Prospects for new physics in $\tau \to \ell \mu \mu$ at current and future colliders

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We investigate constraints on the branching fractions of the lepton-flavour-violating decays $\tau \to \ell \mu \mu$ at current and future $pp$ and $e^+e^-$ colliders. The constraints are translated into those on parameters of the Type-II Seesaw Model, the Left-Right-Symmetric Model, and the Minimal Supersymmetric Standard Model.
1. Introduction

While flavour violation has been observed in quark and neutrino processes, charged-lepton flavour violation (LFV) has not been observed and is not present at tree level in the Standard Model (SM). Many models predict observable LFV rates, such as those with additional Higgs triplet fields or with supersymmetry. It is indeed a typical feature of seesaw models that can explain the small neutrino masses.

Experiments at current and future colliders will substantially improve sensitivity to the LFV decay $\tau \rightarrow \ell \mu \mu$. The LHC experiments should give the best sensitivity to $\tau \rightarrow 3\mu$ over the next few years, and Belle II at the SuperKEKB collider and experiments at the Future Circular Collider can further improve sensitivity by more than an order of magnitude.

We present the prospects [1] for experimental $\tau \rightarrow \ell \mu \mu$ limits and the corresponding constraints on parameters in the Type-II Seesaw Model, the Left-Right Symmetric Model (LRSM), and the Minimal Supersymmetric Standard Model (MSSM). Figure 1 shows example Feynman diagrams for the $\tau^- \rightarrow \mu^- \mu^- \mu^+$ decay in these models.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{example_feynman_diagrams.png}
\caption{Example Feynman diagrams for the decay $\tau^- \rightarrow \mu^- \mu^- \mu^+$ in the Type-II Seesaw Model (left), the LRSM (middle) and the MSSM (right).}
\end{figure}

2. Experimental sensitivity

The best sensitivity to $\tau \rightarrow \ell \mu \mu$ decays comes from experiments at $e^+e^-$ colliders due to the low background and clean environment. The Belle experiment at KEK and the BaBar experiment at SLAC set limits on all six $\tau \rightarrow 3\ell$ decays using 720 million [2] and 430 million [3] tau-lepton pairs, respectively. The background is $\lesssim 0.1$ event in each decay channel, while the selection efficiency ranges from 7.6% to 10.1%. The 90% confidence level (C.L.) upper limit on the branching fraction for $\tau \rightarrow 3\mu$ (\$B_{\tau \rightarrow 3\mu}$) is $2.1 \ (3.3) \times 10^{-8}$ from the Belle (BaBar) experiment.

Future running at KEK will provide Belle II with 50 times its current luminosity by 2025. Conservatively scaling the background by 50 gives an expected upper limit on the branching fraction of $10^{-9}$. More optimistically assuming that future analyses will maintain the current level of background while losing a modest relative 10% of acceptance, the projected limit is $4.7 \times 10^{-10}$.

Recent searches for $\tau \rightarrow 3\mu$ at the LHC have demonstrated the potential to exceed the sensitivity of Belle before Belle II produces tighter constraints. The LHCb experiment has obtained the constraint $\mathcal{B}_{\tau \rightarrow 3\mu} < 4.6 \times 10^{-8}$ with $\sqrt{s} = 7$ and 8 TeV data from Run 1 [4]. With an inclusive $\tau$-lepton production cross section of 85 $\mu$b, more than 300 billion $\tau$ leptons were produced.
in 3 fb$^{-1}$ of integrated luminosity collected by the experiment. At the HL-LHC the yield will be increased by a factor of 24 due to the 15-fold increase in luminosity and the factor of 1.6 larger cross section at $\sqrt{s} = 13$ TeV. We expect the final constraint on $B_{\tau\to3\mu}$ from LHCb to be in the range $(1.5 - 11) \times 10^{-9}$.

ATLAS has also performed a search for $\tau\to3\mu$ using $\sqrt{s} = 8$ TeV data, and constrained the branching fraction to be less than $3.8 \times 10^{-7}$ [5]. To reduce background to less than an event, the analysis uses $W$ boson decays and a boosted decision tree. The HL-LHC will increase the integrated luminosity by a factor of 100, and there will be an additional factor of 1.6 from the increase in cross section. The backgrounds and triggering will be particular challenging at the HL-LHC, but assuming that analysis improvements can maintain current efficiencies the final constraint on the branching fraction will be in the range $(1.8 - 8.9) \times 10^{-9}$.

