PROCEEDINGS OF SCIENCE

CoLoRFulNNLO for LHC processes

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In my talk I gave a status update on the extension of the CoLoRFulNNLO subtraction method for computing QCD jet cross sections with hadrons in the initial state. The scheme has been fully worked out previously for electron-positron collisions and recently important steps have been made towards generalizing it to be able to deliver corrections of the same order for LHC processes as well. In particular, the important bottleneck of regularizing multiple real emissions has been addressed. We demonstrate the numerical stability of the CoLoRFulNNLO method by computing the doubly real contribution for Higgs-boson production in gluon-gluon fusion and for W production.

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1. Introduction

As data taking continues at LHC there is an increasing interest in high precision predictions needed for interpreting these data. In recent years several advances were made in computing multi-loop amplitudes, involving both new techniques and tools. These amplitudes are important ingredients of higher-order computations, hence they are essential in making precise predictions. These developments pave the road towards automated NNLO calculations and beyond.

Besides the calculation of multi-loop amplitudes, another major issue of higher-order calculations must be addressed, namely the regularization of multiple emissions. When extra massless particles appear in the final state at higher orders, collinear or soft configurations result in kinematic singularities which must be regularized. Several different regularizations are possible: the region of phase space close to the kinematic singularity can be cut out (slicing) [1, 2, 3] or the kinematic behavior of the contribution can be mimicked near the singular limits and can be subtracted (subtraction) [4, 5, 6, 7, 8, 9, 10, 11]. Either way, in order to get a physical result the sliced away or subtracted contributions have to be added back after summing and integrating over the unresolved degrees of freedom. The CoLoRFulNNLO method was developed as a subtraction scheme using completely local counterterms derived from the infrared factorization properties of QCD squared matrix elements. To date, it is worked out in full detail for electron-positron annihilation. It was used to obtain predictions for standard event shapes with unprecedented numerical precision and to compute observables for which predictions at NNLO accuracy had not been available in the literature [12, 13, 14]. However, in order to handle LHC processes, the method must be extended to be applicable when colored particles are present in the initial state. In this proceedings a status update is given on extending the scheme and its numerical implementation to LHC processes. In particular, the correct regularization of doubly real emission is demonstrated for Higgs-boson production in gluon-gluon fusion and W-boson production.

2. The Method

For an infrared-safe observable J computed in some hadron-initiated process the cross section takes the form of:

$$\sigma[J](p_A,p_B) = \sum_{a,b}\int_0^1 \mathrm{d}x_a\,\mathrm{d}x_b f_{a/A}(x_a;\mu_F^2) f_{b/B}(x_b;\mu_F^2)\sigma_{ab}[J](p_a,p_b;\mu_F^2)\,. \tag{2.1}$$

Here p_A and p_B are the momenta for the incoming hadrons, x_a and x_b are Bjorken's xs, $f_{a/A}$ and $f_{b/B}$ are the parton distribution functions (PDFs), μ_F is the factorization scale and σ_{ab} is the partonic cross section defined with partons a and b as incoming with their momenta obtained as $p_a = x_a p_A$ and $p_b = x_b p_B$.

The partonic cross section has the usual perturbative expansion:

$$\sigma_{ab}[J] = \sigma_{ab}^{\mathrm{LO}}[J] + \sigma_{ab}^{\mathrm{NLO}}[J] + \sigma_{ab}^{\mathrm{NNLO}}[J] + \ldots \tag{2.2}$$

where $\sigma_{ab}^{\mathrm{LO}}[J]$, $\sigma_{ab}^{\mathrm{NLO}}[J]$ and $\sigma_{ab}^{\mathrm{NNLO}}[J]$ represent the contributions at increasing orders in terms of α_S. If the lowest order contribution has m partons in the final state the first term in the expansion can be written as:

$$\sigma_{ab}^{\rm LO}[J] = \int_m {\rm d}\sigma_{ab}^{\rm B} J_m \tag{2.3}$$

where ${\rm d}\sigma_{ab}^{\rm B}$ is the fully differential Born cross section with a and b as initial state partons.

