

## Two-neutron removal from $^{11}\text{Li}$ in a (p,t) reaction at low incident energy

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The reaction  $^{11}\text{Li}(p,t)^9\text{Li}(\text{g.s.})$  at an incident energy of 4.4A MeV is discussed in terms of a simplistic zero-range distorted-wave Born approximation. The dominant contribution to the transfer reaction is known to originate with the  $(2s_{1/2})^2$  configuration of the valence neutrons in the ground state of  $^{11}\text{Li}$ , and this is confirmed. The cross section angular distribution appears to be largely insensitive to the anomalously large radius of  $^{11}\text{Li}$  caused by its Borromean di-neutron halo. It needs to be understood why a simple model of the reaction is in much better agreement with the experimental angular distribution than a sophisticated treatment that should in principle be superior.

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## 1. Introduction

The structure of halo nuclei, such as  $^{11}\text{Li}$ , is of considerable interest [1]. The two-neutron transfer reaction  $^{11}\text{Li}(p,t)^9\text{Li}$  promises to provide valuable information on the halo-neutron correlation of this exotic Borromean nuclear species.

Unfortunately an early attempt at a theoretical prediction, which comprised the structure properties of  $^{11}\text{Li}$  and reaction dynamics of the transfer reaction as best known, did not reproduce the experimental cross section angular distribution of this reaction at an incident energy of 3 MeV [2] particularly well. Later the hope that a more refined theoretical treatment would be successful, seemed to be somewhat elusive, because a fairly sophisticated theoretical attempt fails spectacularly [3] to reproduce the angular distribution for the same reaction at a slightly higher incident energy of 4.4 MeV. On the other hand, as will be shown in this work, a very simplistic simultaneous transfer, zero-range distorted-wave Born Approximation (DWBA) gives a reasonably good reproduction of the angular-distribution shape. The implication of this very surprising and counter intuitive result needs to be evaluated carefully.

Clearly it needs to be understood why a superior implementation of the reaction mechanism, based on a comprehensive coupled channel theory which includes all secondary components of the participating processes, should yield inferior agreement with the experimental distribution compared to a simple approximation to the same reaction. The most obvious reason, namely that differences in the optical potentials introduced in the DWBA calculations are to blame, will be shown as unlikely to be a correct explanation.

Some of the ideas discussed now have been presented previously at scientific meetings [4, 5]. This presentation serves as additional motivation to understand the apparent inconsistency between the two approaches to the theoretical analysis of the  $^{11}\text{Li}(p,t)^9\text{Li}$  reaction.

## 2. Sequential versus simultaneous two-nucleon transfer

The theoretical results of Thompson [3], shown in Fig. 1, follow from assumption that the two nucleons are transferred partly sequentially in addition to a simultaneous mechanism in the course of the  $^{11}\text{Li}(p,t)^9\text{Li}$  reaction. In this work, on the other hand, (as shown, for example, in Fig. 2), the more usual restriction to purely simultaneous transfer is adopted.

In spite of many investigations, proof of whether simultaneous or sequential transfer dominates is not as clear as one would have hoped. The reaction  $^{208}\text{Pb}(p,t)^{206}\text{Pb}(3^+)$ , which is an unnatural parity transition and therefore forbidden in zero-range DWBA, would appear to be to be a good example to provide guidance on this issue. However, comparison between the result of Igarashi *et al.* [6] (sequential) and Nagarajan *et al.* [7] (simultaneous) could at best be described as inconclusive. This conclusion is independently supported by Charlton [8]. What seems to be beyond dispute is that sequential transfer, which derives from a second order DWBA [9], should to a good approximation theoretically [10] follow the same shape as the first order simultaneous process. This expectation is supported by the similarity, for example, between the calculations of Potel *et al.* [11] and Guazzoni *et al.* [12] for the reactions  $^{118,124}\text{Pt}(p,t)^{116,122}\text{Pt}(\text{g.s.})$  at an incident energy of approximately 25 MeV.

Recent discussions on the issue of implications of the possible multistep character of two-nucleon transfer are provided in Refs. [13, 14]. This issue is ignored as irrelevant to our own calculations in this study, and the question as to whether the absolute magnitude of the transfer reaction could be correctly reproduced is understood as not being of importance anyway when a zero-range DWBA is employed. The issue is whether a consistent insight follows from a simplistic zero-range DWBA description of the  $^{11}\text{Li}(p,t)^9\text{Li}$  reaction at 4.4 MeV to a more realistic transfer treatment which includes all the important ingredients of the reaction mechanism.