Experiments at a Future Circular Collider (FCC) would further increase sensitivity to $\tau\to\ell\mu\mu$ decays. A $pp$ collider at $\sqrt{s} = 100$ TeV would have a $W$ boson cross section $\approx 7$ times that of the LHC. With 3 ab$^{-1}$ of integrated luminosity and a background and efficiency similar to ATLAS, experiments at a $pp$ FCC would provide limits in the range $(3 - 30) \times 10^{-10}$ on $B_{\tau\to3\mu}$. The best potential comes from an $e^+e^-$ collider at the $Z$ boson resonance with 55 ab$^{-1}$ of luminosity at four interaction points, leading to $\approx 300$ trillion $\tau$-lepton pairs. Assuming negligible background and 40-80% acceptance, the constraints on $B_{\tau\to3\mu}$ would be in the range $(5 - 10) \times 10^{-12}$.

A summary of the current and projected 90% C.L. limits on $B_{\tau\to3\mu}$ is given in Table 1.

### Table 1: Current and projected 90% C.L. limits on $B_{\tau\to3\mu}$ [1].

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Current</th>
<th>Projected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belle</td>
<td>$2.1 \times 10^{-8}$</td>
<td>$(4.7 - 10) \times 10^{-10}$</td>
</tr>
<tr>
<td>BaBar</td>
<td>$3.3 \times 10^{-8}$</td>
<td>$-$</td>
</tr>
<tr>
<td>FCC-ee</td>
<td>$-$</td>
<td>$(5 - 10) \times 10^{-12}$</td>
</tr>
<tr>
<td>LHCb</td>
<td>$4.6 \times 10^{-8}$</td>
<td>$(1.5 - 11) \times 10^{-9}$</td>
</tr>
<tr>
<td>ATLAS</td>
<td>$3.8 \times 10^{-7}$</td>
<td>$(1.8 - 8.1) \times 10^{-9}$</td>
</tr>
<tr>
<td>FCC-hh</td>
<td>$-$</td>
<td>$(3 - 30) \times 10^{-10}$</td>
</tr>
</tbody>
</table>

3. **Model parameter constraints**

We translate the current and projected limits on $B_{\tau\to\ell\mu\mu}$ into limits on parameters in the Type-II Seesaw Model, the Left-Right-Symmetric Model, and the Minimal Supersymmetric Model. We focus on the impact of the constraints on $B_{\tau\to3\mu}$, since both $pp$ and $e^+e^-$ colliders contribute to the current and expected constraints.

#### 3.1 Type-II Seesaw Model

The Type-II Seesaw Model adds a single Higgs triplet, whose Yukawa terms lead to lepton number violation and LFV. The neutrino masses are given by $m_{\nu} = \sqrt{2}Y_{\mu}v_{\Delta}$, where $Y_{\mu}$ are the relevant Yukawa couplings and $v_{\Delta}$ is the vacuum expectation value (vev) of the triplet. The vev is related to the $\mu_{\Delta}$ parameter describing the interaction between the triplet and the SM Higgs.
doublet by $v_\Delta = \mu_\Delta \phi_\Delta^2 / (\sqrt{2} m_\Delta^2)$, where $\phi_\Delta$ is the vev of the SM Higgs field and $m_\Delta$ is the mass of the doubly-charged Higgs boson in the Higgs triplet. The partial decay width is given by

$$\Gamma(\tau^+ \to \mu^+ \mu^+ \mu^-) = \frac{m_\tau^2}{192 \pi^4} |C_{\tau \mu \mu \mu}|^2,$$

where $C_{\tau \mu \mu \mu} = m_\nu(\tau, \mu) m_\nu(\mu, \mu) / (2v^2_\mu m_\Delta^2)$ and the $m_\nu$ factors are entries in the neutrino mass matrix before diagonalization. For a neutrino mass after diagonalization of $m_1 = 0.1$ eV, the current and optimistic expected limits in the $v_\Delta$-$\mu_\Delta$ plane for $\tau \to 3\mu$ are shown in Figure 2 (left). Limits in the plane of the Dirac CP-violating phase $\delta$ and the mixing angle $\theta_{12}$ in the diagonalizing matrix are shown in Figure 2 (right) for $v_\Delta = 10^{-10}$ GeV and $m_\Delta = 8$ TeV. Belle and the experiments at the FCC could constrain $\delta$ for these model parameters.

3.2 Left-Right Symmetric Model

The Left-Right Symmetric Model extends the Type-II Seesaw Model by adding SU(2)$_R$ Higgs doublet and triplet fields. The pair of doublets form a bidoublet with zero B-L charge. Here we assume that the lowest mass doubly charged Higgs boson is from the triplet ($\delta_R^{\pm \pm}$), while that from the SU(2)$_L$ triplet ($\delta_L^{\pm \pm}$) has a large mass to avoid flavour changing neutral currents from the neutral Higgs boson in the triplet (which must have a lower mass).

