

Application of a Coupled Harmonic Oscillator Model to Solar Activity and El Niño Phenomena

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Abstract

Solar activity has an important impact not only on the intensity of cosmic rays but also on the environment of Earth. In the present paper, a coupled oscillator model is proposed in order to explain solar activity. Using this model the 89-year Gleissberg cycle can be naturally reduced. Furthermore, as an application of the coupled oscillator model, we attempt to apply the proposed model to the El Niño-Southern Oscillation (ENSO). As a result, the 22-year oscillation of the Pacific Ocean is naturally explained. Finally, we search for a possible explanation for the coupled oscillators in actual solar activity.

1. Introduction: Motivation of the present research

The present study was conducted considering the following background. An 11-year periodicity and a 22-year periodicity have been found in the growth rate of tree rings in trees that have survived on Yaku Island for 1,924 years [1]. Yaku Island is located at 30°E20'N 130°30'E in southern Kyusyu, Japan. Quite surprisingly, the 11-year and 22-year periodicities are found during the Wolf Minimum (1290-1350 CE), the Spörer Minimum (1440-1550 CE), the Maunder Minimum (1650-1725CE), and the Dalton Minimum (1810-1840 CE). These periods are well known as grand minima of solar activity.

In order to investigate why the 11-year and 22-year periodicities are found in tree rings, we analyzed the meteorological data of Yaku Island. Fortunately, records since 1938 exist for temperature, rainfall, and daylight hours. These data are available from the Japan Metrological Agency [2]. When we analyzed the data, we could find no periodicity in the temperature or rainfall data, except for the one-year periodicity. However, in the daylight hours (insolation) dataset, we have found an 11-year and a 22-year periodicity. The 11-year periodicity was found in June when the island is covered by a thick cloud due to a monsoon, whereas the 22-year periodicity was observed in July and August when the island is usually covered by subtropical high atmospheric pressure. Cloudiness affects the growth rate of cedar trees in terms of photosynthesis. Therefore, we considered that the amount of clouds over the island may be affected by solar activity. However, the origin of the 22-year periodicity still remained unclear.

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The remainder of the present paper is organized as follows. In the next section, we explain the coupled harmonic oscillator model and present an application to solar activity. The coupled oscillator model is then applied to the El Niño-Southern Oscillation (ENSO). We demonstrate that the 26-year periodicity found in the Yaku cedar tree rings may arise from the ENSO. Finally, we will discuss the origin of the coupled harmonic oscillator model by comparing recent helioseismological observations.

2. Application of the coupled oscillator model to solar activity

First, we assume that there are two oscillators with slightly different periodicities in the Sun. The solar magnetic activity can be described by the superposition of each amplitude of the oscillators.

Let us refer to the two oscillators having slight different angular frequencies of ω_A and ω_B as Oscillator-A and Oscillator-B, respectively. Then, the respective oscillations may be described by $\sin(\omega_A \cdot t)$ and $\sin(\omega_B \cdot t)$ waves. Taking into account each phase (α and β), these oscillators may be expressed as $\sin(\omega_A \cdot t + \alpha)$ and $\sin(\omega_B \cdot t + \beta)$, respectively. For simplicity, we take $\beta = 0$, and the absolute amplitude of each oscillator is set at 1.0. Then, the combined amplitude of the two oscillations is given by

$$\psi = \sin\{(\omega_A + \alpha) \cdot t\} + \sin(\omega_B \cdot t) = 2\sin\{(\omega_A + \omega_B + \alpha)/2 \cdot t\} \cdot \cos\{((\omega_A - \omega_B + \alpha)/2) \cdot t\}.$$

Herein, we let T_A and T_B denote the longest solar cycle and the shortest solar cycle observed between 1700 and 2015 CE ($\omega_A = 2\pi/T_A$, $\omega_B = 2\pi/T_B$), which are 12.5 years and 9.5 years, respectively. These values were selected based on the Fourier analysis of the 315-year sunspot activity. The results of the Fourier analysis are shown in **Figure 1**. The angular frequencies ω_A and ω_B are 0.502 and 0.661, respectively.