The NLO contribution can be written as the sum of two terms:

$$\sigma_{ab}^{\rm NLO}[J] = \int_{m+1} {\rm d}\sigma_{ab}^{\rm R} J_{m+1} + \int_m \left\{ {\rm d}\sigma_{ab}^{\rm V} + {\rm d}\sigma_{ab}^{\rm C} \right\} J_m \,. \tag{2.4}$$

Here ${\rm d}\sigma_{ab}^{\rm R}$ stands for the real-emission contribution, ${\rm d}\sigma_{ab}^{\rm V}$ is the one-loop term and ${\rm d}\sigma_{ab}^{\rm C}$ is the collinear counterterm. In the CoLoRFulNNLO method local subtraction terms are used to regularize kinematic singularities arising from unresolved real emission. These subtraction terms are derived from the infrared factorization properties of QCD amplitudes. With subtractions the NLO contribution takes the form of:

$$\sigma_{ab}^{\rm NLO}[J] = \int_{m+1} \left[{\rm d}\sigma_{ab}^{\rm R} J_{m+1} - {\rm d}\sigma_{ab}^{\rm R,A_1} J_m \right]_{d=4} + \int_m \left[\left\{ {\rm d}\sigma_{ab}^{\rm V} + \int_1 {\rm d}\sigma_{ab}^{\rm R,A_1} + {\rm d}\sigma_{ab}^{\rm C} \right\} J_m \right]_{d=4} \,. \tag{2.5}$$

At NNLO the contribution is composed of three different terms:

$$\begin{aligned} \sigma_{ab}^{\rm NNLO}[J] &= \sigma_{ab,m+2}^{\rm NNLO}[J] + \sigma_{ab,m+1}^{\rm NNLO}[J] + \sigma_{ab,m}^{\rm NNLO}[J] = \\ &= \int_{m+2} {\rm d}\sigma_{ab}^{\rm RR} J_{m+2} + \int_{m+1} \left\{ {\rm d}\sigma_{ab}^{\rm RV} + {\rm d}\sigma_{ab}^{\rm C_1} \right\} J_{m+1} + \int_m \left\{ {\rm d}\sigma_{ab}^{\rm VV} + {\rm d}\sigma_{ab}^{\rm C_2} \right\} J_m \end{aligned} \tag{2.6}$$

where ${\rm d}\sigma_{ab}^{\rm RR}$, ${\rm d}\sigma_{ab}^{\rm RV}$ and ${\rm d}\sigma_{ab}^{\rm VV}$ are the double-real, real-virtual and double-virtual contributions. These contain two extra real partons, one extra real parton with one more loop and two extra loops as compared to the Born process. ${\rm d}\sigma_{ab}^{\rm C_1}$ and ${\rm d}\sigma_{ab}^{\rm C_2}$ are the collinear counterterms. As in the case of the NLO correction we define local subtraction terms to regularize kinematic singularities in the $m+2$ and $m+1$ parton contributions. In the double-real piece up to two partons can become unresolved. This is mirrored by the structure of our subtractions:

$$\sigma_{ab,m+2}^{\rm NNLO}[J] = \int_{m+2} \left[{\rm d}\sigma_{ab}^{\rm RR} J_{m+2} - {\rm d}\sigma_{ab}^{\rm RR,A_2} J_m - {\rm d}\sigma_{ab}^{\rm RR,A_1} J_{m+1} + {\rm d}\sigma_{ab}^{\rm RR,A_{12}} J_m \right]_{d=4} \tag{2.7}$$

where the last term is introduced to remove the overlap between singly (A_1) and doubly (A_2) unresolved subtractions. As for the real-virtual piece the structure is very similar to the one we have already encountered at NLO due to the presence of only one extra parton. The only difference is the presence of the integrated A_1 terms necessitating a further subtraction:

$$\sigma_{ab,m+1}^{\rm NNLO}[J] = \int_{m+1} \left[\left\{ {\rm d}\sigma_{ab}^{\rm RV} + \int_1 {\rm d}\sigma_{ab}^{\rm RR,A_1} + {\rm d}\sigma_{ab}^{\rm C_1} \right\} J_{m+1} - \left\{ {\rm d}\sigma_{ab}^{\rm RV,A_1} + \left(\int_1 {\rm d}\sigma_{ab}^{\rm RR,A_1} \right)^{\rm A_1} \right\} J_m \right]_{d=4} \,. \tag{2.8}$$

Above ${\rm d}\sigma_{ab}^{\rm RV,A_1}$ is understood to include subtraction terms regularizing the real-virtual and the C_1 counterterm as well. The m parton contribution is free from kinematic singularities due to the infrared finiteness of observable J but because of the two extra loops it contains explicit ε poles. These poles are cancelled by the integrated forms of the various subtraction terms:

$$\sigma_{ab,m}^{\rm NNLO}[J] = \int_m \left[{\rm d}\sigma_{ab}^{\rm VV} + {\rm d}\sigma_{ab}^{\rm C_2} + \int_2 \left[{\rm d}\sigma_{ab}^{\rm RR,A_2} - {\rm d}\sigma_{ab}^{\rm RR,A_{12}} \right] + \int_1 \left\{ {\rm d}\sigma_{ab}^{\rm RV,A_1} + \left(\int_1 {\rm d}\sigma_{ab}^{\rm RR,A_1} \right)^{\rm A_1} \right\} \right]_{d=4} J_m \,. \tag{2.9}$$

