

# A Proportional Fair Resource Allocation Algorithm for Hybrid Hierarchical Backhaul Networks

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In order to improve the total system throughput by analyzing the resource allocation problems in backhaul links and access links of base stations in ultra-dense network (UDN), a proportional fair resource allocation (PFRA) algorithm was designed for the wireless hybrid hierarchical backhaul network in this paper. First, the base stations were stratified and the frame structure of each base station in different layers was re-designed. Then, the objective function was constructed based on the proportional fair utility function to optimize the system throughput. Finally, an iterative algorithm was derived based on Lagrange Multiplier algorithm to obtain the optimal solution of spectrum resource allocation in hybrid hierarchical backhaul network. The performance of the PFRA algorithm has been simulated and the results show that the PFRA algorithm can improve network throughput effectively at the cost of losing a certain network coverage rate. In the future, PFRA needs to be optimized to reach the best balance between network throughput and network coverage rate.

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## 1. Introduction

Complex networks have been widely studied in recent years and different interconnected network models are proposed to solve the cascading failures caused by overload [1-4]. As a new heterogeneous network architecture, ultra-dense network (UDN) is different from the deployment scenario of traditional macro-cell. UDN provides a efficient energy-saving solution to the wireless mobile data services that grow rapidly in the future [5]. Under this network architecture, the number of small-cell base stations (SBSs) is increased and the coverage area of single cell is reduced with denser deployment. Therefore, the load of macro-cell base stations (MBSs) can be shared, the physical distance between the sender and the receiver shortened and low latency and reliable user experience achieved. Meanwhile, UDN reduces the installation cost of the network infrastructure deployment and the operating cost of the base stations significantly [6]. However, the complexity of the network structure makes inter-cell interference more serious. Besides, the spectrum resource and backhaul network capacity is constrained [7]. How to allocate the spectral resource in backhaul and access links of base stations effectively is an important problem in UDN.

How to allocate resource to reach the most total network throughput and spectral highest efficiency will be studied in this paper. Thus, a proportional fair resource allocation (PFRA) algorithm is designed. The objective function to maximize the network throughput in hybrid hierarchical backhaul network is constructed. And an iterative algorithm is derived based on Lagrange Multiplier algorithm to obtain the optimal allocation of spectrum resource. In addition, PFRA algorithm is compared with traditional static allocation algorithms and simulated by MATLAB software respectively. The performances in terms of total network throughput and edge user throughput are compared and the experimental results are analysed.

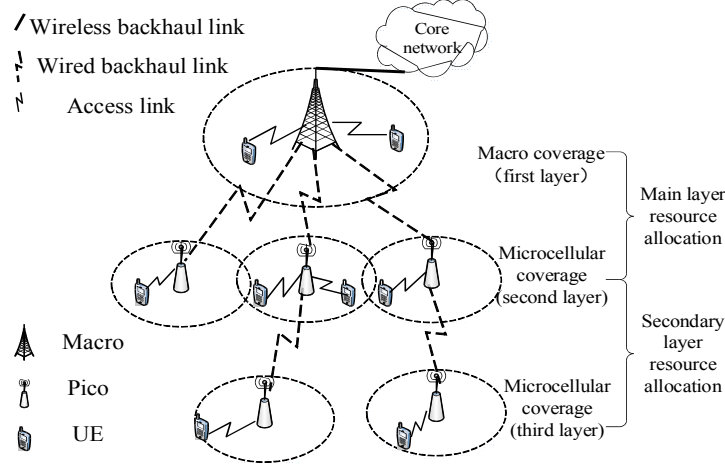
## 2. System Model

Hybrid hierarchical backhaul network is a new network structure as proposed in UDN. In actual deployment, the base stations in the next layer only need to establish a backhaul link with the base station in the upper layer to achieve plug and play. In order to simplify and keep general nature, three-hop hybrid hierarchical backhaul network scenario is used in this paper, as shown in Figure 1. MBSs in the first-level backhaul layer (the following is called first layer) is denoted by  $SN_0$  and SBSs in the second-level backhaul layer (the following is called second layer) is denoted by  $SN_s$  ( $s \in S, s \neq 0$ ).  $S$  is the set of base stations in the first and second layer. SBSs in the third-level backhaul layer (the following is called third layer) is denoted by  $SN_{sc}$  ( $s \in S, c \in C$ ).  $C$  is the set of base stations in the third layer. The set of User Equipment (UE) is denoted by  $U$ . The set of UE connected to the base stations in the first and the second layer is  $U_s$ , in the third layer is  $U_{sc}$ .

