Precision calculations in BSM theories — Higgs mass and Muon $g - 2$

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Two recent results in the field of BSM precision calculations are reviewed: (1) For 3-loop calculations of the MSSM Higgs mass, the problem of regularization is considered. It is proven that regularization by dimensional reduction is consistent with supersymmetry for such calculations in the gaugeless limit. (2) The general 2-Higgs doublet model and the muon $(g - 2)$ is considered. Constraints on the possible values of Yukawa couplings derived. It is then shown that the current deviation in the muon $(g - 2)$ can be explained in a small but interesting part of the 2-Higgs doublet model parameter space.

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1. Introduction

In spite of the absence of direct signals for physics beyond the Standard Model (BSM) at the LHC, important experimental results indirectly constrain BSM physics via loop effects. Two examples are the mass of the Higgs boson $M_h$ and the anomalous magnetic moment of the muon $a_\mu$. The Higgs boson mass $M_h$ is measured very precisely and can be predicted in SUSY models, leading to very nontrivial constraints on the possible SUSY parameter space. The measurement of $a_\mu$ shows a 3–4σ deviation from the corresponding SM prediction which is significant in terms of the uncertainty and large in absolute terms when compared to typical SM and BSM contributions. Trying to explain this deviation in concrete BSM scenarios therefore again leads to nontrivial constraints on BSM parameters.

In the following two sections we review first a proof that regularization by dimensional reduction is consistent with SUSY calculations of $M_h$ at the 3-loop level [1]; then we review an analysis of $a_\mu$ in the 2-Higgs doublet model showing that the measured value of $a_\mu$ can be explained only in a very small, specific part of the 2-Higgs doublet model parameter space and the maximum achievable value is only slightly larger than the measured value [2].

2. Higgs boson mass in the MSSM and regularization by dimensional reduction

A hallmark of renormalizable SUSY theories is that quartic scalar interactions are no free parameters but related to gauge and/or Yukawa couplings via SUSY relations. As a result, the Higgs boson mass is predictable. Specifically, in the minimal supersymmetric Standard Model (MSSM) the tree-level Higgs mass is related to electroweak gauge couplings and predicted to be smaller than $M_Z$. Calculable loop corrections can push the Higgs mass up to the observed value; hence the comparison of the measured value of $M_h$ to its SUSY prediction provides important information on details of the SUSY parameters which enter the loop corrections to $M_h$.

Evaluating the MSSM prediction for $M_h$ with ever increasing accuracy is an intense ongoing effort. Recent progress includes progress on fixed-order computations [3, 4, 5, 6, 7, 8, 9], the development of EFT-based evaluations [10, 11, 12, 13] and hybrid calculations [14, 15, 16]. Interestingly, the code FlexibleSUSY provides a common platform for implementations of all three kinds of calculations [15, 5, 9, 13]. Comparing these recent state-of-the-art calculations shows that the theory uncertainty of the MSSM predictions is still significantly larger than the experimental error. Hence further future improvements on the theory predictions are mandatory.

Here we focus on one question relevant for these theory predictions: the question whether regularization by dimensional reduction is consistent with SUSY at the required level for fixed-order 3-loop calculations of $M_h$ in the MSSM in the limit of vanishing electroweak gauge couplings (the so-called “gaugeless limit”). The corresponding question for the 2-loop level has been discussed and answered in Ref. [17] and further progress on this question has been made in the meantime in Refs. [18, 19].

The question is nontrivial since for ordinary dimensional regularization it is well known that SUSY is broken, and even for regularization by dimensional reduction there is no all-order proof of the consistency with SUSY [20]. On a technical level the question is equivalent to the question
whether the counterterm structure generated by the usual renormalization transformation of fields and parameters of the MSSM is correct. In order to answer it, two main steps are carried out:

- Step 1: derive suitable SUSY Slavnov-Taylor identities which determine the counterterms in question.
- Step 2: verify that these Slavnov-Taylor identities are valid on the regularized level in dimensional reduction.

These two steps then imply that dimensional reduction is indeed consistent with SUSY and the usual renormalization transformation generates the correct counterterm structure.