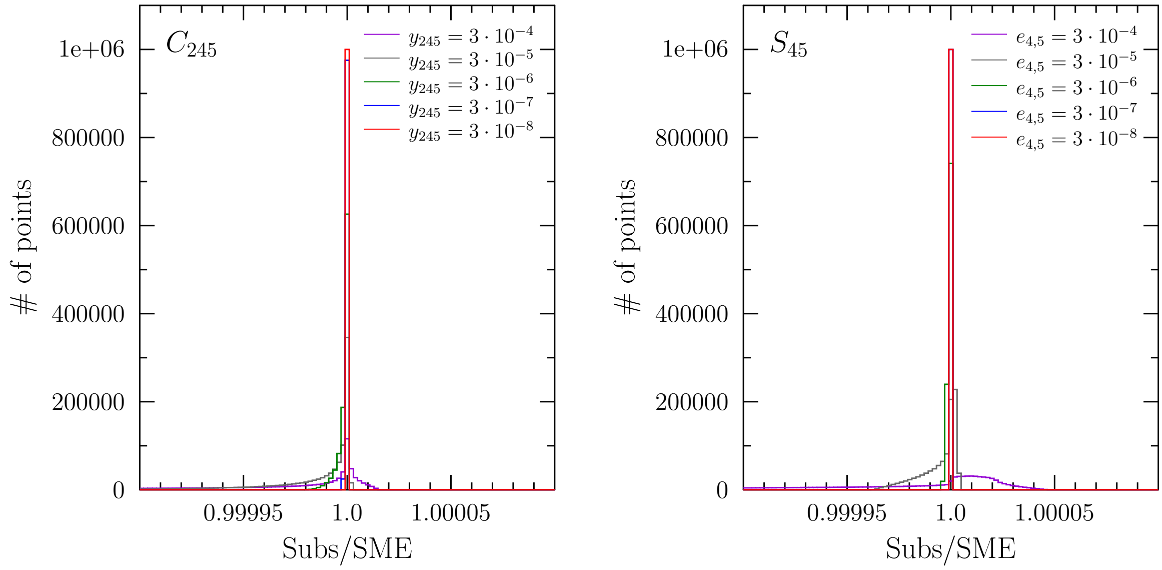

Figure 1: The rate of cancellation of kinematic singularities for the process of $u(p_1) + \bar{d}(p_2) \to W^+(p_3) + g(p_4) + g(p_5)$ for a triple collinear (C_{245}) and a double soft (S_{45}) limit. The scaleless quantities describing the limit are the scaleless three-particle invariant, $y_{245} = p_{245}^2/Q^2$, and scaleless energies, $e_i = E_i/Q$, respectively.

The cancelation of the ε poles coming from loop contributions against those of the integrated subtractions can serve as a powerful check of the subtractions and the method. However, the drawback of this check is that it requires all the subtractions to be defined and to carry out the integrations over the unresolved degrees of freedom. Nonetheless, the subtractions applied in the $m+2$ and $m+1$ parton pieces can be checked separately even before carrying out the tedious integrations. In the following we summarize those checks we applied to the $m+2$ parton contribution in order to validate our subtraction terms.

3. Checking the Subtraction Terms

If for a given contribution all subtraction terms are defined and a physical limit is approached one subtraction term should cancel the kinematic singularity coming from the squared matrix element while the other subtractions should cancel among each other. This provides means for testing the method. Phase space points approaching any specific unresolved configuration can be generated and the ratio of the sum of all subtractions and the squared matrix element can be computed. If the number of points is plotted as a function of the corresponding ratio as the limit is approached the spread of points should decrease and ultimately close to the limit all the points should scatter around one. We illustrate this for a triple collinear and a double soft limit on Fig. 1 where we present such spike plots in case of W production. We checked that all the other limits behaved in a similar way. This provides a local check of the subtraction terms.

Furthermore a global test of convergence can be performed. In any computation beyond leading order accuracy there is a technical cut[1] on the phase space due to the finite precision of numbers

[1] This cut is not to be confused with the cut used throughout in computations performed with the slicing method.