Here, we slightly modify the term $\sin\{(\omega_A + \omega_B + \alpha)/2 \cdot t\}$ to express an exact 11.0-year periodicity, choosing the phase factor $\alpha/2 = -0.0089$. Then, from the term $\cos\{((\omega_A - \omega_B + \alpha)/2) \cdot t\}$, an 89-year periodicity is reduced. The 89-year periodicity is known as the Gleissberg periodicity [3]. Quite interestingly, this periodicity has been independently detected in another sample of Yaku cedar tree rings based on the C13 measurement [4]. However, we cannot reproduce the 22-year periodicity. In **Figure 2**, we compare our simple expression with actual sunspot data.

3. Application of the Coupled Oscillator Model to the El Niño-Southern Oscillation (ENSO)

The El Niño-Southern Oscillation (ENSO) is observed as the oscillation of the equatorial ocean temperature between the east coast of Indonesia and the west coast of Peru. We examine the possibility that the ENSO could produce the 22-year periodicity by the coupled oscillator model. Details on the ENSO may be found elsewhere [5]. A 5.3-year periodicity and a 3.5-year periodicity are chosen as the two fundamental oscillation frequencies [6]. These numbers are obtained from actual observation data.

The combined amplitude of the two oscillations can then be expressed numerically as follows:

$$\psi = \sin\{(\omega_A \cdot t)\} + \sin(\omega_B \cdot t) = 2\sin\{(\omega_A + \omega_B)/2 \cdot t\} \cdot \cos\{((\omega_A - \omega_B)/2) \cdot t\}.$$

Here, the angular frequencies of ω_A and ω_B are 1.795 and 1.185, respectively. From the term $\sin\{(\omega_A - \omega_B)/2 \cdot t\}$, we can obtain the 20.6-year periodicity. Therefore, we believe that the 22-year periodicity found in the Yaku cedar tree rings may arise from ocean oscillation rather than solar activity.

By the teleconnection process from the equator to the Pacific Ocean, several oscillations may be induced over the ocean surface, one of which is the Pacific Decadal Oscillation (PDO) [7]. Another oscillation reported by Nitta is an oscillation arising from the La Niña phenomenon [8].

When La Niña appears, high tropical atmospheric pressure covering the Japanese coastal region will be depressed. The frequent appearance of clouds is then expected over Yaku Island. This effect gives rise to variation in the growth rate of the tree rings via photosynthesis.

4. Meridional circulation of solar plasma and the coupled oscillator model

In this section, we discuss the correspondence between the two coupled harmonic oscillators and actual solar activity. Until now, we have extended discussions based on a complete phenomenological examination. Here, we compare the proposed model with actual observations of solar activity and attempt to understand which dynamics of the Sun may be related to the coupled oscillator model.

Zhao et al. reported a very interesting result on the solar convection zone based on the helioseismological data obtained by the Solar Dynamics Observatory (SDO)/Michelson Doppler Imager [9]. According to Zhao et al., the meridional circulation of the plasma flow is formed by double layers. The plasma flow of the outer layer is carried from the equator poleward, and perhaps the flow returns back through the shallow interior path from the polar side toward the equator, the corresponding depth of which is $r = 0.82 - 0.91R_{\odot}$. At the deeper layer of the convection zone ($r = 0.70 - 0.82R_{\odot}$), the plasma is assumed to flow from the equator poleward ($r = 0.70R_{\odot}$) and probably takes the return path through the middle region of the convection zone ($r = 0.85R_{\odot}$). In the northern hemisphere, the plasma gas flows anticlockwise in the upper cell and clockwise in the lower cell. To this point, only observational results have been considered. Based on the above observation results, we hereinafter propose a model.

We assume that the currents in the upper cell and the inner cell constitute an oscillator and interact. In the northern hemisphere of the Sun, the toroidal current of the upper cell produces the poloidal magnetic field providing south polarity (S) around the north-pole region of the Sun. The toroidal currents of the outer and inner cells may constitute the coupled harmonic oscillator.