The proof is given in detail in Ref. [1]; here we briefly illustrate the main steps. As a first example of a SUSY Slavnov-Taylor identity consider

$$0 = \sum \phi_i \Gamma \tilde{H}_{Lk} Y_{\phi_i} + \sum \lambda \Gamma \phi_a \phi_b \phi_c \bar{\epsilon} \Gamma \tilde{H}_{Lk} \phi_i + \text{perm. + fin.} \quad (2.1)$$

Here $\phi_i$, $\tilde{H}_i$, $\lambda$ denote Higgs, Higgsino and gaugino fields; $Y_i$ denote sources of BRST transformations; the abbreviation “fin.” summarizes terms which vanish at tree-level and which don’t receive $n$-loop counterterm contributions at $n$-loop order; “perm” denotes terms corresponding to all possible permutations of $\phi_{a,b,c}$. This identity describes the fundamental SUSY relation between the quartic Higgs-boson self-coupling and the electroweak gauge couplings. The gauge couplings are reflected in the second term of Eq. (2.1). In the gaugeless limit this identity (which has to hold in the renormalized theory, i.e. after adding counterterms) unambiguously determines the counterterm for the quartic Higgs self interaction [17].

As a second example of SUSY Slavnov-Taylor identities consider

$$0 = -\Gamma \tilde{u} R \bar{\epsilon} Y u_R - \Gamma u_R \bar{\epsilon} Y \tilde{u} R + \text{...} \Rightarrow \text{Yukawa couplings} \quad (2.2)$$

$$0 = \Gamma \tilde{u} R \bar{\epsilon} Y u_R + \text{known} \Rightarrow \text{SUSY transformations} \quad (2.3)$$

$$0 = -\Gamma \tilde{u} R \bar{\epsilon} Y u_R - \Gamma u_R \bar{\epsilon} Y \tilde{u} R \Rightarrow \text{self energies} \quad (2.4)$$

As indicated, this set of identities correlates Yukawa couplings (either quark–quark–Higgs or quark–squark–Higgsino) with SUSY transformations, SUSY transformations with one another, and SUSY transformations with self energies. Combining identities of these kinds allows to eliminate the SUSY transformations and to relate counterterms to Yukawa couplings to field renormalization counterterms defined via self energies.

Now we illustrate the second main step, for the case of the Slavnov-Taylor identity (2.1). The identity must hold after renormalization, but the question is whether it holds already on the regularized (and subrenormalized) level in dimensional reduction. In order to check this we can apply the quantum action principle in dimensional reduction [20], which relates the potential breaking of the identity on the regularized level to the quantity

$$\left(\Delta^{\leq 2L} \cdot \Gamma^{DRED}\right) \phi_{a,b,c} \tilde{H}_{Lk} \bar{\epsilon} \quad (2.5)$$

i.e. the 1PI Green function with the indicated external fields and one insertion of the operator $\Delta^{\leq 2L}$, given by applying the Slavnov-Taylor operator to the classical action (including up to 2-loop
counterterms for subrenormalization). This Green function is represented by Feynman diagrams like the ones in Fig. 1; the insertion $\Delta_{\leq 2L}$ is given by particular 4-fermion operators determined in Ref. [20]. In spite of the complicated structure of these Feynman diagrams one can show that all diagrams in Fig. 1 and all further contributing diagrams vanish due to the numerator algebra in the appropriate limit. Hence Eq. (2.5) vanishes and the identity (2.1) is valid on the regularized level (in the appropriate order and limit).

Ref. [1] presents this kind of analysis for all relevant Slavnov-Taylor identities. All Slavnov-Taylor identities were found to be valid on the regularized level in dimensional reduction at the appropriate order. As a result, the usual renormalization transformation generates the correct counterterm structure and no explicit SUSY-restoring counterterms are required in the following sectors of the MSSM in the gaugeless limit:

- quartic Higgs couplings and Higgs mass counterterms at the 3-loop level
- Yukawa interactions between Higgs/Higgsino and quark/squark at the 2-loop level
- quartic interactions between Higgs bosons and squarks at the 2-loop level
- quartic interactions between Higgs bosons and $\epsilon$-scalars at the 2-loop level

3. Muon $g - 2$ in the 2-Higgs doublet model

The anomalous magnetic moment of the muon $a_\mu$ constitutes one of the very few observables with a significant deviation between the SM prediction and the measured value [21]. The most up-to-date SM theory evaluations [22] find deviations with a significance of 3–4$\sigma$; the absolute deviation is in the range $(26.8 \ldots 30.6) \times 10^{-10}$. This deviation might be a hint for BSM physics. Future measurements at Fermilab and J-PARC promise to increase the accuracy substantially.

