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Cosmic-ray propagation in the light of the Myriad model

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A hardening of the proton and helium fluxes is observed above a few hundreds of GeV/nuc. The distribution of local sources of primary cosmic rays has been suggested as a potential solution to this puzzling behavior. Some authors even claim that a single source is responsible for the observed anomalies. But how probable these explanations are? To answer that question, our current description of cosmic ray Galactic propagation needs to be replaced by the Myriad model. In the former approach, sources of protons and helium nuclei are treated as a jelly continuously spread over space and time. A more accurate description is provided by the Myriad model where sources are considered as point-like events. This leads to a probabilistic derivation of the fluxes of primary species, and opens the possibility that larger-than-average values may be observed at the Earth. For a long time though, a major obstacle has been the infinite variance associated to the probability distribution function which the fluxes follow. Several suggestions have been made to cure this problem but none is entirely satisfactory. We go a step further here and solve the infinite variance problem of the Myriad model by making use of the generalized central limit theorem. We find that primary fluxes are distributed according to a stable law with heavy tail, well-known to financial analysts. The probability that the proton and helium anomalies are sourced by local SNR can then be calculated. The p-values associated to the CREAM measurements turn out to be small, unless somewhat unrealistic propagation parameters are assumed.

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1. Gambling with the discreteness of cosmic ray sources

The proton and helium spectra are well described by a power-law distribution up to an energy of $\sim$ 350 GeV/nuc, above which a hardening is observed. This anomaly was reported by the PAMELA collaboration [1] and has been recently confirmed by the precision AMS-02 measurements [2, 3]. The proton and helium fluxes measured by the CREAM balloon borne detector [4] are clearly in excess of a simple power-law behavior.

Various explanations have been proposed to account for the observed hardening, such as the existence of distinct populations of primary cosmic ray (CR) accelerators with different injection spectra [5], sources characterized by a double injection spectrum like the magnetized winds of exploding Wolf-Rayet and red supergiant stars [6], or the possibility of a retro-action of cosmic rays themselves on the properties of the plasma, with the consequence of a softer diffusion coefficient K at high energies [7].

Another possibility lies in the presence of local accelerators which, if considered as point-like objects, may yield a contribution somewhat different from what is expected from sources continuously spread in space and time inside our neighborhood. According to [8], a few nearby remnants can be responsible for the observed spectral changes. In a more consistent analysis [9] where all sources, either remote or local, have the same power-law injection spectrum, the proton and helium data are well fitted with local injectors borrowed from the Green catalog and the ATNF pulsar database. The canonical CR transport model MED [10] yields already a χ^2 of 1.3 per dof. The fit improves considerably if model A from [9] is assumed.

CR model	δ	K_0 [kpc^2/Myr]	L [kpc]	V_C [km/s]
model A [9]	0.85	0.0024	1.5	13.5
MED [10]	0.70	0.0112	4	12

Table 1: Two examples of CR parameters used herefater. The transport of charged particles inside the magnetic fields of the Galaxy is a random walk, well described by diffusion with coefficient $K = K_0 \, \beta \, \mathscr{R}^{\delta}$, where β and $\mathscr{R}$ stand for the CR velocity and rigidity. The half-height L of the magnetic halo is of order a few kpc. A Galactic wind with velocity V_C blows particles away from the disk at GeV energies. It has little effect on CR propagation above a few tens of GeV.

In this proceedings, the question is to know whether the explanation proposed by [8] or [9] is natural or not. In the conventional approach, sources are described by a continuous jelly in space and time. Here, the local sources are treated as point-like objects and their distribution is such that they yield a larger flux. But is this probable? To address this question and the more general problem of the stochasticity of CR fluxes yielded by injectors localized in space and time, we need the so-called Myriad model.