### 3. Results from two different theoretical formulations

Results from Thompson [3] for the reaction  $^{11}\text{Li}(p,t)^9\text{Li}(\text{g.s.})$  are reproduced in Fig. 1. Details of the calculation [3] appear to be very similar to those for the same reaction at a lower incident energy [2].

The wave function  $\varphi$  of the two halo neutrons in the ground state of  $^{11}\text{Li}$  is expressed [15, 16] as

$$\varphi = 0.45 \left| s_{1/2}^2(0) \right\rangle + 0.55 \left| p_{1/2}^2(0) \right\rangle + 0.04 \left| d_{5/2}^2(0) \right\rangle, \quad (3.1)$$

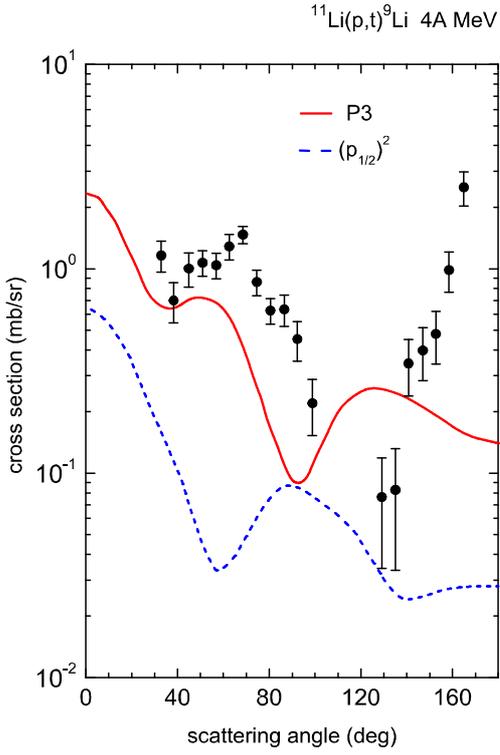
where the base wave functions are indicated in a standard notation. In Fig. 1 the predicted angular distribution is shown as a dashed curve when the  $(1p_{1/2})^2$  component of the bound state is assumed to be dominant (The dashed curve has only a 3% contribution from  $(2s_{1/2})^2$ ). Clearly the major contribution comes from the  $s$ -occupation, as indicated by the continuous curve in Fig. 1. The theoretical results in Fig. 1 will be referred to as finite-range with exact strong couplings (FRESCO) [17, 18] calculations in the remainder of this paper.

In Fig. 2 results are displayed for zero-range DWBA calculated with the code DWUCK4 [19]. The bound-state wave functions of target valence neutrons were generated for Fig. 2 in the standard way of adjusting the depth of a Woods-Saxon potential with radius  $R=1.25A^{1/3}$ , where  $A$  is the mass number of the core, and diffuseness  $a = 0.65$  for each nucleon to reproduce the correct binding energy and wave function attributes. The bound-state geometric parameters adopted here do not take the extended mass range of the halo target into account at this, but the influence of this neglect will be addressed later.

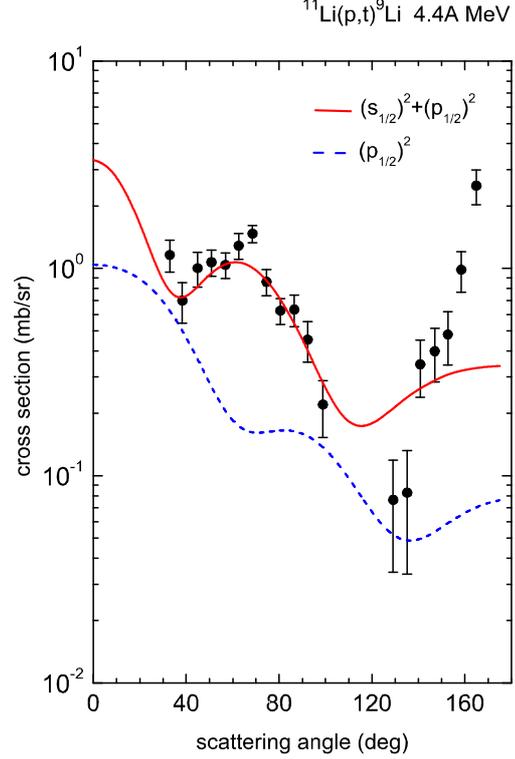
As was shown in Ref. [4, 5], essentially the same angular distribution was obtained if the two transferred neutrons were treated as a di-neutron cluster bound in the target system. This result is consistent with the prediction following from Refs. [20, 21, 22]. Of course, in the present case where the bound-state wave function is expected to be a superposition of configurations with two valence neutrons in the same intrinsic shell model orbital, the di-neutron approximation should be especially appropriate.