If we assume that the strength of the poloidal magnetic field at the north-pole region is on the order of 10^{-4} T, then the strength of the toroidal current can be estimated to be as large as 1×10^{11} A by the following equation: $B = \mu I / 2r$. (Here, we adopt a value of $\mu = 4\pi \times 10^{-7}$. Note, however, that as denser matter exits in the convection zone, μ requires a correction.) In the parallel flow region at radius $r = 0.85R_{\odot}$, the distance between the two cells (plasma loops) may be rather short in comparison with the other region. When we assume the distance between two cells is approaching 700 km, a much higher magnetic field of approximately 1,000 gauss is expected to be induced, which may be on the same order as the magnetic field of the sunspots. The 22-year variation of the solar magnetic field may be induced by the competition of the two currents between the inner cell and the outer cell, and the overall feature of the solar magnetic field is determined. The coupled oscillator model is pictorially represented in **Figure 3**.

Figure 3 shows the solar minimum of 1986. Here, the N- and S-magnetic poles, which correspond to the “plumbs” of the coupled harmonic oscillator, are pictorially represented. In Figure 3, the plumbs of the inner loop approach the closest distance, while the plumbs of the outside loop remain at the farthest distance. At the equatorial region of the Sun, the embryos of the magnetic poles collide with each other and may result in pair annihilation (cancellation). The energy of the cancellation in the region may be estimated as to be $\sim 3 \times 10^{22}$ Mx [gauss·cm²]. Some part of them is expired as the Coronal Mass Ejection (CME). The remaining embryos will be pulled back by the attractive force between the N and S poles.

The correspondence described here may be compared with the relation between the thermodynamics and the statistical mechanics of solar activity. The thermodynamics describes thermal phenomena from a macroscopic point of view, while the statistical mechanics explains the phenomena from a microscopic point of view.

The plasma circulation crossing the inhomogeneous magnetic field may be an origin of the

toroidal current. However in this paper we will not join in the historical debates on the origin of the magnetic field [10-11]. Instead we simply assume the toroidal currents in the inner cell and the outer cell. For example the current in the outer cell becomes stronger, according to the law of Lenz, an inverse current may be induced in the inner cell, and a poloidal magnetic field with opposite polarity may be generated by the inner cell. Thus, the currents in the inner cell and outer cell constitute a coupled harmonic oscillator respectively. These oscillators are shown pictorially in **Figure 4** and **5**.

In closing, we consider two things. First, we consider the average speed of the meridional flow of the inner and outer loops. Taking the average speed of the inner flow as $\langle v_{in} \rangle \approx 6.0$ m/s and that for the outer flow as $\langle v_{out} \rangle \approx 5.3$ m/s, the 9.5-year and 12.5-year circulation periodicities of the plasma flow can be reproduced. According to the actual observation, the meridional flow is very slow near the equator, whereas, at higher latitudes, the meridional flow increases to approximately 10 m/s [9]. Second, the sunspot embryos may be produced at $r = 0.85r_{\odot}$ at the middle latitude of the Sun when the two cells approach to the nearest distance. The embryos of the magnetic poles (a tiny magnet composed of plasma vortex) may be produced by the interactions of the two flows (turbulence). These embryos surface by the magnetic levitation mechanism [12].

5. Variation of cosmic ray intensity observed by neutron monitors and the proposed model

Here, we provide an interpretation of the cosmic ray intensity measured for a long period with the use of neutron monitors. As known by cosmic ray physicists, the shapes of the intensity distributions of the solar minimum during the periods of 1969-1980 and 1992-2000, and those of the periods of 1960-1968, 1980-1991, and 2004-2012 are quite different [13]. The data obtained between 1969 and 1980 and between 1992 and 2000 have a “flat” distribution of the cosmic ray intensity, whereas the intensity of the data obtained during the other periods exhibited a sharp peak. We point out this difference based on the coupled oscillator model presented herein. This difference may arise from the difference in the position of the oscillators, i.e., whether the solar minimum is induced by the outer oscillator or the inner oscillator. The solar minimum produced by the outer oscillator, as in 1986, the magnetic poles are formed by the outside loop. The magnetic field produced by the outer oscillator may be stronger than those produced by the inner oscillator due to the difference in the radius of the current.