2. The Myriad model: a framework for a statistical approach

Once injected in the interstellar medium, primary cosmic rays propagate inside the turbulent Galactic magnetic fields. CR transport is basically understood as a diffusion process taking place within a magnetic halo which is generally pictured as a circular slab, matching the shape of the

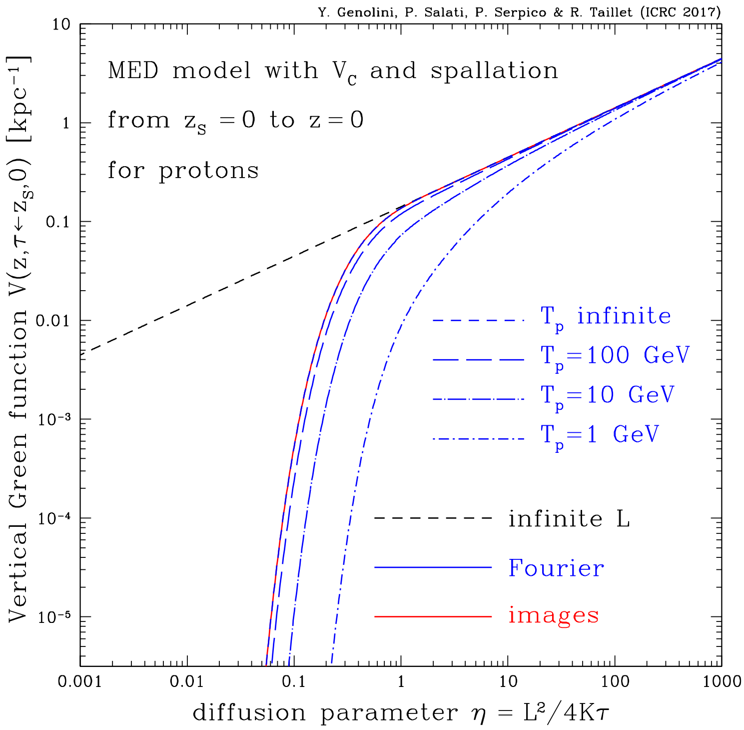

Figure 1: The CR propagator $G(\mathbf{x},t \leftarrow \mathbf{x}_S,t_S)$ gauges the probability for a particle emitted at the source $\mathbf{x}_S$ and time t_S to reach location $\mathbf{x}$ at time t. As the magnetic halo behaves as a slab with virtually no radial boundaries, the Green function G is the product of (i) a radial part that accounts for horizontal diffusion along an infinite plane and (ii) a vertical contribution $V(z,t \leftarrow z_S,t_S)$ which is attenuated by the leakage taking place at the boundaries $z = \pm L$ when the duration $\tau = t - t_S$ exceeds the diffusion timescale $\mathscr{T} \sim L^2/K$. The short-dashed black line is calculated as if L were infinite. The red curve is derived using the infinite series of images of the source yielded by the boundaries. This method cannot deal with convection and spallation. These mechanisms are taken care of with the Fourier approach, for which the CR proton kinetic energy has been varied. As $T_{\rm p}$ increases, diffusion becomes dominant, and the blue curves behave asymptotically as the red one.

Milky Way, inside which a thin Galactic disk of gas and stars is sandwiched. The CR density $\psi = dn/dE$ is related to the CR flux $\Phi \equiv (v_{CR}/4\pi)\,\psi$ and fullfills the diffusion equation

$$\frac{\partial \psi}{\partial t} - K\Delta\psi = q_{\rm acc}(\mathbf{x},t)\;, \tag{2.1}$$

where $q_{\rm acc}(\mathbf{x},t)$ accounts for the *continuous* space-time distribution of accelerators. The solution to the master equation (2.1) is given by the convolution, over the magnetic halo (MH) and the past, of the CR propagator $G(\mathbf{x},t \leftarrow \mathbf{x}_S,t_S)$ with the source distribution $q_{\rm acc}(\mathbf{x}_S,t_S)$

$$\psi(\mathbf{x},t) = \int_{-\infty}^{t} dt_S \int_{\rm MH} d^3\mathbf{x}_S\, G(\mathbf{x},t \leftarrow \mathbf{x}_S,t_S)\, q_{\rm acc}(\mathbf{x}_S,t_S)\;. \tag{2.2}$$

The propagator describes the probability that a CR species injected at point $\mathbf{x}_S$ and time t_S diffuses at location $\mathbf{x}$ at time t. Assuming that $q_{\rm acc}$ does not vary in time leads to the steady state canonical solution produced by CR numerical codes, where the flux at the Earth is constant in time.