The mass of the $\delta_R^{\pm \pm}$ is related to the parameters in the Higgs potential via $m_{\delta_R^{\pm \pm}}^2 \approx 2\rho_2 v_R^2 + \alpha_3 k_2^2 / 2$, where $\rho_2$ is the coefficient of the term containing $\text{Tr}[\Delta_R^\dagger \Delta_R]$, $\alpha_3$ is the coefficient of the term containing $\text{Tr}[\Phi^\dagger \Phi \Delta_R^\dagger \Delta_R^\dagger]$, $v_R$ is the vev of the SU(2)$_R$ Higgs triplet, and $k_2$ is the quadrature difference between the vacuum expectation values of the SU(2)$_L$ and SU(2)$_R$ Higgs doublet fields (which are represented as a bidoublet $\Phi$). We choose two benchmark scenarios where the $\delta_R^{\pm \pm}$ has an $\mathcal{O}(\text{TeV})$ mass: $(\alpha_3, v_R) = (1, 30 \text{ TeV})$ and $(18.88, 8.68 \text{ TeV})$.  

**Figure 2:** Left: The current and projected limits on $R_{\tau \to 3\mu}$ translated to the plane of the Higgs triplet vev ($v_\Delta$) and the $\mu_\Delta$ parameter describing the interaction between the triplet and the SM doublet [1]. The parameters are related by $v_\Delta = \mu_\Delta \phi_\Delta^2 / (\sqrt{2} m_\Delta^2)$. Lines of constant $m_\Delta$ are shown in the plane. Right: Projected limits in the plane of the Dirac CP-violating phase $\delta$ and the mixing angle $\theta_{12}$ for $v_\Delta = 10^{-10}$ GeV and $m_\Delta = 8$ TeV [1].
The partial decay width is given by the same equation as in the Type-II Seesaw Model, except with SU(2)_R parameters in C_{\mu \mu \mu}. The current and projected experimental constraints on $B_{\tau \rightarrow 3\mu}$ are translated into constraints in the $\rho_2$-$m_N$ plane for the two benchmark scenarios in Figure 3, where $m_N$ is the right-handed neutrino mass (equal for all generations).

3.3 Minimal Supersymmetric Standard Model

In the MSSM the soft supersymmetry-breaking terms generically induce generational mixing among sleptons and lead to LFV processes. The mass matrix can be explicitly separated into the symmetry-breaking flavour-violating terms and the SM flavour-conserving terms. Writing the supersymmetric lepton field as a 6-dimensional vector with three generation indices and two handedness indices, the mass matrix has blocks of left-left, right-right, and left-right terms:

$$
M^2_{L_{Li}} = M^2_{E_{ij}} + \left[ m_i^2 + \left( -\frac{1}{2} + \sin^2 \theta_W \right) m_2^2 \cos 2\beta \right] \delta_{ij}
$$

$$
M^2_{R_{Rij}} = M^2_{E_{ij}} + (m_i^2 - \sin^2 \theta_W m_2^2 \cos 2\beta) \delta_{ij}
$$

$$
M^2_{L_{Rij}} = v_1 \delta^L_{ij} - m_j \mu \tan \beta \delta_{ij},
$$

where the first term in each equation is the supersymmetric mixing term that causes LFV, $i$ and $j$ are generational indices, $\tan \beta = v_2/v_1$, and $v_1$ and $v_2$ are the vacuum expectation values of the two Higgs doublets. We parameterize the off-diagonal elements as $\delta^L_{ij} = M^2_{E_{bij}}/(m_A m_B)$ and choose the following set of benchmark parameters: $\tan \beta = 10$, $\mu = -100$ GeV, $m_A = 1$ TeV, $m_1 = 250$ GeV, $m_2 = 500$ GeV, $m_3 = 2$ TeV, $m_{L_i} = m_{E_j} = 1$ TeV, and $A_\tau = 200$ GeV. Figure 4 shows the translation of the constraints on $B_{\tau \rightarrow 3\mu}$ into the $\delta^L_{23}$-$\delta^R_{23}$ and $\delta^L_{23}$-$m_L$ planes, where the latter varies $m^*_{L_i} = m_{E_j}$ instead of fixing it to 1 TeV. The figure shows little dependence on $\delta^R_{23}$ since the decay is mediated by neutralinos with small gauge couplings to right-handed fermions. The branching fraction increases with increasing $\delta^L_{23}$ and decreasing $m_{L_i}$, so tighter constraints on
References