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Optimal measurement of charged particle momentum from multiple scattering: Bayesian analysis of filtering innovations

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Novel gamma-ray telescope schemes are under development so as to bridge the 0.1-100 MeV sensitivity gap of gamma-ray astronomy (Compton, pair creation), (silicon wafer stacks, emulsions, gas detectors). The lower average density with respect, e.g. to the tungsten/silicon active target of the Fermi-LAT makes square-meter effective area telescopes voluminous objects, for which the photon energy measurement by conventional means (calorimeter, magnetic spectrometer, transition radiation detector) is a challenge for the mass budget.

We present an optimal measurement of track momentum by the multiple measurement of the angular deflections induced by multiple scattering in the active target itself, using a Bayesian analysis of the filtering innovations of a series of Kalman filters applied to the track.

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This work originated from a theoretical and experimental development of low density, homogeneous detectors such as a gas time projection chamber as a high-performance gamma-ray telescope and polarimeter in the $\gamma \rightarrow e^+e^-$ conversion regime, that is above 1 MeV [1]. In these "active targets", the incident γ converts and the produced electrons are tracked in the same structure. The single track angular resolution of these detectors, if tracking is performed with an optimal method in the presence of multiple scattering such as a Kalman filter [2], is so good that polarimetry has been predicted to be possible despite the dilution of the polarization asymmetry induced by multiple scattering [3], and has actually been demonstrated by the characterization of a TPC prototype on beam [4, 5, 6]. Other projects plan to use all-silicon active targets, that is, without any additional tungsten converters [8, 7]. Whatever the method used, the lower density and the lower average atomic number of the active target, with respect to the tracker of the Fermi-LAT, induce a larger volume at a given value of the effective area: measuring the photon energy, be it with a calorimeter or by a measurement of the electron momenta with a magnetic spectrometer or with a transition radiation detector, is a challenge to the mission mass budget.

Lower energies are the domain where Compton telescopes are most efficient. When one scatter is detected, from the measurement of the position and the energy of the recoil electron and of the absorbed scattered photon, the reconstruction of the direction of the incident photon constrains it to a large arc, that is a large point spread function (psf). If in addition the direction of the recoil electron is measured, the extention of the arc is much reduced and the pdf is improved [9]. Alas, the angular coverage of the calorimeter is an issue: it must be low enough that the incident photon can enter the detector and large enough to measure the energy of the scattered photon: in this ETCC (electron tracking Compton camera) scheme the effective area undergoes a sharp drop for photon energies above 0.5 MeV [9].

We develop an optimal method to measure the electron momentum from the multiple measurement of multiple scattering induced deflections, based on a Bayesian analysis of the innovation residues of a set of momentum-dependent Kalman filters applied to the track [10]. This work has been performed under a number of assumptions/approximations. We assume relativistic particles ($\beta \approx 1$) without loss of generality. Only the first-order term (angle deflection) of multiple scattering is taken into account which is legitimate for the thin detectors considered here; the 2nd-order transverse displacement is neglected. Continuous ($\mathrm{d}E/\mathrm{d}x$) and discrete (BremsStrahlung radiation) energy losses are also neglected. In TPCs in which the signal is sampled, most often the electronics applies a shaping of the pulse before digitisation, that creates a short scale longitudinal correlation between successive measurements that we neglect too. Also the limitations of pattern recognition, that is, in the case of γ-ray telescopes, of the assignment of each hit to one of two close tracks, are not addressed.

The distribution of the probability density function $p(s)$ is shown in Fig. 1 for one simulated 50 MeV/c track [10]. s is the average multiple-scattering angle variance per unit track length and is related to the track momentum p

$$p = p_0 \sqrt{\frac{\Delta x}{l X_0 s}}. \tag{1}$$

l is the longitudinal sampling along the track, X_0 the detector radiation length, Δx is the scatterer thickness and $p_0 = 13.6\,\mathrm{MeV}/c$ is the "multiple-scattering constant": in the case of a homogeneous detector such as a TPC, $\Delta x = l$.

