We find a new $CP$ violating effect in charmed hadron decays into neutral kaons, which is induced by the interference between the Cabibbo-favored and doubly Cabibbo-suppressed amplitudes with the $K^0 \rightarrow \bar{K}^0$ mixing [1]. It is estimated to be of order of $O(10^{-3})$, much larger than the direct $CP$ asymmetry, but missed in the literature. To reveal this new $CP$ violation effect, we propose a new observable, the difference of the $CP$ asymmetries in the $D^+ \rightarrow \pi^+ K^0_S$ and $D^+_s \rightarrow K^+ K^0_S$ modes. Once the new effect is determined by experiments, the direct $CP$ asymmetry then can be extracted and used to search for new physics.
CP asymmetry plays an unique role in understanding the matter-antimatter asymmetry and searching for new physics. It has been well established in the kaon and B meson systems \cite{2, 3, 4, 5}, but not yet in the charm sector. In the past decade, many efforts have been devoted to study the CP violation in the singly Cabibbo-suppressed (SCS) D meson decays. The most precise experimental result of CP violation in SCS decays is \cite{6}

\[
\Delta A_{CP} \equiv A_{CP}(D^0 \to K^+ K^-) - A_{CP}(D^0 \to \pi^+ \pi^-) = (-0.10 \pm 0.08 \pm 0.03)\%
\] (1)

With the precision lower than $10^{-3}$, the CP violation in charm decays has not been observed.

CP asymmetry can also occur in $D \to f K_S^0$ decays, where $f$ is a final-state particle. For example, the CP violation in $D^+ \to \pi^+ K_S^0$ has been measured by Belle collaboration with 3.2$\sigma$ from zero \cite{7}. In this work, we point out a new CP-violation effect, which is induced by the interference between the Cabibbo-favored (CF) and doubly Cabibbo-suppressed (DCS) amplitudes with the mixing of final-state mesons \cite{1}. It is estimated to be of order of $10^{-3}$, much larger than the direct CP asymmetry, however, missed in the literature \cite{7, 8, 9, 10}. We propose a new observable, the difference between the CP asymmetries in the $D^+ \to \pi^+ K_S^0$ and $D^+_t \to K^+ K_S^0$ decays, to measure the new CP violation effect. Once the new effect is obtained, the direct CP asymmetry in charm decays can be extracted correctly and used to search for new physics.

In experiments, the $K_S^0$ state is reconstructed by $\pi^+ \pi^-$ final state. The time-dependent CP violation in $D$ meson decays into neutral kaons is defined by

\[
A_{CP}(t) \equiv \frac{\Gamma(D \to K(t) (\to \pi^+ \pi^-) f) - \Gamma(\overline{D} \to |K(t) (\to \pi^+ \pi^-)| \overline{f})}{\Gamma(D \to K(t) (\to \pi^+ \pi^-) f) + \Gamma(\overline{D} \to K(t) (\to \pi^+ \pi^-) \overline{f})},
\] (2)

where $K(t)$ donates the immediate state of neutral kaons. The mass eigenstates of neutral kaons, $|K_S^0\rangle$, of mass $m_S$ and width $\Gamma_S$ and $|K_L^0\rangle$ of mass $m_L$ and width $\Gamma_L$, are linear combinations of the flavor eigenstates $|K^0\rangle$ and $|\overline{K}^0\rangle$, $|K_{S,L}^0\rangle = p_K|K^0\rangle \mp q_K|\overline{K}^0\rangle$, with $p_K = (1 + \epsilon) / \sqrt{2(1 + |\epsilon|^2)}$ and $q_K = (1 - \epsilon) / \sqrt{2(1 + |\epsilon|^2)}$, and $\epsilon$ is a small parameter characterizing the indirect CP violation in neutral kaon mixing \cite{5}. For convenience, the ratio between DCS and CF amplitudes is set as

\[
\mathcal{A}(D \to K^0 f) / \mathcal{A}(D \to \overline{K}^0 f) = r_f e^{i(\phi + \delta_f)},
\] (3)