In the Myriad model, the jelly of sources $q_{\rm acc}$ of the conventional approach is replaced by a constellation of point-like objects located each at $\mathbf{x}_i$ and t_i. The CR flux at the Earth yielded by a population $\mathscr{P}$ of such a myriad of injectors becomes

$$\Phi_{\mathscr{P}}(\mathbf{x}_{\odot},0) = \frac{v_{CR}}{4\pi} \sum_{i\in\mathscr{P}} G(\mathbf{x}_{\odot},0 \leftarrow \mathbf{x}_i,t_i)\, q_{\rm acc}(\mathbf{x}_i,t_i) \equiv \sum_{i\in\mathscr{P}} \varphi_i \, . \tag{2.3}$$

Although we can have some information on the closest and youngest sources, we have little knowledge of the actual population $\mathscr{P}$ in which we live. This is a problem insofar as the flux $\Phi_{\mathscr{P}}$ depends precisely on how the sources are distributed in space and time around us. To tackle the calculation of the flux in these conditions, we can adopt a statistical point of view and consider the ensemble of all possible populations $\mathscr{P}$ of sources. Each source of a given population lies within the magnetic halo and is younger than some critical value $\mathscr{T}$ that sets the size of the phase space volume over which the analysis is carried out. Sources older than $\mathscr{T}$ have no effect on the flux at the Earth if that age is taken to be of order the CR confinement time L^2/K, as shown in figure 1. Without loss of generality, we may assume that each source accelerates the same CR yield $q_{\rm SN}$. Assuming also a constant explosion rate $\nu \sim 1$ to 3 per century, we find that each population contains a number $\mathscr{N} = \nu \times \mathscr{T} \sim \nu L^2/K$ of sources yielding each a flux φ_i at the Earth whose sum is the flux $\Phi_{\mathscr{P}}$.

To simplify the statistical treatment, we may finally assume that each source is *independently* and *randomly* distributed in phase space according to the probability distribution function (pdf) $\mathscr{D}(\mathbf{x}_S,t_S)$. This is certainly correct as long as the correlation length between sources is smaller than the typical distance over which the CR propagator G varies. In the Myriad model, the individual flux φ yielded by a single source is a random variable whose pdf $p(\varphi)$ is the key of the statistical analysis. It is related to the phase space pdf $\mathscr{D}(\mathbf{x}_S,t_S)$ through

$$dP = p(\varphi)\,d\varphi = \int_{\mathscr{V}_\varphi} \mathscr{D}(\mathbf{x}_S,t_S)\; d^3\mathbf{x}_S\; dt_S \, , \tag{2.4}$$

where $\mathscr{V}_\varphi$ denotes the space-time region inside which a source contributes a flux in the range between φ and $\varphi + d\varphi$. The mapping between the phase space distribution $\mathscr{D}(\mathbf{x}_S,t_S)$ and the single-source flux pdf $p(\varphi)$ is realized through the CR propagator G. The pdf $p(\varphi)$ yields the total flux $\langle\Phi\rangle$ averaged over the ensemble of all possible populations $\mathscr{P}$ of sources

$$\langle\Phi\rangle = \mathscr{N} \times \langle\varphi\rangle \equiv \frac{v_{\rm CR}}{4\pi}\,\langle\psi\rangle \, . \tag{2.5}$$

We find that this statistical average is indeed equal to the canonical result obtained with the standard approach based on steady state and on sources spanning continuously the available phase space region

$$\langle\psi(\mathbf{x})\rangle = \int_{-\mathscr{T}}^{0} dt_S \int_{\rm MH} d^3\mathbf{x}_S\, G(\mathbf{x},0 \leftarrow \mathbf{x}_S,t_S)\,\{q_{\rm acc}(\mathbf{x}_S) \equiv \nu\, q_{\rm SN}\, \mathscr{D}(\mathbf{x}_S)\} \, . \tag{2.6}$$

In the Myriad model, the source term $q_{\rm acc}$ is equal to $\nu\, q_{\rm SN}\, \mathscr{D}(\mathbf{x}_S)$, where the space distribution of sources is defined by $\mathscr{D}(\mathbf{x}_S) \equiv \mathscr{T} \times \mathscr{D}(\mathbf{x}_S,t_S)$ as the explosion rate ν is constant over time.