6. Summary

The conclusions of the present study are summarized as follows:

1. We have investigated a possible explanation of the solar activity by a coupled oscillator model.
2. By taking the periods of the two oscillators as 12.5 years and 9.5 years, the 89-year Gleissberg cycle can be reproduced.
3. We speculate that the two oscillators represent the ring currents induced by the plasma motions of the inner loop and the outer loop of the meridional circulations.
4. Two types of variation of the cosmic ray intensity during the solar minimum may be related to the position of the oscillators, whether they are induced by the outer layer or the inner layer of the convection zone.
5. The coupled harmonic oscillator model can be applied to the ENSO. The 22-year oscillation of the solar-environment can be reproduced. The 22-year periodicity may be related to the ENSO, not the solar activity. The coupled oscillator model has successfully explained other astrophysical objects [14, 15].

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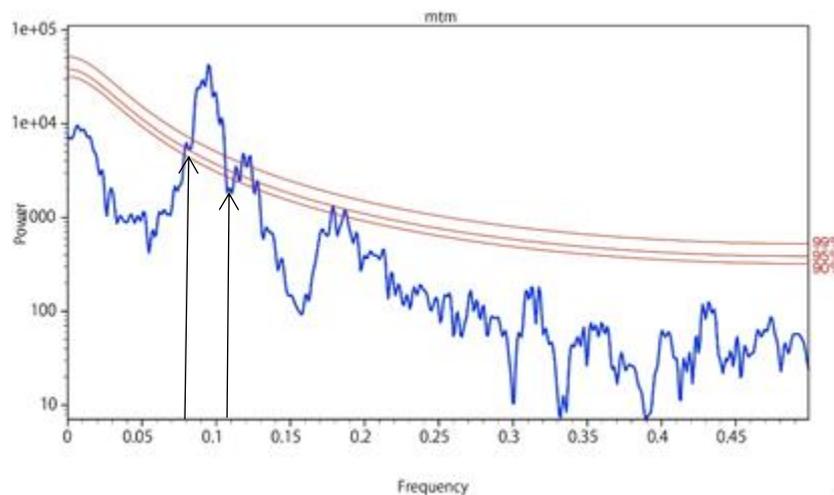


Figure 1. Result of Fourier analysis for the sunspots observed between 1700 and 2005. A sharp peak occurred far beyond the 99% confidence level that corresponds to the 10.7-year periodicity. The two arrows indicate the periods of 12.5 years (left-hand side) and 9.5 years (right-hand side).