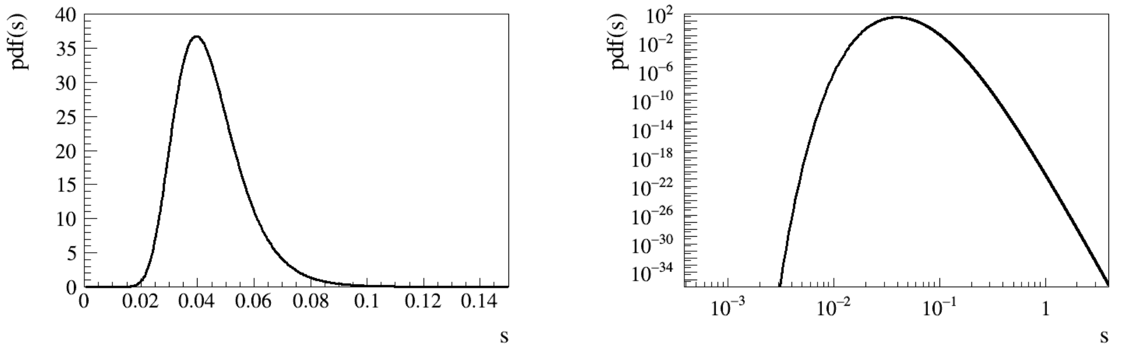

Figure 1: $p(s)$ distribution for a $50\,\text{MeV}/c$ track in a silicon detector. On that track, the momentum is measured to be equal to $49.9\,\text{MeV}/c$ [10].

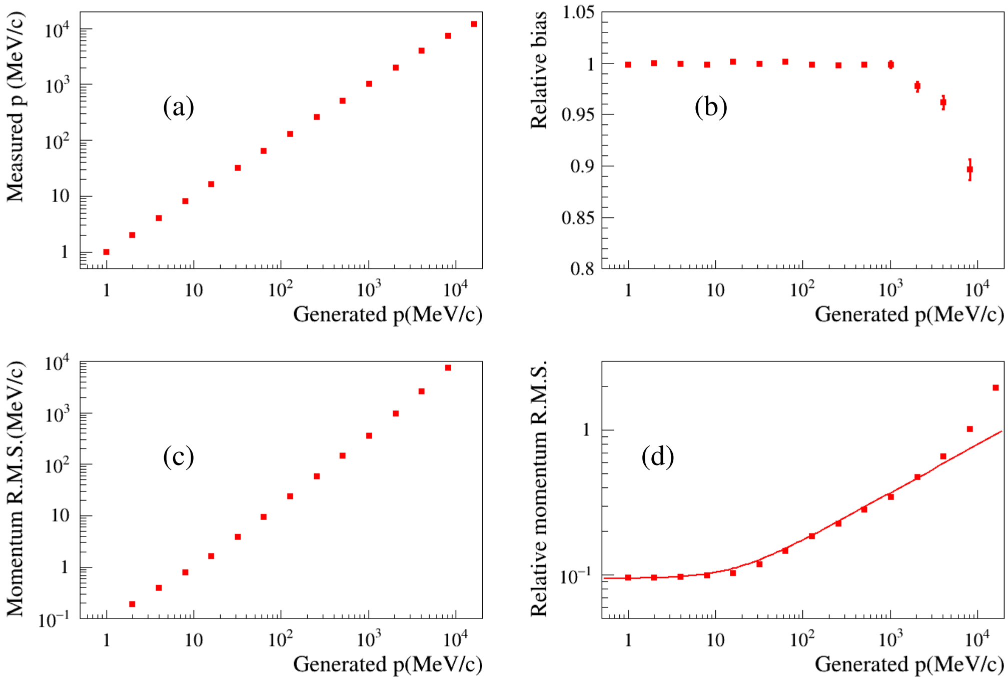

Figure 2: Performance of the momentum measurement for the silicon detector: Variation as a function of the true (generated) particle momentum of (a) the average measured momentum; (b) the average measured normalized to the generated momentum; (c) R.M.S of the measured momenta; (d) the relative R.M.S of the measured momenta [10]. The curve is from eq. (2).

A numerical characterisation of the method shows that for a given detector the method is reliable up to some limit momentum above which the relative precision σ_p/p becomes larger than unity. For lower momentum tracks, the momentum estimation is found to be unbiased (Fig. 2). The method is found to be usable at low energy, below 1 GeV for silicon detectors such as e-ASTROGAM [8] or AMEGO [7].

We perform a parametric study of the estimator from which we extract a heuristic analytical description of the relative uncertainty of the momentum measurement. A good representation of these data is obtained with the following expression [10]:

$$\frac{\sigma_p}{p} \approx \frac{1}{\sqrt{2N}} \sqrt[4]{1+256\left(\frac{p}{p_0}\right)^{4/3}\left(\frac{\sigma^2 X_0}{N\Delta x\, l^2}\right)^{2/3}}, \tag{2}$$

σ is the single measurement spatial RMS precision and N the number of measurements. The method is also of interest for the high-precision measurement of muon momentum in large, finely instrumented liquid argon detectors for long-range neutrino studies such as DUNE [11].

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