We would like also to derive the variance of the flux Φ from the Myriad model. This quantity gauges how probable the flux of a given population is far from the average. It is related to the moment of second order of the pdf $p(\varphi)$. As we shall see in the next section, this moment diverges since $p(\varphi) \propto \varphi^{-8/3}$ for large values of the flux φ. The canonical central limit theorem does not apply. This apparent problem was tentatively solved by replacing the nearby and recent sources, which are at the origin of the divergence, by a catalog of sources [8] or [9]. Some other attempts focused on trying to regularize the divergence by imposing a cut-off at larges fluxes (see for instance [11] for a review and [12] for CR anisotropy).

3. Generalized central limit theorem

According to the generalized central limit theorem (GCLT), the statistical behavior of the total flux Φ is controlled by the evolution of the single-source pdf in the high-flux regime. Large values of φ are produced by recent and nearby sources. In that case, CR transport to the Earth is well described by pure diffusion and the vertical boundaries of the magnetic halo have no effect. The flux produced by a single source that exploded a time τ ago at a distance d can be expressed in that limit as

$$\varphi = \frac{v_{\rm CR}}{4\pi} \frac{q_{\rm SN}}{(4\pi K\tau)^{3/2}} e^{-d^2/4K\tau} \equiv \frac{a}{\tau^{3/2}} e^{-d^2/4K\tau} \, . \tag{3.1}$$

The heavy tail behavior which the distribution $p(\varphi)$ exhibits is encapsulated by the survival function

$$R(\varphi) = \int_{\varphi}^{+\infty} p(\varphi')\, d\varphi' = \int_0^{\tau_{\rm M}} d\tau \, \frac{4\pi}{3} \, d_\tau^3 \, \mathscr{D}(\mathbf{x}_\odot, 0) \, . \tag{3.2}$$

The maximal age $\tau_{\rm M}$ over which the previous integral runs is defined by $\varphi \equiv a/\tau_{\rm M}^{3/2}$ and is a decreasing function of the flux. The region of phase space over which the source pdf $\mathscr{D}(\mathbf{x}_\odot, 0)$ must be integrated corresponds to the domain of figure 2 extending below the thick blue line. At fixed age τ lying between 0 and $\tau_{\rm M}$, the space integral extends over a sphere with radius d_τ such that

$$\frac{d_\tau^2}{6K\tau_{\rm M}} = -\frac{\tau}{\tau_{\rm M}} \ln\left\{\frac{\tau}{\tau_{\rm M}}\right\} \, . \tag{3.3}$$

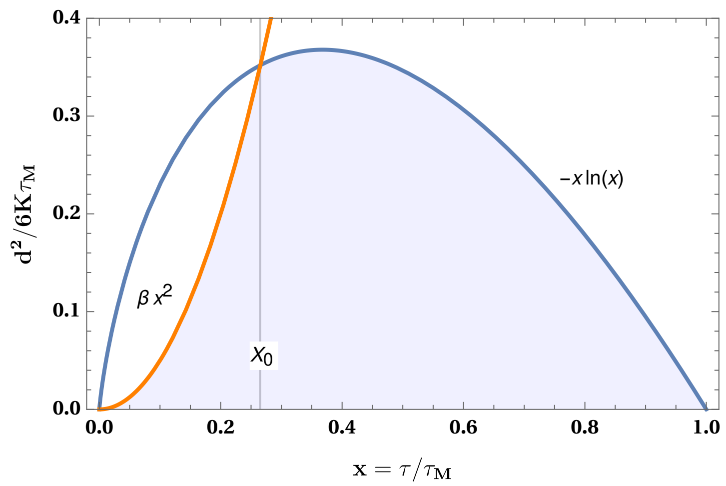

Figure 2: Below the thick blue line: space and time region where a source contributes a flux $\varphi' > \varphi = a/\tau_M^{3/2}$ in the diffusion approximation. Light blue shaded region: space and time domain which also respects the causal constraint.