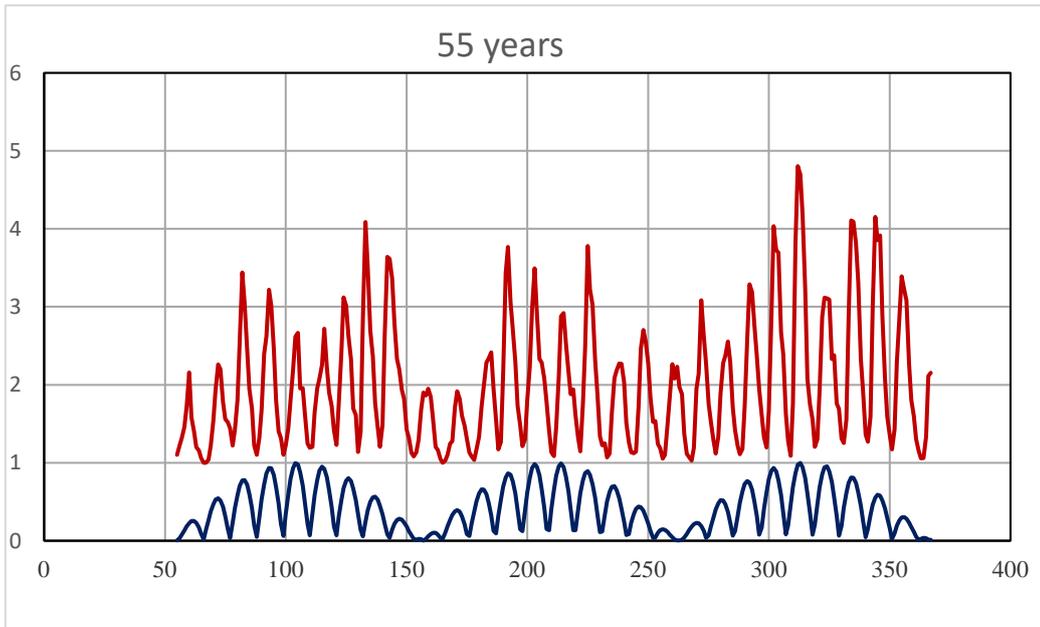


Figure 2. Observed sunspot number (red) and sunspot number predicted by the coupled harmonic oscillator model (blue). In order to adjust the start time, the prediction was shifted 55 years. The fundamental oscillator was produced with 22.0-year periodicity. However, for the purpose of comparison, the absolute value of the amplitude of ψ is taken.

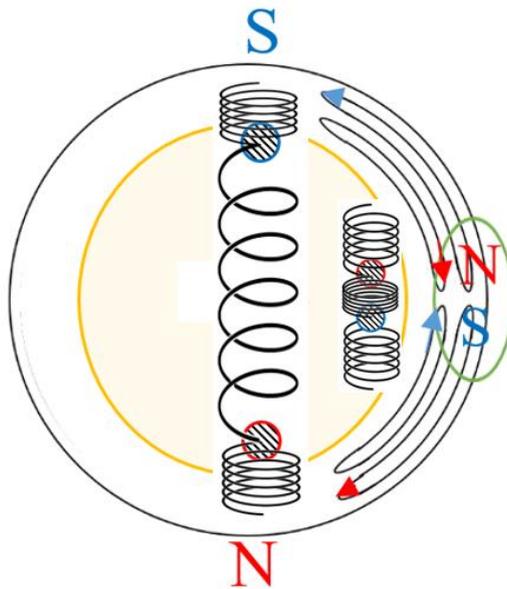


Figure 3. Schematic diagram of the double coupled oscillator model. The position of the plumbs of the oscillator corresponds to the magnetic poles of the solar magnetic field induced by the internal ring-current and outside ring-current. The image represents the solar minimum of 1986. The inner oscillator (inner cell) approached the minimum distance, whereas the outer oscillator (outer cell) takes the longest distance.

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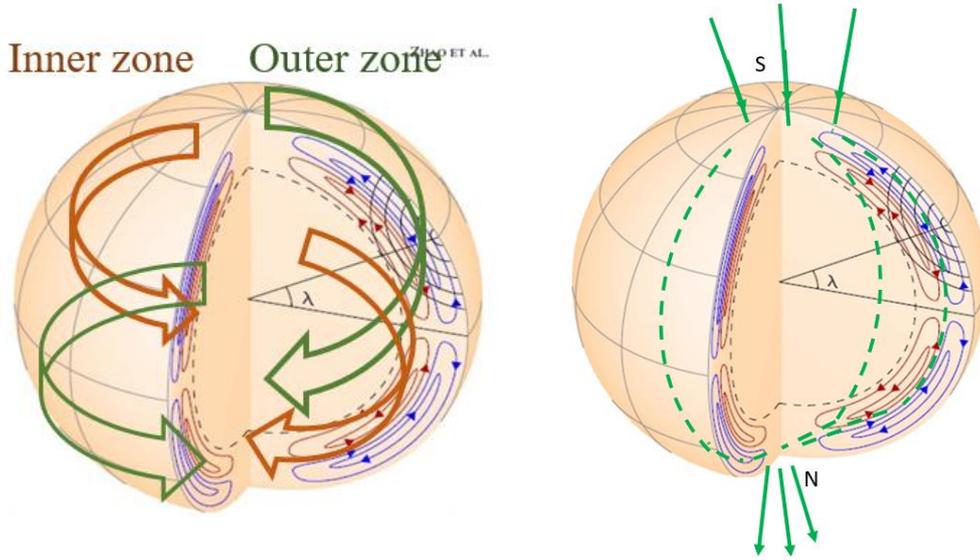


Figure 4. Pictorial representation of the current model on the figure of Zhao et al. The **brown arrows** correspond to the inner stream, while the **dark green curves** represent the toroidal currents in the outer cell. (Original image by Zhao et al. [ApJL 774 (2013) L29].) The right-hand side of the figure shows the poloidal field of the Sun induced by the outer cell.