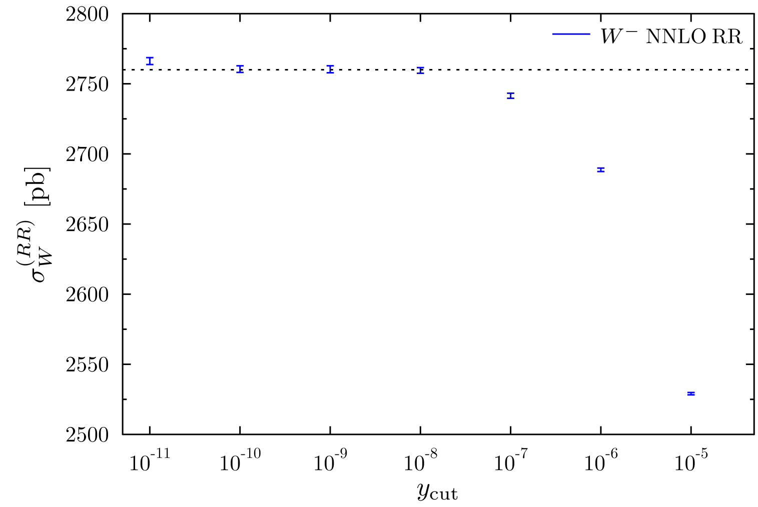

Figure 2: The cross section contribution from the double-real line as a function of y_{cut} for W^- production.

in computer arithmetics. In practice this means that there is a lower limit defined for all two-particle invariants such that

$$y_{\mathrm{min}} = \min_{i,j} \frac{(p_i + p_j)^2}{Q^2} \geq y_{\mathrm{cut}} \,, \tag{3.1}$$

where Q is the center-of-mass energy of incoming partons and y_{cut} should be selected small enough such that the cross section is unaffected by further decreasing its value. In case of initial state hadrons those invariants also have to be incorporated in the set of pairs that are formed with the initial state partons.

If kinematic singularities are regularized in the double-real and real-virtual contributions and the resulting cross section contribution is plotted as a function of y_{cut} at small values a saturation of the result has to occur. If this does not happen at least one kinematic singularity is not regularized adequately. Plotting the cross section as a function y_{cut} can thus act as a very powerful check even in the early stages of developing a new subtraction scheme since it only requires the definition of subtractions for the double-real or real-virtual contribution but not their integrated forms.

The CoLoRFulNNLO framework was first developed for electron-positron annihilation, hence, its extension to hadronic initial states requires that several new subtraction terms are introduced. As the subtraction terms are defined on the whole phase space, cancellations must happen between them in various limits, which can only be achieved if the Sudakov parameterizations (e.g.: explicit definitions of momentum fractions appearing in the splitting kernels) used in the terms are delicately tuned to each other. Thus it was essential to perform saturation tests for all the subprocesses of a given process in order to ensure correct behavior even if all subtraction terms showed correct limiting behavior in their respective limits.

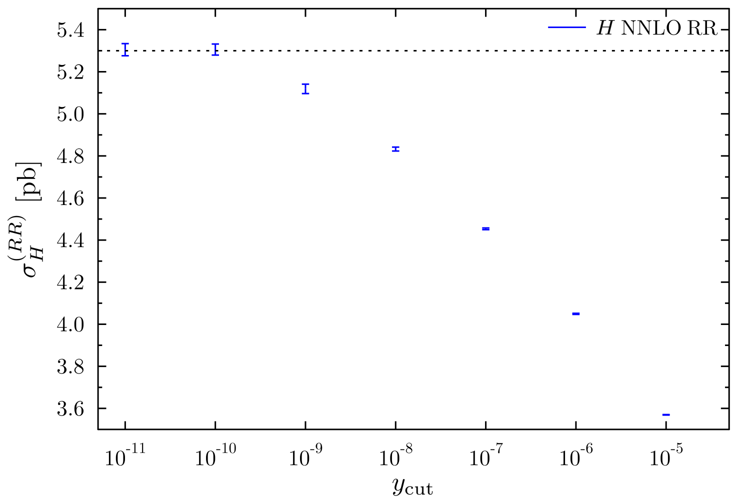

Figure 3: Same as Fig. 2 but for Higgs-boson production.

4. Preliminary Results

The CoLoRFulNNLO subtraction method was implemented in the `MCCSM` (Monte Carlo for the CoLoRFulNNLO Subtraction Method) numerical code [15] that is being extended to treat processes with hadronic initial states as well. The first processes we implemented are $W^{\pm}$ production and Higgs-boson production in gluon-gluon fusion. For both processes we computed the cross section contribution coming from the regularized double-real line as a function of y_{cut}.