As implied before, in the present work magnitudes for different bound-state contributions, as well as the overall magnitude, are based on shape reproduction of the experimental angular distribution cross sections. As in Fig. 1, contributions from the  $p$ -component and the overall cross section are shown as dashed and continuous curves, respectively. Apart from the fact that both curves in Fig. 2 have a somewhat higher absolute magnitude than the corresponding ones of Fig. 1, the relative magnitudes are roughly consistent.

The interesting difference between Figs. 1 and 2 is that the simplistic zero-range DWBA is clearly superior in its reproduction of the experimental angular distribution. As was already



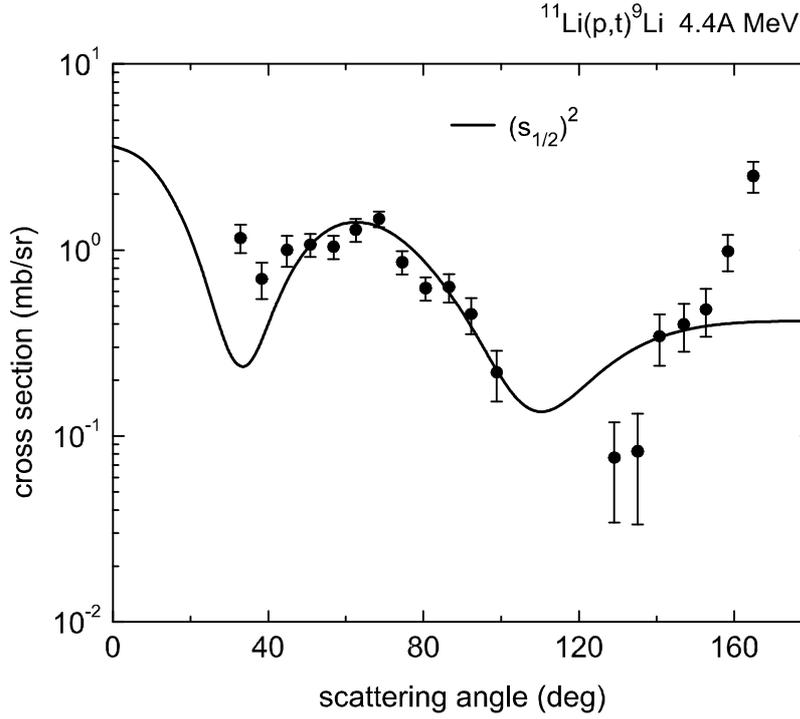
**Figure 1:** Cross section angular distribution for the reaction  $^{11}\text{Li}(p,t)^9\text{Li}(g.s.)$  at an incident energy of 4.4A MeV. Figure adapted from Ref. [3]. Experimental data from TRIUMF are also available in Ref. [3]. Theoretical calculations [3], [1] are based on a multistep transfer formalism. The curve P3 (solid line) has a  $(2s_{1/2})^2$  neutron component of 45% added to the  $(1p_{1/2})^2$  wave function. The dashed line is a calculation for a  $(1p_{1/2})^2$  occupation and it includes only a negligible contribution of 3% from  $(2s_{1/2})^2$ . Furthermore, it excludes a N-N potential to correlate the neutrons.