UE is connected to one base station at most at some point to communicate without considering the cooperative transmission. Since only downlink transmission is considered in this paper, the data packets that SBSs transmit to UE are generally sent by MBSs to SBSs. During this period, the data packets transmitted from the upper layer are decoded by SBSs, and queued in the buffer of SBSs, waiting for the scheduler to be transmitted to the corresponding target UE in turn. According to the definition of frame structure in 3GPP LTE [8], the reception and transmission of SBSs can't be carried out at the same time so as to avoid the signal interference.

Therefore, a radio frame is divided into the access subframe and the backhaul subframe. The MBSs and SBSs in each layer simultaneously multiplex the subframe frequency band and transmit data to their own UE at the same time in the access subframe. The MBSs orthogonally allocates frequency band to UE for backhaul link in backhaul subframe. The subframe allocation changes upon the scene, the UE's need or higher configuration.

A basic OFDMA resource allocation unit can be named as a resource block (RB), which is composed of resource elements in column [9]. RB is used as the smallest independent resource allocation unit in this paper.



**Figure 1:** Hybrid Hierarchical Network Framework

### 3. Proportional Fair Resource Allocation Algorithm

The PFRA algorithm re-designs the frame structure of base stations in different layers based on hybrid hierarchical network framework. The frame structure of SBSs in the second layer is the same as MBSs and opposite to SBSs in the third layer. How many RBs are allocated to each base station is derived from the following algorithm.

The algorithm takes a utility function based on proportional fairness [10]:

$$\max \sum_{u \in U} \log R_{u,s} = \max \sum_{u \in U} \sum_{s \in S} \rho_{u,s} \log \left( \sum_{j \in J} w_{j,u} r_{j,u} \right) \quad (3.1)$$

$R_{u,s}$  is the received data rate of a UE (denoted by  $u$ ) which connected to the  $SN_s$  on a radio frame. If  $u$  is directly connected to the  $SN_s$ ,  $\rho_{u,s}=1$ , otherwise  $\rho_{u,s}=0$ .  $w_{j,u}=1$  means that the  $j$ th RB is assigned to  $u$ , otherwise  $w_{j,u}=0$ . The  $r_{j,u}$  refers to the instantaneous data rate received by  $u$  at the  $j$ th RB.  $J$  is the sum of RBs in a radio frame. (3.1) meet following conditions:

$$\begin{aligned} \sum_{s \in S} \rho_{u,s} &= 1, \rho_{u,s} \in \{0,1\}, \forall u \in U \\ \sum_{u \in U} w &\leq 1, w \in \{0,1\}, \forall j \in J, \forall s \in S \end{aligned} \quad (3.2)$$

#### 3.1 Main Layer Resource Allocation

The main layer resource allocation is that the MBSs allocate the RBs to their own UE and the SBSs in the second layer. Only the access resource of the MBSs and the access and backhaul resource of the SBSs in the second layer are considered. We assume that the data services and service queues of the UE under the MBSs are of full buffer. Therefore, UE belonging to the MBSs can consume all the allocated access resource  $J_a$ . The average data rate of  $u$  connected to  $SN_0$  is

$$\bar{R}_{u,0} \approx \frac{\lceil J_a \rceil}{U_0} f(\bar{\gamma}_{u,0}) G(U_0) \quad (3.3)$$

The scheduling gain  $G(U_0) = 1$  due to the Round-Robin (RR) scheduling scheme used. The key idea of the RR scheduling is that the UEs in the cell have the same priority, and each UE is periodically scheduled with equal opportunity.  $f(\bar{\gamma}_{u,0})$  denotes the linear mapping function of signal to interference plus noise ratio (SINR) in the access link that from  $u$  to  $SN_0$ , calculated by

$$f(\bar{\gamma}_{u,0}) = \log_2 \left( 1 + \frac{R}{N_0 + I} \right) \quad (3.4)$$

$R$  refers to the signal UE received.  $N_0$  is the Gaussian white noise.  $I$  means the interference UE received which is an accumulated value. And the SBSs in the second layer not only provide access services to their own UE, but also provide backhaul services to the SBSs in the third layer.

