

## Indications for a high-rigidity break in the cosmic-ray diffusion coefficient

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The repeated deflections of cosmic rays (CR) onto magnetic irregularities turn their propagation into a diffusive process. This increases their permanence time in the Galaxy, increasing their interaction probability with the interstellar (IS) medium. Also, due to their energy, their collision byproducts contain species which are otherwise rare or absent in the IS medium, such as fragile elements like Li-Be-B or antiparticles, like antiprotons ( $\bar{p}$ ). These so-called “secondary” species have long been used to set constraints on propagation parameters in the generalized diffusion-loss equations linking the CR injection to the observable fluxes at the Earth. Using cosmic-ray boron to carbon ratio (B/C) data recently released by the AMS-02 experiment, we find tantalizing indications (*decisive evidence*, in Bayesian terms) in favor of a diffusive origin for the broken power-law spectra found in protons ( $p$ ) and helium nuclei (He). The result is robust with respect to currently estimated uncertainties in the cross sections, and in the presence of a small component of primary boron, expected because of spallation at the acceleration site. Reduced errors at high energy as well as further cosmic ray nuclei data (as absolute spectra of C, N, O, Li, Be) may definitively confirm this scenario. This proceeding is based on [1].

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## 1. Introduction

The last decade has witnessed a major improvement in the precision of direct CR measurements, as well as a significant extension of the dynamical range covered by data, culminating with the AMS-02 experiment on board the ISS. Traditional theoretical models available are under serious strain, when faced to the challenge of matching the precision of recent observations. On the one hand, the experimental error bars have shrunk to such a level that the intrinsically statistical nature of the predictions provides an irreducible limitation to their predictive power, an effect that should be certainly taken into account (see [2] for a recent study in that sense). On the other hand, the observations have revealed subtle features that demand some explanation, such as the broken power-law spectra measured in  $p$  [3] and He fluxes [4], and also probably present in heavier nuclei, confirming earlier indications by PAMELA [5] and CREAM [6]. Theoretical studies should thus aim at reducing (or at least better assessing) uncertainties, while enlarging the range of phenomena to explain, i.e. should do “more and better”. For instance, a number of explanations have been put forward for the broken power laws. As reviewed in [7], the most promising tool to disentangle among different classes of models resides in the study of secondary species, or alternatively of the ratio of a secondary species, like B, to a (mostly) primary one, like C. If breaks are already present in the spectra accelerated at the sources, such a ratio should appear featureless, since the daughter nucleus “inherits” the one of its parent. If these features are due to propagation phenomena (as suggested by the similar rigidity at which it is seen in different species) the effect should be roughly twice as pronounced in a secondary species, thus emerging in a secondary over primary ratio, provided that a sufficiently high-precision measurement extending up to high rigidities is available. In that context, the ratio of B/C fluxes has recently been released by the AMS-02 experiment [8] up to  $\sim 2$  TV.

In this study, we investigate several hypotheses in the framework of a 1D diffusion model [9]. This geometry is sufficient to capture the physics encoded in the B/C ratio; the simplicity of the model is an asset to test the diffusion coefficient break and the robustness of our conclusions. We follow a strategy which is complementary to recent trends: we restrict the theoretical framework to a sufficiently simple scenario with few fit parameters, and compare different hypotheses without introducing additional ones. To that aim, we make use of break parameters determined by the  $p$  and He analysis from AMS-02, thus performing a test “a priori”. With this setup, we find a *decisive evidence* (in Bayesian terms) in favor of a hardening in the B/C data, hence pointing to a diffusion origin for the phenomenon. This evidence is robust against what we believe are the dominant high-rigidity shape uncertainties: this includes the behavior of the cross sections (XS) at high-energy, notably the ones entering the B production from C and O progenitors, and the impact of a sub-dominant, but theoretically expected fraction of “primary” B component [10, 11, 12], i.e. the B produced via spallation at Galactic sources and further accelerated in situ.