where $r_f$ is the size of the ratio, $\phi$ and $\delta_f$ are relative weak and strong phases respectively. In the SM, $r_f \sim |V_{cd}V_{us}/V_{cs}V_{ud}| \sim 0(10^{-2})$ and $\phi = \text{Arg} \left[ -V_{cd}^* V_{us}/V_{cs}^* V_{ud} \right] = (-6.17 \pm 0.43) \times 10^{-4}$ \cite{5}. With the small parameters $\epsilon$, $r_f$ and $\phi$, we obtain the time-dependent CP violation as

\[
A_{CP}(t) \simeq \left[ A_{CP}^{\epsilon 0}(t) + A_{CP}^{\text{dir}}(t) + A_{CP}^{\text{ind}}(t) \right] / D(t),
\] (4)

in which $D(t) = e^{-T_f t} (1 - 2r_f \cos \delta_f \cos \phi + e^{-T_f t}|\epsilon|^2)$. The $A_{CP}^{\epsilon 0}(t)$ term is the indirect CP violation in $K^0 - \overline{K}^0$ mixing,

\[
A_{CP}^{\epsilon 0}(t) = -2e^{-T_f t} \left( Re(\epsilon) \cos(\Delta m t) + Im(\epsilon) \sin(\Delta m t) \right) + 2Re(\epsilon)e^{-T_f t}.
\] (5)

The $A_{CP}^{\text{dir}}(t)$ term is the direct CP violation in charm decay induced by the interference between the CF and DCS amplitudes,

\[
A_{CP}^{\text{dir}}(t) = 2e^{-T_f t} r_f \sin \delta_f \sin \phi.
\] (6)
The $A_{CP}^{int}(t)$ term is the interference effect between the CF and DCS amplitudes with $K^0 - \bar{K}^0$ mixing,

$$A_{CP}^{int}(t) = -4r_f \cos \phi \sin \delta_f \left( \text{Im}(\epsilon) e^{-i \tau t} - e^{-i \phi} \cos(\Delta m t) - \text{Re}(\epsilon) \sin(\Delta m t) \right).$$

The parameters $r_f$ and $\delta_f$ for the $D^+ \to \pi^+ K^0_S$ and $D_s^+ \to K^+ K_S^0$ decays have been estimated in the factorization-assisted topological-amplitude (FAT) approach [11, 12]. The dependences of the $CP$ asymmetry in the $D^+ \to \pi^+ K(t) (\to \pi^+ \pi^-)$ decay on $t/\tau_S$ are displayed in Fig. 1. It is found that the total $CP$ violation dominated by $A_{CP}^{int}(t)$, and the deviation from $A_{CP}^{int}(t)$ mainly comes from $A_{CP}^{int}(t)$. The direct $CP$ asymmetries are too small to be seen in Fig. 1, being of order of $\mathcal{O}(10^{-5})$. According to Eqs. (5) and (7), $A_{CP}^{int}(t = 0) = A_{CP}^{int}(t) = 0$, resulting in $A_{CP}(t = 0) = A_{CP}^{int}(t = 0)$. Both the forthcoming experiments cannot measure the direct $CP$ asymmetries, unless the large weak phase differences are provided by new physics. Thereby, an observation with nonvanishing $A_{CP}(t = 0)$ indicates new physics. Compared to the SCS processes, in which the $CP$ asymmetry cannot discriminate new physics due to the ambiguities in estimating the penguin amplitudes, the direct $CP$ asymmetry in neutral kaon modes would give a more unambiguous new physics signal.