The absolute deviation should be compared to the contribution of the electroweak SM of 15.36(10) $\times 10^{-10}$ [23]. Typically, BSM contributions are suppressed compared to the electroweak SM contributions by a factor $(M_W/M_{BSM})^2$; hence it is not obviously straightforward to explain the deviation in terms of new physics. In view of this situation it is motivated to ask: what are the possible $a_\mu$ values in any given BSM scenario, and in which parameter region can a given BSM

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1In addition, Refs. [18, 19] have shown the corresponding results for triple interactions between gluon/glueino and quark/squarks.
scenario accommodate an $a_\mu$ deviation as large as the observed one? An example of a past study of this kind is the one by Ref. [24], which showed that the MSSM can accommodate the observed deviation even if the lightest SUSY particle mass is as heavy as $M_{LSP} = 1$ TeV.

Here we review the analysis of Ref. [2] of $a_\mu$ in the 2-Higgs doublet model (2HDM). The 2HDM provides a much less promising explanation of the $a_\mu$ deviation than SUSY since the leading contributions arise only at the 2-loop level and are thus suppressed. Nevertheless, a series of recent works [25, 26, 27, 28, 29, 30, 31, 32, 33] have demonstrated that the 2HDM remains a viable explanation in a certain part of its parameter space. The goal of the analysis of Ref. [2] was to identify precisely the viable parameter space and to find the overall maximum contribution to $a_\mu$ that could be explained in the 2HDM.

The analysis takes as a basis the flavour-aligned 2HDM with general Higgs potential, but assuming CP conservation. This model has the following important input parameters: the four physical Higgs masses $M_h, M_H, M_{H^\pm}, M_A$ and three Yukawa coupling parameters $\zeta_l,u,d$ (which specify the $A$-couplings to leptons, up- and down-type quarks in units of the SM Yukawa couplings).\footnote{The more common 2HDM types I, II, X, Y correspond to special cases: e.g. in type II, $\zeta_l = \zeta_d = -\tan \beta = -1/\zeta_u$, and hence in type II, $|\zeta_d/\zeta_u| = 1$ always. In type X or the so-called lepton-specific model, $\zeta_l = -\tan \beta = -1/\zeta_{u,d}$.}

Further input parameters are the mixing angle $\cos(\beta - \alpha)$ (which must be very small because of LHC Higgs coupling measurements) and further Higgs potential parameters (which enter the $a_\mu$ analysis only via the Higgs self couplings $C_{hAA}$ and $C_{HH^+H_-}$). In terms of these parameters, useful approximations for the most important contributions to $a_\mu$ in the flavour-aligned 2HDM (in terms of variables $\delta_S \equiv M_S/100$ GeV and for $M_{H^\pm} = M_H$) are given by [2]

\begin{align*}
\alpha_{\mu,1}^{2HDM} & \simeq \left( \frac{\zeta_l}{100} \right)^2 \frac{-3 - 0.5 \ln(\delta_A)}{A_A^2} \times 10^{-10} \quad & \text{1-loop contributions} \\
\alpha_{\mu,FZ} & \simeq \left( \frac{\zeta_l}{100} \right)^2 \left\{ 8 + 4\delta_A^2 + 2\ln(\delta_A) \right\} \times 10^{-10} \quad & \text{2-loop $\tau$-loop Barr Zee} \\
\alpha_{\mu,2} & \simeq \left( \frac{\zeta_l}{100} \right)^2 \left\{ 54 - 14\ln(\delta_A) - 15\ln(\delta_H) \right\} \times 10^{-10} \quad & \text{2-loop top-loop Barr Zee} \\
|a_\mu|^2 & \simeq \rho \left| \frac{C_{HH^+H_-}}{1 \text{GeV}} \right| |\zeta_l| \times 10^{-15}, \quad & \text{where } \rho = (1 \ldots 6) \quad & \text{2-loop bosonic loops}
\end{align*}

These formulas show that very small $M_A < 100$ GeV and large lepton Yukawa coupling $\zeta_l = 0'(100)$ are needed to come close to the observed deviation of $\Delta a_\mu \sim 30 \times 10^{-10}$.