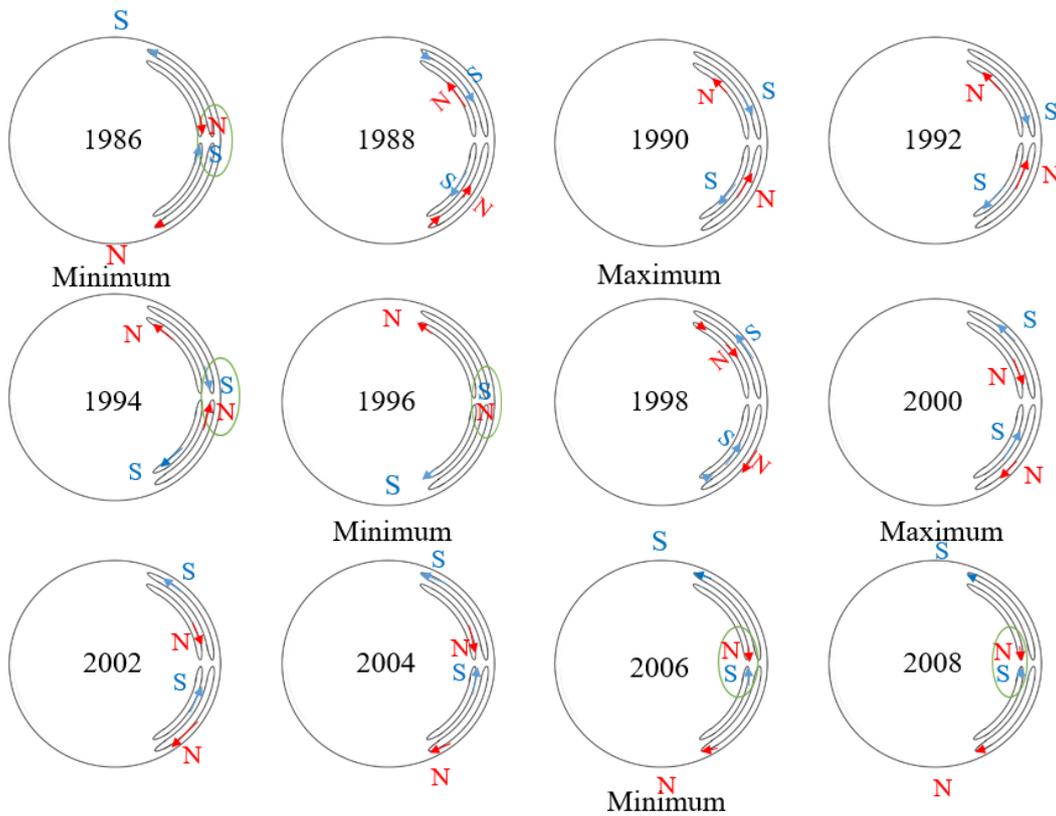


Figure 5. Schematic diagram of the solar magnetic field between 1986 and 2008. Two solar maxima and three solar minima occur during this period. In 1988 and 1998, the two magnetic poles approached the nearest distance at the middle latitude, being possibly induced by the current movement for the longitudinal direction. Many sunspot embryos may be produced during this time at $r \approx 0.85R_{\odot}$. This is like a current eddy that is often seen in a strait. The embryos of the sunspot is produced in-between the two plasma flows at the middle of the convection zone. The butterfly diagram may be realized by the equatorward plasma flow. “Plasma eddy” merges as the sun spot