As demonstrated below, the average data rate of the  $u$  connected to  $SN_s$  ( $s \in S, s \neq 0$ ) is

$$\bar{R}_{u,s} \approx \frac{\lceil J_a^s \rceil}{|U_s + U'_s|} f'(\bar{\gamma}_{u,s}) \quad (3.5)$$

The SBSs in the second layer can't use  $J_a$  fully unless the link from  $SN_0$  to  $SN_s$  can provide enough transfer rate. Therefore, the access resource  $J_a^s$  that SBSs allocated is a part of  $J_a$ .  $U'_s$  represents the set of UE connected to SBSs in the third layer,  $U'_s = \sum_{s \in S} U_{sc}$ . If  $u \in U_s$ ,  $f'(\bar{\gamma}_{u,s}) = f(\bar{\gamma}_{u,s})$ . If  $u \in U'_s$ ,  $f'(\bar{\gamma}_{u,s}) = f(\bar{\phi}_{sc})$ .  $f(\bar{\phi}_{sc})$  is the linear mapping function of the SINR in the backhaul link that goes from  $SN_{sc}$  to  $SN_s$ , calculated as (3.4).

Therefore, the cumulative average data rate  $\bar{R}_a^s$  obtained on the allocated access subframe of  $SN_s$  is calculated by  $\bar{R}_a^s = \sum_{u \in (U_s + U'_s)} \bar{R}_{u,s}$ . Correspondingly, the cumulative average data rate  $\bar{R}_b^s$  obtained on the allocated backhaul subframe of  $SN_s$  is calculated as  $\bar{R}_b^s = \lceil J_b^s \rceil f(\bar{\phi}_s)$ .

$J_b^s$  is the required backhaul resource where  $SN_0$  transmits data to  $SN_s$ . In order to avoid packet congestion, the cumulative average data rate of each base station should match each other on the access subframe and the backhaul subframe, so,  $\bar{R}_a^s = \bar{R}_b^s$ .

We substitute (3.3) and (3.5) into (3.1) to obtain the object function of PFRA:

$$\max \left( \underbrace{\sum_{s \in S} |U_s + U'_s| \times \log \lceil J_a^s \rceil}_{\text{sub-formula 1}} + \underbrace{\sum_{u \in U} \sum_{s \in S} \rho_{u,s} \times \log \frac{f'(\bar{\gamma}_{u,s})}{|U_s + U'_s|}}_{\text{sub-formula 2}} \right) \quad (3.6)$$

When the SINR of the communication link is unchanged at the period of resource allocation, the ratio  $\eta_s$  of the backhaul resource to the access resource of each base station can be determined. So, the sub-formula 2 in (3.6) is a definite value. (3.6) can be solved only when the sub-formula 1 is considered and two conditions are met as below:

$$\begin{aligned} \sum_{s \in S} \eta_s \lceil J_a^s \rceil &= \lceil J \rceil \\ \lceil J_a^s \rceil &\leq \lceil J_a \rceil, \forall s \in S \end{aligned} \quad (3.7)$$

The sub-formula 1 can be transformed by using the equal sign constraint:

$$\max \sum_{t \in D} |U_t + U'_t| \times \log \lceil J_a \rceil + \sum_{s \in S, s \notin D} |U_s + U'_s| \times \log \lceil J_a^s \rceil \quad (3.8)$$

The subset  $D$  is defined as a set of base stations that can fully use the access resource. The Lagrange multiplier formula is used to solve (3.8) and  $\lambda$  ( $\lambda \geq 0$ ) is the Lagrange multiplier.

$$L(|J_a^s|, \lambda) = \sum_{t \in D} |U_t + U_t'| \times \log |J_a| + \sum_{s \in S, s \notin D} |U_s + U_s'| \times \log |J_a^s| - \lambda \left( \sum_{t \in D} \eta_t |J_a| + \sum_{s \in S, s \notin D} \eta_s |J_a^s| - |J| \right) \quad (3.9)$$