**Figure 2:** Cross section angular distribution for the reaction  $^{11}\text{Li}(p,t)^9\text{Li}(g.s.)$  at an incident energy of 4.4A MeV. The source of the experimental data is the same as that of Fig. 1. Theoretical calculations are from zero-range simultaneous transfer DWBA formulations. Results with a combination of  $(2s_{1/2})^2$  and  $(1p_{1/2})^2$  configurations in the bound state is shown as a solid line. The relative contributions from the two configurations are adjusted as described in the text. A dashed line represents a calculation in which only the  $(1p_{1/2})^2$  component is used.

mentioned, the zero-range DWBA angular distributions with detail-dependant absolute magnitude, were arbitrarily normalised to the experimental values. The FRESCO calculations, on the other hand, are expected to give absolute cross sections. From Fig. 1, this is not exactly true, but the shape difference between theoretical and experimental angular distributions is a more serious concern.

Exactly the same optical potential parameters are used for the distorted waves in the entrance- and exit channels of the theoretical results in Fig. 2 as those listed for Ref. [1], therefore this as an obvious explanation of the shape difference in the theoretical predictions obtained in Figs. 1 and 2, is unlikely to be valid.



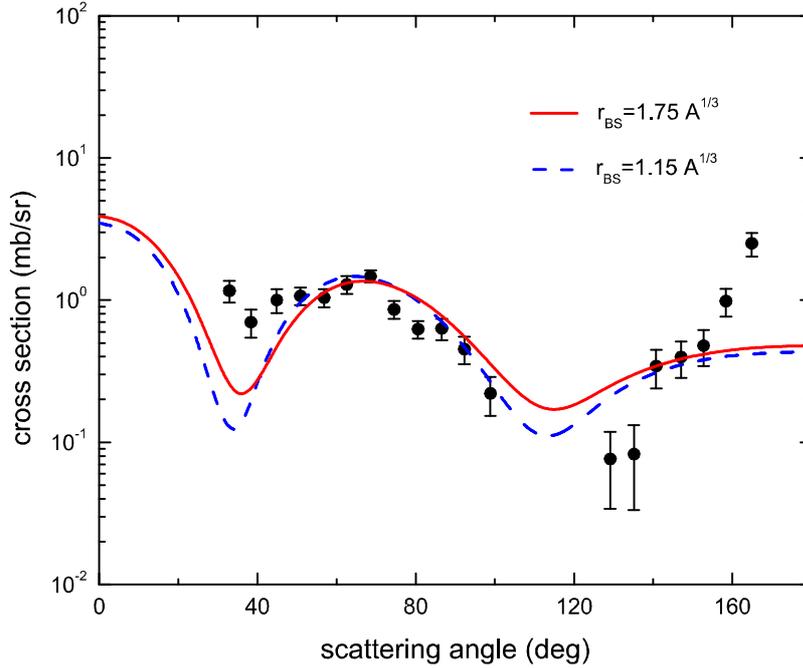
**Figure 3:** See caption to Fig. 2. The theoretical curve represents a DWBA calculation with both valence neutrons in the ground state of  $^{11}\text{Li}$  bound in the  $2s_{1/2}$  orbital. The theoretical curve is normalised to the experimental angular distribution.

#### 4. Dominant $(2s_{1/2})^2$ -configuration of the $^{11}\text{Li}$ structure

One must keep in mind that, apart from the rather drastic differences in the theoretical treatments, the corresponding curves in Figs. 1 and 2 share roughly the same significance. The distinction that the overall cross section in Fig. 1 is based on a coherent sum of  $(2s_{1/2})^2$  and  $(1p_{1/2})^2$  wave functions, whereas those are added incoherently in Fig. 2, is diluted greatly by the clear dominance of the  $(2s_{1/2})^2$ -configuration. This is shown in Fig. 3 where the shape resulting from the  $(2s_{1/2})^2$ -component by itself is compared with the experimental data. The observed agreement is already much better than for the overall FRESCO results.

#### 5. Size effect of $^{11}\text{Li}$ on transfer reaction

As is well known, the rms radius of  $^{11}\text{Li}$  is much larger than normal. Variation of the geometry of the bound state serves as a crude approximation to the influence of the extended range of the halo mass distribution of the  $^{11}\text{Li}$  nucleus. This was investigated in Ref. [13] where the halo neutrons were treated as a di-neutron cluster. This is reproduced in Fig. 4. The radius of the bound state is increased from  $R = 1.15A^{1/3}$  to  $R = 1.75A^{1/3}$ , where  $A$  is the mass number of the core system  $^9\text{Li}$ . The larger radius value corresponds roughly to the actual rms radius of  $^{11}\text{Li}$ . Clearly no appreciable difference is observable. Similar insensitivity is shown to the diffuseness parameter.