A straightforward calculation yields the survival function which, in the large flux limit, behaves as $\varphi^{-5/3}$ since

$$\varphi^{5/3}\,R(\varphi) \,=\, \pi^{3/2}\,(2/5)^{5/2}\,\mathscr{D}(\mathbf{x}_{\odot},0)\,(6K)^{3/2}\,a^{5/3} \,\equiv\, c\,. \tag{3.4}$$

All the conditions are now met to use the central limit theorem in order to derive the pdf of the total flux Φ. As $p(\varphi)$ scales as $\varphi^{-8/3}$ in the large flux limit, the generalized version of the theorem must be applied, with the consequence that the variable $(\Phi - \langle\Phi\rangle)/\Sigma_\Phi$ is distributed according to the stable law $\mathrm{S}(5/3,1,1,0;1)$. Here, $\langle\Phi\rangle$ denotes the *statistical ensemble* average flux which has been shown to be equal to the flux yielded by the smooth distribution of sources $q_{\mathrm{acc}}(\mathbf{x}_S) \equiv \nu\, q_{\mathrm{SN}}\, \mathscr{D}(\mathbf{x}_S)$ of the conventional approach. The flux "variance" Σ_Φ is no longer given by the moment of second order of the pdf $p(\varphi)$, which is clearly divergent. According to the GCLT, it is expressed now as

$$\Sigma_\Phi \,=\, \left\{ \frac{\pi\,\mathscr{N}\,c}{2\Gamma(5/3)\sin(5\pi/6)} \right\}^{3/5}\,, \tag{3.5}$$

where c is defined through the survival function and $\mathscr{N}$ denotes the number of sources of each population. The "variance" Σ_Φ scales like $q_{\mathrm{SN}}\,K^{-3/5}\,\nu^{3/5}$. Notice that a similar result was found by [13] in a pioneering analysis of the CR electron and positron fluxes produced by discrete stochastic sources.

To summarize, we have found that the heavy tail behavior of $p(\varphi) \propto \varphi^{-8/3}$ leads to a peculiar probability distribution for the total flux. The stable law $\mathrm{S}(5/3,1,1,0;1)$ which controls the fluctuations of Φ with respect to $\langle\Phi\rangle$ has a much slower decrease than the canonical Gaussian law in the high-flux limit. Large excursions of Φ are possible in the Myriad model, depending on the relative value of Σ_Φ with respect to the average

$$\frac{\Sigma_\Phi}{\langle\Phi\rangle} \propto L^{-1} \times K^{2/5} \times \nu^{-2/5}\,. \tag{3.6}$$

Large flux fluctuations are possible in models of CR transport where the magnetic halo is thin and the explosion rate low. The higher the CR energy, the larger the diffusion coefficient K and the farther the fluctuations of the flux.

4. Computing the odds of the Galactic lottery

We can apply the Myriad model to calculate the pdf of the proton flux at the Earth and estimate the p-values associated to the AMS-02 and CREAM measurements. Each p-value is calculated according to the relation

$$\mathrm{p}_{\mathrm{value}} \,=\, \int_{\Phi_{\mathrm{exp}}}^{+\infty} d\Phi' \int_0^{+\infty} d\Phi_{\mathrm{th}} \times q(\Phi'|\Phi_{\mathrm{th}}) \times p(\Phi_{\mathrm{th}}|\langle\Phi\rangle)\,, \tag{4.1}$$

where $q(\Phi'|\Phi_{\mathrm{th}})$ is a Gaussian law whose spread σ_{exp} is the experimental variance, while $p(\Phi_{\mathrm{th}}|\langle\Phi\rangle)$ denotes the probability to get the theoretical flux Φ_{th} in the Myriad model. The 1-σ (pale yellow) and 2-σ (dark yellow) theoretical uncertainty bands are featured in figure 3 for two different CR models. The corresponding p-values are listed in table 2 where the explosion rate has been varied in the case of the MED model.