## 2. Methodology

Within a very large class of models, CR fluxes observed at the Earth in the high-rigidity regime (tens of GV to hundreds of TV) are expected to depend only on the source term and the diffusion properties. Moving toward lower energies, additional effects kick in, such as convective winds,

reacceleration, solar modulation, and energy losses. Given our primary goal to isolate features in the (effective homogeneous and isotropic) diffusion coefficient  $K = K(R)$ , we focus in the following on the rigidity range above  $\mathcal{O}(10)$  GV and keep as primary fit parameters its normalization  $K_0$  and power-law index  $\delta$ . We also fix the diffusive halo height  $L$  to 10 kpc, since it is a parameter largely degenerate with  $K_0$ . We emphasize that our goal here is not to find “the best fit” parameters for the description of the data over the whole energy range, but identify and use the key physical variables the high-rigidity data depend on, for the purpose of our study. In this context, we test for two models, with the same number of free parameters. The conventional diffusion model

$$K(R) = K_0 \beta (R/\text{GV})^\delta, \quad (2.1)$$

vs

$$K(R) = K_0 \beta \frac{(R/\text{GV})^\delta}{\left\{1 + (R/R_b)^{\Delta\delta/s}\right\}^s} \quad (2.2)$$

where  $s, \Delta\delta$ , and  $R_b$  are not extra parameters adjusted to the B/C data, but result from a fit on the breaks in the AMS-02  $p$  and He spectra. In practice, we treat the break parameters as *nuisance parameters*, whose best fit values and errors are extracted from  $p$  and He. To do such an analysis correctly, it is necessary to take into account degeneracies between the parameters. This can be done thanks to their covariance matrix, which is unfortunately not provided by the AMS-02 collaboration. Hence, we perform a new *simultaneous* fit to the  $p$  and He data, taking into account statistical and systematic uncertainties as described in Ref. [13]. Our results ( $R_b = 312$  GV,  $\Delta\delta = 0.142$ ,  $s = 0.040$ ) are consistent with both sets of values found by AMS-02. In fact, we checked that adopting the best fit values found in their publication would not affect our conclusions. Note that the hypothesis (2.2) means attributing the breaks in  $p$  and He to diffusion. There are several proposals in the literature to realize such an effective behavior with microphysical mechanisms (e.g. [14]) or more complicated geometries and functional forms for the relevant quantities [15], but we shall not indulge here on these more model-dependent specifics. The role played by the velocity parameter  $V_a$  (as implemented in USINE [16]) and the convective speed  $V_c$  lessens as the rigidity increases. For instance, the data prove to be insensitive to the convective velocity  $V_c$  as the fit yields a result consistent with zero when limited to higher and higher  $R$ . However, because of parameter degeneracies, we treat  $V_a$  and  $V_c$  as nuisance parameters whose variation range (from 0 to 10 km/s) is estimated via a preliminary fit over the full B/C data. We treat the solar modulation in the force field approximation and set the Fisk potential to 0.730 GV, the average value<sup>1</sup> over AMS-02 data taking period [17]. We work in a 1D approximation since, apart from a renormalization in the effective value of the diffusion parameters (and particularly  $K_0$ ), moving to a 2D geometry leads to similar fitting performances [18, 16]. We checked that, assuming a different low- $R$  dependence of the diffusion coefficient ( $K \propto \beta^0$  instead of  $\beta^1$  as discussed in [19]), does not affect the statistical significance of the results obtained below. Needless to say, the best-fit values of the propagation parameters such as  $\delta$  *do depend* on the theory framework (and the range of  $R$ ) one is fitting to [19], and we warn the readers that a comparison of the propagation parameters obtained in our setup with similar parameters obtained in parametrically extended global fits to all data would be misleading, just like an extrapolation of our model to very low  $R$ , where it is expected a priori to fail.

<sup>1</sup>Value retrieved from the online  $\phi$  value calculation on CRDB (<http://lpsc.in2p3.fr/crdb/>).