The precise analysis of Ref. [2] then proceeds in two steps:

- Step 1: derive experimental constraints on the relevant parameters (Higgs boson masses, Yukawa couplings, Higgs self couplings). These constraints are also interesting in their own right in view of LHC searches for extra Higgs bosons.

- Step 2: derive the possible range of $a_\mu$ in the 2HDM, depending on relevant input parameters, and derive the parameter range in which the current deviation can be accommodated.

Figure 2 shows the most important results of the analysis of the relevant parameter constraints. Most importantly, a light pseudoscalar Higgs boson mass $M_A$ is experimentally allowed. But for
each value of $M_A$, there are maximum values of the Yukawa parameters $\zeta_l$ and $\zeta_u$.

The constraints on $\zeta_l$ arise from $\tau$-decays, $Z$-boson decays and from LEP $ee \rightarrow \tau\tau A$ searches; interestingly, the upper limit is in the range $50 \ldots 100$, which is just the interesting range for $a_\mu$. The constraints on $\zeta_u$ arise from the B-decays $b \rightarrow s\gamma$ and $B_s \rightarrow \mu\mu$ and from LHC Higgs searches for either $gg \rightarrow A \rightarrow \tau\tau$ or $gg \rightarrow H \rightarrow \tau\tau$. Depending on the parameter region, the bounds from B-physics or from LHC can be stronger. In either case, $\zeta_u$ is constrained to be at most of the order of 0.5.

Figure 2c shows final results for the possible values of $a_\mu$. For each value of $M_A$, the Yukawa couplings are set to the maximum allowed values as determined before. $a_\mu$ is then evaluated either in the 2HDM type X (where only the 1-loop and 2-loop $\tau$-contributions are relevant) or in the general flavour-aligned 2HDM with or without bosonic contributions computed in Ref. [33].

We see that the type X model barely explains the current deviation in $a_\mu$, and only in a very small parameter space with $M_A = 20 \ldots 40$ GeV and with $\zeta_l = -\tan \beta$ set to its maximum allowed value. On the other hand, in the general flavour-aligned 2HDM the 2-loop top-contributions (and to a smaller extent the bosonic contributions) can significantly increase $a_\mu$ if the top Yukawa coupling is of the order of its maximum value around 0.5.

By maximizing the $\tau$- and top-Yukawa couplings, the currently observed value for $a_\mu$ can be explained for Higgs masses up to $M_A = 100$ GeV. Interestingly, there is an absolute maximum value of $a_\mu^{2HDM} \approx 45 \times 10^{-10}$ which can be accommodated in the flavour-aligned 2HDM. This value is not far above the currently observed value. Hence the future $a_\mu$ measurements will significantly

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3There is also a maximum value of $\zeta_l$, which is not shown since it is not of importance for $a_\mu$. However, the combination of these constraints rules out the entire low-$M_A$ region in the 2HDM type II. On the other hand, the 2HDM type X trivially fulfills the constraints on $\zeta_u$ for large $\tan \beta$. 

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Figure 2: (a,b) Maximum possible values of the lepton and up-quark Yukawa parameters $\zeta_l$ and $\zeta_u$ in the flavour-aligned 2HDM, given experimental constraints discussed in the text. (c) The overall maximum $a_\mu$ (including one-loop and all two-loop contributions) as a function of $M_H$, for several fixed values of $M_H = M_{H^\pm}$, in the flavour-aligned 2HDM. The result without top-loop and bosonic contributions (which would correspond to the maximum in the 2HDM type X) is shown in blue; the result without bosonic two-loop contributions in red; the total maximum result, including the maximum bosonic contributions in black. The yellow band indicates the current $a_\mu$ deviation. The plots are taken from Ref. [2].
constrain the 2HDM parameter space and might even exclude the model entirely. At the same time the result in Fig. 2c also highlights the importance of further dedicated searches for low-mass pseudoscalar Higgs bosons at the LHC.

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References