The access resource  $|J_a^s|$  of  $\text{SN}_s$  can be obtained with derivative in (3.9).

$$|J_a^s| = \begin{cases} \frac{|U_s + U_s'|}{\eta_s} \cdot \frac{|J|}{U}, & s \in S, s \notin D \\ \frac{\sum_{t \in D} |U_t + U_t'|}{\sum_{t \in D} \eta_t} \cdot \frac{|J|}{U}, & t \in D \end{cases} \quad (3.10)$$

The result of resource allocation must be in the form of integer, so the access resource of  $\text{SN}_s$  is  $|J_a^s| = \lceil \max(1, |J_a^s|) \rceil, \forall s \in S, (U_s + U_s') \neq 0$ .

The iterative algorithm is used to find the smallest number of SBSs in subset  $D$ . First, the subset  $D$  only contains the MBSs. Then, make  $P$  as a set of SBSs in the second layer when access resource calculated by (3.10) do not satisfy the constraint condition (3.7). Finally, if the set  $P$  is nonempty, the SBS with most access resource is selected from the set  $P$  and added to the subset  $D$  which is updated. The process repeats until the set  $P$  is empty.

### 3.2 Secondary Layer Resource Allocation

The secondary layer resource allocation is that the SBSs in the second layer allocate RBs to their UEs and the SBSs in the third layer. After the access resource  $J_a^s$  of  $\text{SN}_s$  has been calculated, the backhaul resource  $J_b^s$  is obtained by  $|J_b^s| = \eta_s |J_a^s|, s \in S$ .

The backhaul link cumulative rate  $\bar{R}_b^{sc}$  of  $u$  connected to  $\text{SN}_{sc}$  is

$$\bar{R}_b^{sc} = \sum_{u \in U_{sc}} \bar{R}_{u,sc} \approx \sum_{u \in U_{sc}} \frac{|J_a^s|}{|U_s + U_s'|} f(\bar{\phi}_{sc}) \quad (3.11)$$

The backhaul resource  $J_b^{sc}$  of  $\text{SN}_{sc}$  is

$$|J_b^{sc}| = \frac{\bar{R}_b^{sc}}{f(\bar{\phi}_{sc})} = \sum_{u \in U_{sc}} \frac{|J_a^s|}{|U_s + U_s'|} \quad (3.12)$$

The access resource  $J_a^{sc}$  of  $\text{SN}_{sc}$  is

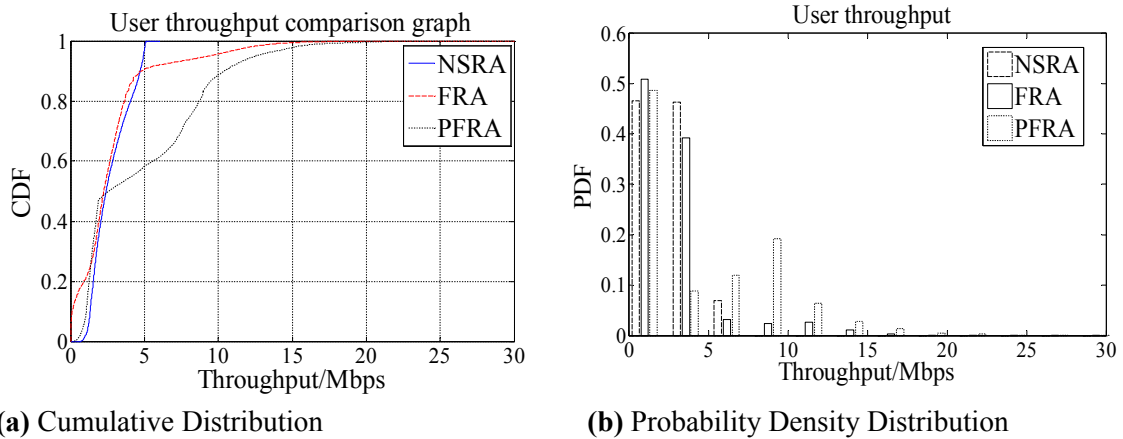
$$|J_a^{sc}| = \frac{f(\bar{\phi}_{sc})}{\sum_{u \in U_{sc}} f(\bar{y}_{u,sc})} \cdot \frac{|U_{sc}|^2}{|U_s + U_s'|} |J_a^s| \quad (3.13)$$

It should be ensured that  $J_a^{sc}$  is less than backhaul resource  $J_b$  of  $\text{SN}_0$ , otherwise, reduce the  $J_a^{sc}$ . Thus by now, the resource allocation of the access and the backhaul links have been completed.