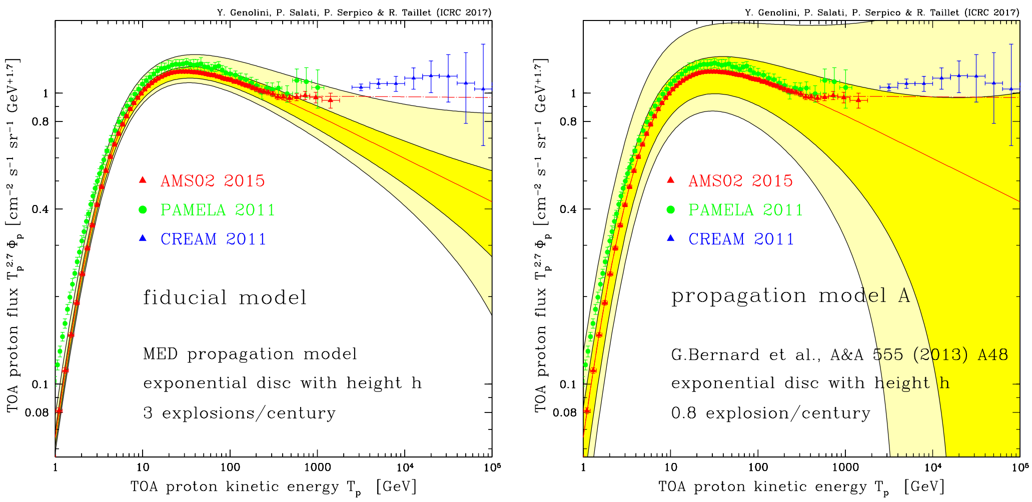

Figure 3: In both panels, the TOA proton flux is plotted as a function of TOA proton kinetic energy. The 1-σ (pale yellow) and 2-σ (dark yellow) bands predictions of the Myriad model have been derived assuming a stable law with index $5/3$. The average flux is featured by the solid red line and is assumed to follow a simple power-law. We have used here a fit to the AMS-02 data from which the hardening has been removed. Measurements from AMS-02 [2], PAMELA [1] and CREAM [4] are also shown for comparison. The dotted long-dashed red curve is a fit to the AMS-02 data with hardening taking place at a rigidity of 355 GV. The left panel features the fiducial case of the MED CR propagation model [10] with 3 explosions per century. In the right panel, the results for model A of [9] are derived, with 0.8 explosion per century. As the number of sources $\mathcal{N} \sim \nu L^2/K$ is 5 to 6 times smaller than previously, the uncertainty bands widen. In both panels, sources are exponentially distributed along the vertical direction with scale height 100 pc.

We can conclude from these results that the observed CR proton and helium anomalies have little chance to originate from a statistical fluctuation in the positions of the local sources. The p-values are larger in the last row of table 2 where the CR propagation model A of [9] is assumed with a half-height of the magnetic halo of 1.5 kpc and a diffusion spectral index of 0.85. Both values are now in tension with recent observations [14].

Notice finally that the statistical analysis sketched above needs to be refined in two respects: (i) the behavior of the pdf $p(\varphi)$ can be dominated for intermediate values of the flux φ by sources distributed along the Galactic plane, hence a stable law with index 4/3 instead of 5/3 and (ii) a light cone cut-off needs to be implemented in relation (2.4) since CR diffusion cannot be faster than light. The region extending above the solid red curve in figure 2 becomes excluded and is removed. These refinements have been studied in detail in a comprehensive analysis [16] where stable laws are shown to provide a robust description of the statistical behavior of the CR flux.

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Kinetic energy [TeV]	0.724	0.96	1.41	3.16	5.02	7.94	12.6
MED model with $\nu = 3$	10.2	8.68	7.67	1.6	1.23	1.18	0.98
MED model with $\nu = 2$	12.3	10.6	9.34	2.14	1.64	1.57	1.31
MED model with $\nu = 1$	16.2	14.2	12.6	3.52	2.7	2.59	2.16
model A [9] with $\nu = 0.8$	27.2	25.8	24.5	14.3	12.8	13.3	12.9

Table 2: The p-values in % of the three last AMS-02 (red) and four first CREAM (blue) proton flux measurements have been calculated in the framework of the Myriad model. They are obtained from the convolution of the experimental uncertainty with a stable law with index 5/3. The three first rows correspond to the MED CR propagation model [10] where the SN explosion rate has been decreased from 3 to 1 per century. The last row refers to model A found in [9] to explain the CREAM data as resulting from known local sources.

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