The cross section is unphysical due to the missing real-virtual and double-virtual pieces. The setup employed in the runs was the following: a 13 TeV LHC configuration was used with the `NNPDF30_nnlo_as_0118` PDF set as provided by `LHAPDF` [16], for $W^{\pm}$ production $m_W = 80.385\,\mathrm{GeV}$, $m_Z = 91.1876\,\mathrm{GeV}$ and $\alpha_{\mathrm{EM}}(m_Z) = 1/128$ and for Higgs-boson production $m_H = 125\,\mathrm{GeV}$ and vacuum expectation value $v = 246.219\,\mathrm{GeV}$ were used.

The saturation plot for W^{-} production is depicted on Fig. 2 while for Higgs-boson production on Fig. 3. The saturation of the cross section contribution at small values of y_{cut} is visible for both processes indicating that our subtraction terms cancel all kinematic singularities correctly.

With the double-real contribution regulated by subtractions it is possible to calculate the contribution to a physical observable from that part. One natural observable for both processes is the rapidity of the vector/scalar boson. The rapidity distribution for $W^{\pm}$ is shown on Fig. 4 while the Higgs-boson rapidity is depicted on Fig. 5. In both cases the relative smoothness of the double-real contribution shows that the CoLoRFulNNLO subtraction method can produce numerically stable predictions for the most computationally demanding part.

5. Conclusions

In my talk I gave a status report on the application of the CoLoRFulNNLO subtraction method

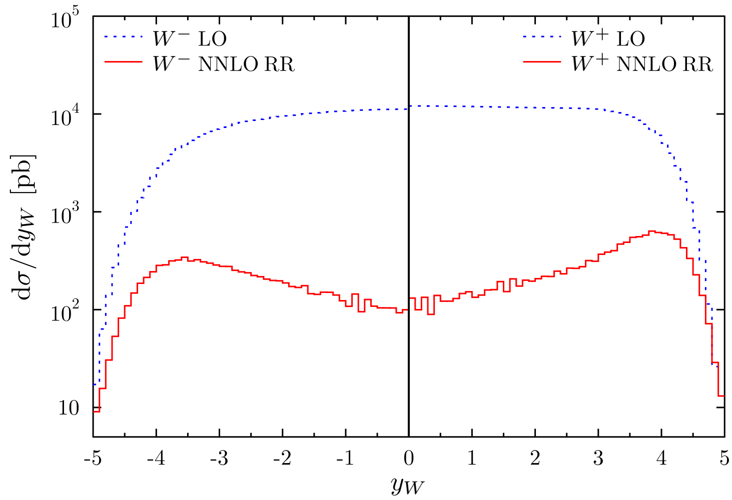

Figure 4: Rapidity distribution of W^- (left side) and W^+ (right side) at LO (blue dotted) and the regulated double-real contribution (red) to the NNLO result.

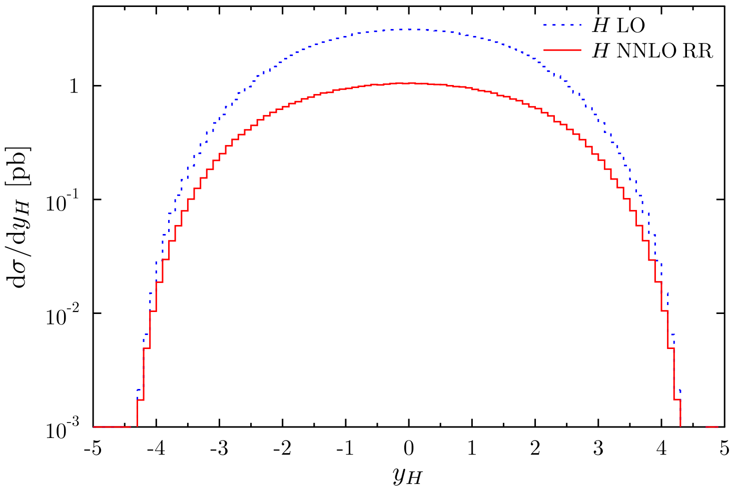

Figure 5: Same as Fig. 4 but for Higgs-boson production.

to key LHC processes. The method is implemented in the MCCSM numerical code, which has gone through a major revision and now includes facilities to use any PDF provider and makes possible to use an arbitrary number of different scales in a single run. This can be extremely beneficial when dealing with hadron collisions where it is not at all trivial how to choose the best scale for a process *a priori*. As a demonstration of the method and of the code I presented spike and saturation plots for the double-real contribution to $W^\pm$ production and Higgs-boson production in gluon fusion demonstrating that the newly defined subtraction terms are proper regulators. I also showed predictions for the computationally most demanding double-real contribution to indicate that numerically stable predictions for differential quantities can be achieved.

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