The time-integrated $CP$ asymmetry is

$$A_{CP} = \int_0^\infty F(t) \frac{A_{CP}^{int}(t) + A_{CP}^{int}(t) + A_{CP}^{int}(t)}{F(t) \text{d}t},$$

where $F(t)$ is a function to take into account relevant experimental effects. With the approximation of $F(t) = 1$ in the interval $[t_1, t_2]$ and $F(t) = 0$ elsewhere [9], quation (8) yields

$$A_{CP}(t_1, t_2) = \frac{2\text{Re}(\epsilon) - 4\text{Im}(\epsilon) r_f \cos \phi \sin \delta_f}{1 - 2r_f \cos \delta_f \cos \phi} \left[ 1 - \frac{\cos((t_1) - \cos((t_2))}{\tau_S (1 + x^2)} \right] + 2r_f \sin \delta_f \sin \phi,$$

where $x = \Delta m / \Gamma$, $c(t) = e^{-i \phi} \cos(x \sin(\Delta m t))$, and $s(t) = e^{-i \phi} [\cos(\Delta m t) + \sin(\Delta m t)]$. In the first line, those terms proportional to $r_f$ represent the new effect $A_{CP}^{int}(t_1, t_2)$, and those without
CP violation in charmed hadron decays into neutral kaons

Figure 2: The plot of time-integrated CP asymmetries in the $D^+ \rightarrow \pi^+ K_S^0$ given in [1], where the left plot is the total CP asymmetry and the right one is the new CP-violation effect. The dashed lines is the theoretical uncertainties of our predictions.

$r_f$ are the CP violation in the neutral kaon mixing. The second line, which is independent of $t_1, t_2$, corresponds to the direct CP asymmetry in charm decays. The time-integrated CP asymmetries in the $D^+ \rightarrow \pi^+ K_S^0$ and the new CP violating effect are exhibited in Fig. 2. In some ranges of $t_1$ and $t_2$, these two quantities are relatively larger than other ranges. The experimental investigations could choose the favorable time intervals. In some experiments, including Belle and LHCb, the new CP violation effect is in absence [7, 13, 14, 15, 16]. However, since this new effect is of the same order as the direct CP asymmetries in the SCS processes, it cannot be neglected in these measurements.

In order to measure the new CP-violation effect in experiments, we propose an observable

$$\Delta A_{\pi^+ K^+} \equiv A_{CP}^{D^+ \rightarrow \pi^+ K_S^0(t_1, t_2)} - A_{CP}^{D_s^+ \rightarrow K^+ K_S^0(t_1, t_2)}.$$ (10)

The CP violation in the kaon mixing cancels in the above difference, and the direct CP violation is negligible. Our global-fit analysis indicates that the $2r_f \cos \phi \cos \delta_f$ term in denominator of Eq. (9) matters little due to the large strong phases $\delta_f$ [12], which is consistent with those derived in the literature [17, 18, 19, 20] and supported by experiment [21]. Then we have

$$\Delta A_{CP}^{\pi^+ K^+} \simeq A_{CP}^{int.D^+ \rightarrow \pi^+ K_S^0(t_1, t_2)} - A_{CP}^{int.D_s^+ \rightarrow K^+ K_S^0(t_1, t_2)}.$$ (11)

The model-independent $SU(3)$ symmetry analysis shows the new effects in two modes are constructive in $\Delta A_{CP}^{\pi^+ K^+}$. The dependencies of $\Delta A_{CP}^{\pi^+ K^+}$ on $t_1$ and $t_2$ are plotted in Fig. 3. $\Delta A_{CP}^{\pi^+ K^+}$ is of order of $10^{-3}$ in most of time intervals, which is accessible at Belle II and LHCb upgrade experiments [6, 15, 22, 23].

In summary, we investigated the time-dependent and time-integrated CP violation in charm decays into neutral kaons. We first pointed out a new measurable CP-violating effect, the interference between charm decays and kaon mixing, exists in these modes. It could be revealed by measuring the difference of CP asymmetries in the $D^+ \rightarrow \pi^+ K_S^0$ and $D_s^+ \rightarrow K^+ K_S^0$ modes on Belle...
II and LHCb upgrade. In addition, an observation with non-zero CP violation at $t = 0$ would signal new physics.

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