## 4. Matlab Simulation Analyses

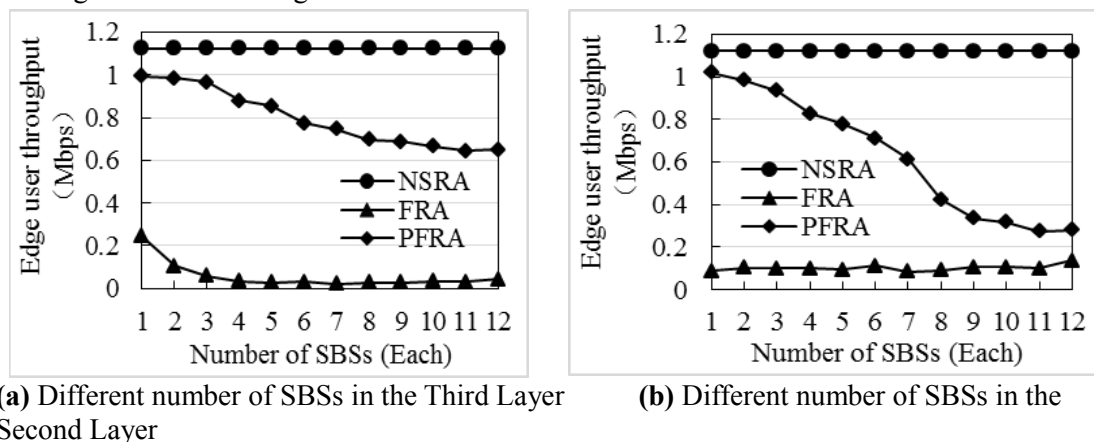
The simulation is performed under a single cell. There is only one MBS in the cell divided into three sectors. The coverage distance of the MBS is  $[-1/3\text{km}, 1/3\text{km}]$ . We mainly simulate the downlink performance gain, focusing on the data from the core network to the UEs.

It can be observed from Figure 2 that the user rate of non-small-cell-station resource allocation (NSRA) algorithm is more concentrated, and that of PFRA algorithm and fixed resource allocation (FRA) algorithm is relatively scattered. PFRA algorithm is the best of the three schemes to improve the user rate. On the one hand, it can reduce the number of users with low rate and increase the number of users with high rate. On the other hand, PFRA algorithm ensures that the number of users in the bottom layer is consistent with the allocation of backhaul resource, and the data rate of the backhaul link is greater than or equal as the sum of rate of the access user. The smallest resource allocation unit used in this scheme is RB with large granularity. Therefore, the rate will significantly be decreased by reducing one RB. If the smallest segmentation unit with smaller granularity is used, such as carrier [11] [12], the user throughput performance will be better.



**Figure 2:** Comparison of User Throughput Distribution for Different Algorithms (with 5 SBSs in the second layer and 2 SBSs in the third layer)

Figure 3 shows that the edge user throughput of PFRA algorithm and FRA algorithm is less than that of the NSRA algorithm. Compared with NSRA algorithm, in PFRA algorithm, while the addition of the small base stations makes total throughput significantly increase, the interference will relatively increase. RR scheduling scheme used in this paper without considering the quality of user channel and the transmission rate may lead to small network coverage rate of PFRA algorithm.



**Figure 3 :** Edge User Throughput of Different Layer and SBSs

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## 5. Conclusion

The PFRA algorithm is designed in this paper for wireless hybrid hierarchical backhaul network to improve the total network throughput. The simulation results show that the PFRA algorithm is effective in improving the network throughput compared with the traditional NSRA algorithm and FRA algorithm. However, the edge user throughput of PFRA algorithm is not very satisfactory when compared with NSRA algorithm.

RR scheduling scheme used may lead to the reduction of the network coverage rate. The proportional fair scheduling scheme which takes user fairness and user rate into account will probably solve this problem and improve the network performance. That's the further study we need to do.

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