The other ingredient the results depend upon is the source spectrum. In conventional models, B is considered to be fully secondary, mostly sourced by spallation of O and C. Fortunately, there is virtually no dependence of the B/C ratio upon the spectral shape of the primary C and O spectra [20, 18], at least as long as they are the same, which seems to be confirmed anyway from preliminary AMS-02 data, showing a flat C/O ratio at high-energy. For definiteness, we fixed the injection power-law index to 2.1, but the specific choice is not essential. Hence, the main uncertain ingredients determining the B source term, and so the the transport parameters, are the spallation cross sections [19, 18]. We compare our results for two choices, the GALPROP (GAL) dataset [21] and Webber 2003 (W03) one [22]. Since both datasets assume a constant extrapolation above some energy, this comparison might not capture the whole uncertainty, in particular on the *shape* of the B/C in the energy range of interest. To assess its importance, we also test a different extrapolation, assuming a very mild growth of all cross sections with  $\ln^2 s$ ,  $s$  being expressed in terms of energy per nucleon. This is certainly the reasonable leading growth behavior for the total and inelastic nucleon-nucleon cross section [23], and leads to a corresponding growth in nucleus-nucleus collisions, as expected based on (generalized) Glauber models and experimentally checked in proton-air cross-section measurements in extensive air-shower detectors, see e.g. [24]. Lacking a more motivated alternative, we further assume that the branching ratio into B is energy-independent. In practice we adopt for C, O, and B cross sections the same rise in energy/nucleon as  $\sigma_{pp}$  vs energy, starting at the energy at which the total  $pp$  cross section starts growing (zero derivative), i.e. around 100 GeV/nuc (Lab frame). Continuity with the low-energy cross section is imposed.

Finally, the hypothesis that all B is secondary implicitly assumes that the acceleration time at the source is instantaneous if compared to diffusion time  $t_K \propto K^{-1}$ . For a given activity timescale of a source capable of accelerating particles at the energy of interest,  $t_A$ , we may expect a primary to secondary fraction of B proportional to  $t_A/t_K$ . For a fiducial gas density value typical of the IS medium  $n \sim 1 \text{ cm}^{-3}$ , the probability for a C nucleus to interact in producing a stable B daughter is of the order of  $r \sigma n c t_A \sim 1\%$  for a cross section  $\sigma \simeq 60 \text{ mb}$  (a slightly larger than 1 correction should be applied to account for nuclear species),  $r = 4$  accounts for the standard strong shock compression factor, and an active lifetime of  $3 \times 10^4 \text{ yr}$  has been assumed. This is of the order of the age of the oldest supernova remnants detected in TeV  $\gamma$ -rays—hence capable of accelerating charged parent CRs to higher energy—such as the one in the W51 complex [25]. It is similar to a benchmark value already considered in past publications, see e.g. Eq. (10) in [26].

At the level of the precision of the data, it is mandatory to account for possible correlations between different energy bins; this is usually captured by the correlation matrix. Lacking this matrix, we focus on two extreme cases: i) completely correlated systematics, and ii) completely uncorrelated systematics. As this study is insensitive to any global normalization factor, a good approximation for the former case is to use the statistical errors only ( $\sigma_{\text{stat}}$ ). For the latter case, the total uncertainty for each data point is defined as the quadratic sum of statistical and systematic uncertainties ( $\sigma_{\text{tot}}$ ). Note that a toy-correlation matrix can be constructed based on the detailed systematic errors in [8] and a model of the energy correlations for each of these systematics. We checked that our qualitative results do not change using this toy-model. However, they indicate the quantitative importance of the covariance matrix, whose publication by the AMS-02 collaboration could prove very useful.

