_3-\frac{5}{2}\zeta_5 \\
&+\Big[\frac{8569}{576}-\frac{11}{2}\zeta_3\Big]L_H+\frac{143}{96}L_H^2\bigg)+n_lC_F^2\bigg(\frac{221}{192}+3\zeta_3-5\zeta_5+\frac{3}{32}L_H\bigg)+n_l^2C_A\bigg(\frac{103327}{3888} \\
&-\frac{173}{24}\zeta_2+\frac{7}{72}\zeta_3+\Big[\frac{9187}{576}-\frac{11}{6}\zeta_2+\frac{1}{6}\zeta_3\Big]L_H+\frac{173}{48}L_H^2+\frac{11}{36}L_H^3\bigg)+n_l^2C_F\bigg(\frac{55}{8}-\frac{13}{24}\zeta_2 \\
&-\frac{15}{4}\zeta_3+\Big[\frac{767}{288}-\zeta_3\Big]L_H+\frac{13}{48}L_H^2\bigg)-n_l^3\bigg(\frac{7127}{5832}-\frac{7}{18}\zeta_2-\frac{1}{27}\zeta_3+\Big[\frac{127}{162}-\frac{1}{9}\zeta_2\Big]L_H \\
&+\frac{7}{36}L_H^2+\frac{1}{54}L_H^3\bigg) \\
&+\frac{m_b^2}{M_H^2}\bigg\{C_A^2\bigg(\frac{991}{2}-\frac{121}{2}\zeta_2-\frac{317}{24}\zeta_3-\frac{35}{8}\zeta_5+\Big[\frac{2675}{12}+22\zeta_3\Big]L_H+\frac{121}{4}L_H^2\bigg)+C_FC_A\bigg(\frac{148921}{192} \\
&-\frac{1415}{8}\zeta_2-\frac{3881}{24}\zeta_3-\frac{685}{24}\zeta_5+\Big[\frac{42767}{96}-\frac{99}{4}\zeta_2-\frac{189}{4}\zeta_3\Big]L_H+\frac{1415}{16}L_H^2+\frac{33}{8}L_H^3\bigg) \\
&+C_F^2\bigg(\frac{18485}{64}-\frac{693}{8}\zeta_2-\frac{675}{8}\zeta_3+\frac{105}{4}\zeta_5+\Big[\frac{5319}{32}-\frac{81}{4}\zeta_2-27\zeta_3\Big]L_H+\frac{693}{16}L_H^2 \\
&+\frac{27}{8}L_H^3\bigg)-n_lC_A\bigg(\frac{20135}{144}-22\zeta_2+\frac{367}{24}\zeta_3+\Big[\frac{845}{12}+4\zeta_3\Big]L_H+11L_H^2\bigg)-n_lC_F\bigg(\frac{13991}{96} \\
&-\frac{121}{4}\zeta_2-\frac{133}{3}\zeta_3+\frac{10}{3}\zeta_5+\Big[\frac{3475}{48}-\frac{9}{2}\zeta_2-9\zeta_3\Big]L_H+\frac{121}{8}L_H^2+\frac{3}{4}L_H^3\bigg)+n_l^2\bigg(\frac{157}{18}-2\zeta_2 \\
&+5L_H+L_H^2\bigg)\bigg\}.
\end{aligned}
\tag{3.7}
$$

Here, C_A and C_F are the quadratic Casimir invariants of $SU(N)$, $L_H=\log(\mu^2/M_H^2)$, n_l is the number of light quarks running in loops and ζ_n is the Riemann Zeta function. Eq. (3.7) agrees with the

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expression presented in [16] if one sets $L_H = 0$, $C_A = 3$, $C_F = 4/3$ and $(m_b^2/M_H^2) = 0$. We also find agreement with [27] if one sets $L_H = 0$ and $(m_b^2/M_H^2) = 0$, which contains an independent calculation of all but the ζ_2 terms of $\Delta_{11}|_{a_s^3}$.

4. Conclusions

By computing the previously-unknown α_s^4 corrections to Δ_{12}, we complete the QCD corrections to the hadronic decay of the Higgs boson at this order. The computation is performed in an effective theory, however additional corrections proportional to powers of $(M_H^2/M_t{}^2)$ are known to be small. The new contributions to $\Gamma(H \to \text{hadrons})$ are of the same order of magnitude as the previously-known contributions, but come with an opposite sign; they do much to improve the convergence of the QCD perturbative series.

Additionally, we provide an independent cross check of the four-loop Higgs boson decay to gluons. Computer-readable files containing the analytic results of these proceedings and Ref. [9] can be found at [28].

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