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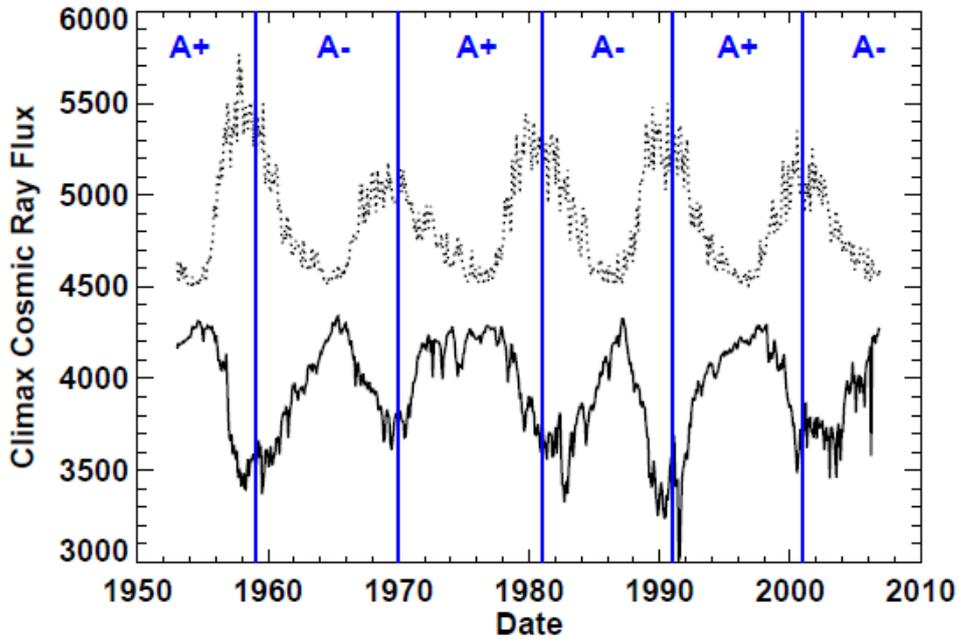


Figure 6. Cosmic ray intensity measured by the Climax neutron monitor and the sunspot number (dotted plots). A+ indicates periods with positive polarity pole, and A- indicates periods with negative polarity. This figure is courtesy of D.H. Hathaway, arXiv:15020792v1 [astroph-SR] (Feb. 2015). We suggest that the sharp rising and falling of the counting rate during A- may be caused by the outer oscillator of the double combined harmonic oscillators.

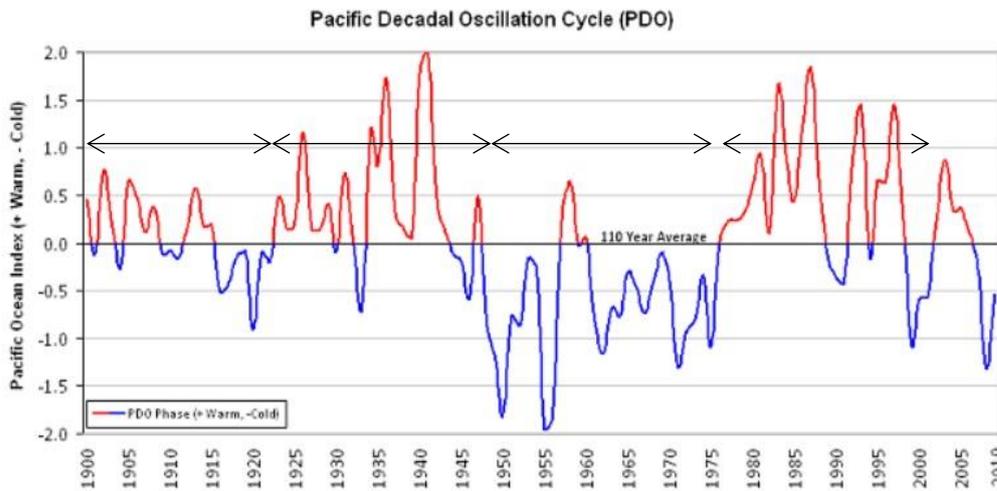


Figure 7. The Pacific Decadal Oscillation between 1900 and 2010. The PDO phenomenon is unfamiliar to cosmic-ray physicists. The surface temperature of the Pacific Ocean oscillates with an approximately 20-year periodicity between the high state (red) and the low state (blue) (difference of approximately ± 2 degrees). When the surface temperature of the California coast is high, the state is defined as being positive (red). This corresponds to the low state of the Japanese side. For further details, see Ref [7]. The horizontal arrows correspond to 22 years.

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