### 3. Results

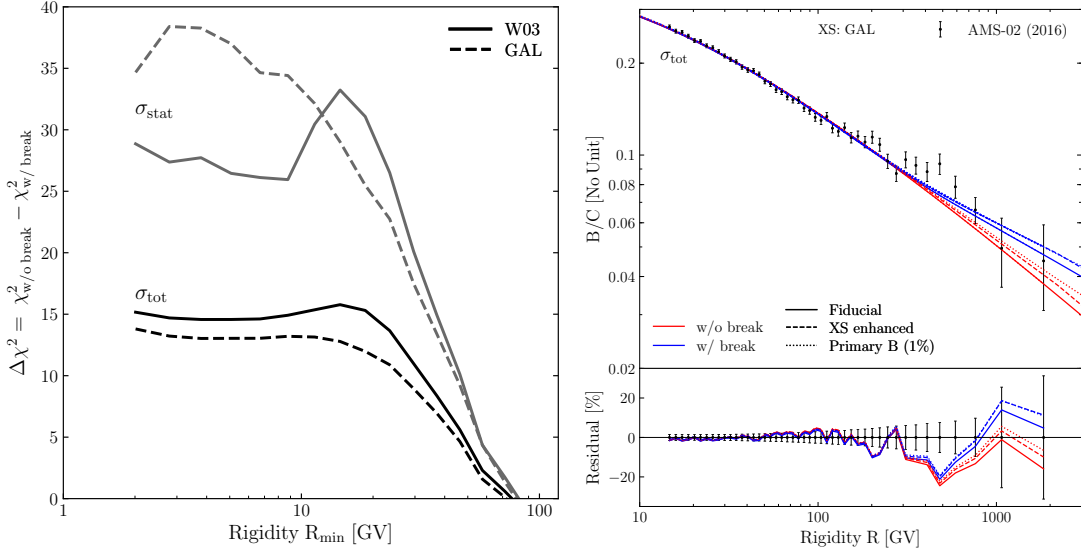
Since we focus on high- $R$ , we fit the B/C data above  $R_{\min}$ , and gauge how the fit is improved by a break in the diffusion coefficient, calculating the  $\Delta\chi^2$  between the best-fit obtained using equations (2.2) and (2.1). To check that the exact choice of  $R_{\min}$  is not crucial we perform a scan on  $R_{\min}$ . The evolution of the  $\Delta\chi^2$  vs  $R_{\min}$  is plotted left panel of Fig. 1, for the Webber (solid) and GALPROP (dashed) cross-section datasets. In all cases, an improvement of the fit is present when the break is introduced. As expected, a larger  $\Delta\chi^2$  is found when statistical errors only are considered, although the nominal quality of the fit (in a frequentist approach) degrades. Within cross-section errors,  $\Delta\chi^2$  is approximately constant up to  $R_{\min} = 20$  GV, and decreases above. This confirms that any choice in the interval  $2\text{GV} \lesssim R_{\min} \lesssim 20\text{GV}$  would lead to similar results of our test, while cutting at too high rigidities would hamper its statistical power since the baseline in  $R$  becomes too short to highlight significant changes in the effective  $\delta$ . Hypothesis tests are better performed by computing the *Bayesian evidence*  $K$  of the two models [27, 28]. In our case, it is straightforward to show that

$$2\log(K) = \Delta\chi^2, \quad (3.1)$$

since both models share *exactly* the same parameters such that the ratio of the priors drops out of  $K$ . The conventional evidence analysis considers a value of  $2\log(K) > 10$  as *decisive evidence* [27, 28, ?]. As shown below, in our analysis this criterion is always satisfied, for all assumptions tested (e.g. different choices for the spallation cross sections). Of course, one may worry that other physical effects could imitate the break in the diffusion coefficient. We test the robustness of our model against two of them: first, we include a different, but physically motivated high-energy extrapolation of the cross sections; second, by adding a reasonable amount of primary B, corresponding to 1% of the C source term. Note that, once again, we do not extend our theory space with extra parameters to be fitted. The best-fit values for each of these models are summarized in Tab. 1 for  $R_{\min} = 15$  GV (we have checked in each case the independence of the results from the exact choice of  $R_{\min}$ ). In all cases, the fits with break are better, yielding a smaller  $\chi^2$ . The inferred  $\delta$  is only modestly altered, by  $\sim 0.01$ , well below the magnitude of the break. None of the potentially degenerate effects mentioned above significantly alters the  $\Delta\chi^2$ : the indication for the break remains “decisive” ( $\Delta\chi^2 \geq 10$ ). Figure 1 displays the best fits reported in Table 1, using GALPROP spallation cross sections and  $\sigma_{\text{tot}}$ . The residuals shed light upon the important weight of the six high-energy data points between 300 GV and 800 GV, and stress the importance of reducing the error bars in this range to tighten the test.

### 4. Discussion & Conclusions

By analyzing AMS-02 B/C data, we have found a *decisive evidence* (in a Bayesian sense) in favor of a high-rigidity break in the cosmic-ray diffusion coefficient, matching the similar features found in  $p$  and He spectra. This suggests that the three observables ( $p$ , He, B/C) may find a *simultaneous* explanation for their spectral features in a model where the break is due to diffusion. We have conducted our study in a rather minimal theoretical setup, although we have tested the robustness of our conclusions with respect to a number of effects, such as the high-energy behavior of the cross sections or the presence of a small primary B component.



**Figure 1:** *Left:* Evolution of  $\Delta\chi^2$  (with and without the break) vs the minimal rigidity  $R_{\min}$  above which the fit is performed. Several cases are reported, using the GALPROP (GAL) or Webber 2003 (W03) cross-section datasets, and considering either statistical ( $\sigma_{\text{stat}}$ ) or total ( $\sigma_{\text{tot}}$ ) uncertainties. *Right:* Best fits and residuals with (blue) and without (red) the break using GALPROP cross sections and  $\sigma_{\text{tot}}$ , for the different models considered in the text.