**Figure 4:** Angular distribution for  $^{11}\text{Li}(p,t)^9\text{Li}(\text{g.s.})$  at an incident energy of 4.4 MeV with two different values of the bound state radius of the halo neutrons treated as a di-neutron cluster. The continuous curve corresponds to a radius which corresponds to the rms size of  $^{11}\text{Li}$ , and the dashed line to a number for stable nuclei in general which is consistent with the microscopic values used in Fig. 2 and 3. The normalisations of the theoretical curves to the experimental angular distribution are arbitrary. Figure reproduced from Ref. [13].

## 6. Influence of distorting optical potential parameters

Optical potential parameters applied to transfer reactions are generally those that give a good account of elastic scattering. In the absence of detailed information, global parameter sets which reproduce the general trend of elastic scattering over the target mass and incident energy of interest are often useful.

As was mentioned before, in my calculations I employ exactly the same optical model potentials as those listed by Tanihata *et al.* [2]. Those are derived from global sets of Becchetti and Greenlees for protons [23] and tritons [24]. Although there is a slight discrepancy in the implementation of Tanihata *et al.* [2] of these same quoted data sets, the difference is of no consequence at all. For consistency of comparison between the FRESCO and zero-range DWBA calculations, the same numerical values [2, 5] of the parameters were used now.

It should be mentioned that in the opinion of Schiffer [25] one should not use optical model parameters for transfer reactions extracted from elastic scattering for the specific nuclei under consideration, but rather those that are representative of the averages for the required vicinity of nuclei. Whether this sentiment applies to something as exotic as  $^{11}\text{Li}$ , however, is unfortunately not clear. Although published information is unavailable, one has to keep in mind that there are

indications that  $p+^{11}\text{Li}$  elastic scattering at the appropriate incident energy is not reproduced well by the standard optical potentials.

A further concern is that, whereas optical model parameters are derived from elastic scattering, a transfer reaction is sensitive to a different radial range of the generated distorted waves. It is not foreseen that our insight into these problems will see rapid progress soon, but fortunately the anticipated difficulties do not appear to be serious in most practical applications.

As previously reported [5], there is some indication that it would in principle be possible to improve the description of the present  $^{11}\text{Li}(p,t)^9\text{Li}$  study by a better selection of distorting parameters. However, clear guidance is not available at this stage.

## 7. Summary and conclusions

The DWBA prediction of a simplistic calculation for the reaction  $^{11}\text{Li}(p,t)^9\text{Li}(\text{gs})$  at an incident energy of 4.4A MeV was compared with an existing result of a sophisticated model with finite-range and exact strong couplings (FRESCO). No attempt was made in the DWBA calculation to extract absolute cross sections, therefore shape comparisons with both the experimental and FRESCO results were explored.

Not only does the zero-range DWBA give a reasonably good reproduction of the cross section angular distribution, but it is found to be significantly superior to the more elaborate theoretical treatment. Current wisdom does not expect that inclusion of additional mechanisms, such as for example core polarisation, will alleviate the deficiency observed in the sophisticated theory.

The response to the transfer reaction is similar to that of nuclear species of normal rms size, and the extended halo structure of  $^{11}\text{Li}$  does not seem to influence the shape of the  $(p,t)$  angular distribution appreciably. This conclusion is based on a treatment of the halo neutrons as a di-neutron cluster, which is known to be a valid approximation, and increasing the size of the bound state radius to provide the correct rms matter value of  $^{11}\text{Li}$ .

The present analysis is in agreement with the FRESCO result that the  $(2s_{1/2})^2$ -component of the neutron halo wave function contributes the dominant share to the cross section yield. This suggests that one should be able to discover the origin of the discrepancy between the two models by carefully considering the angular distributions from initially only this component as a reasonable approximation to the complete calculation.

The simplicity of the current theoretical treatment, combined with its ability to reproduce the main characteristics of experimentally observed angular distributions, is encouraging. Clearly the observed result needs to be investigated further to provide guidance to a better understanding of the mechanism of the two-neutron transfer reaction induced by protons on the halo nucleus  $^{11}\text{Li}$  at low incident energy.

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