Fit cases		Fiducial					Cross section enhanced					Primary boron $Q_B/Q_C = 1\%$				
Error	Spal. XS	w/o break		w/ break		$\Delta\chi^2$	w/o break		w/ break		$\Delta\chi^2$	w/o break		w/ break		$\Delta\chi^2$
		$\delta$	$\chi^2$	$\delta$	$\chi^2$		$\delta$	$\chi^2$	$\delta$	$\chi^2$		$\delta$	$\chi^2$	$\delta$	$\chi^2$	
$\sigma_{\text{stat}}$	W03	0.67	197	0.68	164	<b>33</b>	0.67	190	0.68	160	<b>30</b>	0.69	155	0.69	131	<b>24</b>
	GAL	0.62	160	0.62	131	<b>29</b>	0.62	154	0.62	127	<b>27</b>	0.64	126	0.64	105	<b>21</b>
$\sigma_{\text{tot}}$	W03	0.58	84	0.59	68	<b>16</b>	0.58	80	0.59	65	<b>15</b>	0.60	69	0.61	57	<b>12</b>
	GAL	0.52	62	0.53	50	<b>12</b>	0.52	59	0.53	48	<b>11</b>	0.54	52	0.55	42	<b>10</b>

**Table 1:** Best fit values for  $\delta$  ( $K_0$  is also a fit parameter, see [1] for details), using AMS-02 B/C data above  $R_{\min} = 15$  GV. The number of degrees of freedom is  $46 - 2 = 44$ . For each cases described in the paper, we compare the best  $\chi^2$  with and without the break. Two different spallation cross-section (Spal. XS) datasets are tested, i.e., GALPROP (GAL) and Webber (W03), as well as different choices for the data uncertainties.

It is unclear at the moment if—in a frequentist approach—our results suggest that the underlying models are inadequate to describe the data. Overall, at least for GALPROP cross sections and for the analysis with  $\sigma_{\text{tot}}$ , our fits with the break are of acceptable quality. The fit quality assuming  $\sigma_{\text{stat}}$  is instead quite poor. Lacking AMS-02 information on the error correlations, we may speculate that the actual situation is in between. Even then, it might still be that the simple models considered here provide an acceptable description at high- $R$ : for instance, theoretical predictions are not error-free, but should be at the very least subject (via the primary C) to the kind of space-time source stochasticity effects first assessed in [2], comparable to AMS-02 statistical uncertainties.

None of the conventional parameters in more extended theoretical models (like  $V_c$ ,  $V_a$ , etc.) appears degenerate with the kind of high- $R$  feature discussed here. While their introduction is certainly important in attempts to explain the data over the whole range of  $R$ , it appears unlikely that those effects might significantly alter our conclusions, as confirmed by some preliminary tests.

One may be tempted to achieve a better fit by extending the model space with “non-conventional” free parameters, such as leaving either the diffusion break parameters or the primary B fraction free, as we have checked a posteriori. The price to pay for a nominally better fit, however, would be enormous: allowing for a break significantly larger than the one found in  $p$  and He (or a primary B fraction as high as 5% of the C) would spoil the emerging global understanding of the broken power-law phenomenon. It may also raise additional theoretical problems: e.g. a large primary B typically requires very steep diffusion index  $\delta \simeq 0.8$ , which would be at odds both with Fermi acceleration expectations for the primary spectra, and exacerbate the known problem of a too large expected anisotropy in presence of too large  $\delta$ . Given the current understanding, we deem unwise to unleash such wild speculations. We believe that a global understanding of the key features presented by CR data is preliminary to a detailed “channel-by-channel” modeling, if that is at all possible within current theoretical capabilities. In this spirit, a test of the ideas discussed here will probably benefit more of a first coherent understanding of an enlarged dataset, including absolute flux measurements of primary species like C and O, “intermediate” ones like N, or secondary ones like Li, Be, B notably in the high- $R$  regime, rather than of a complete description of the B/C down to very small rigidities. Needless to say, future results from AMS-02—including information on uncertainty correlations and/or high precision data covering even higher energies (e.g., CALET on ISS, the DAMPE satellite, and ISS-CREAM to be launched soon)—will be determinant in that respect.

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