PROCEEDINGS OF SCIENCE

The Physics of our Universe

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Almost everyone knows the Pythagorean theorem—and is acutely aware also, that $E = mc^2$. But everyone who can do simple algebra should also learn how to *deduce* the fact that $E = mc^2$ from that Pythagorean theorem–or, rather, from a tiny extension of that famous theorem. That tiny extension is usually called Einstein's Theory of Special Relativity—and this paper shows exactly how that deduction can with the greatest of ease be accomplished, although to do so does require using just a tad more than simple high-school algebra: we'll need one small technical mathematical tool, contributed by Isaac Newton. In order to get fully on top of Einstein's relativity, all that is necessary is to copy out this paper (because copying means that each equation is actually absorbed and understood). Copy every bit of it—indeed, it is so important that I recommend that the whole thing be memorized–and to assist you with that, I have provided an appendix, in which every step of the algebra is recapitulated—so no work on your part is needed at all.

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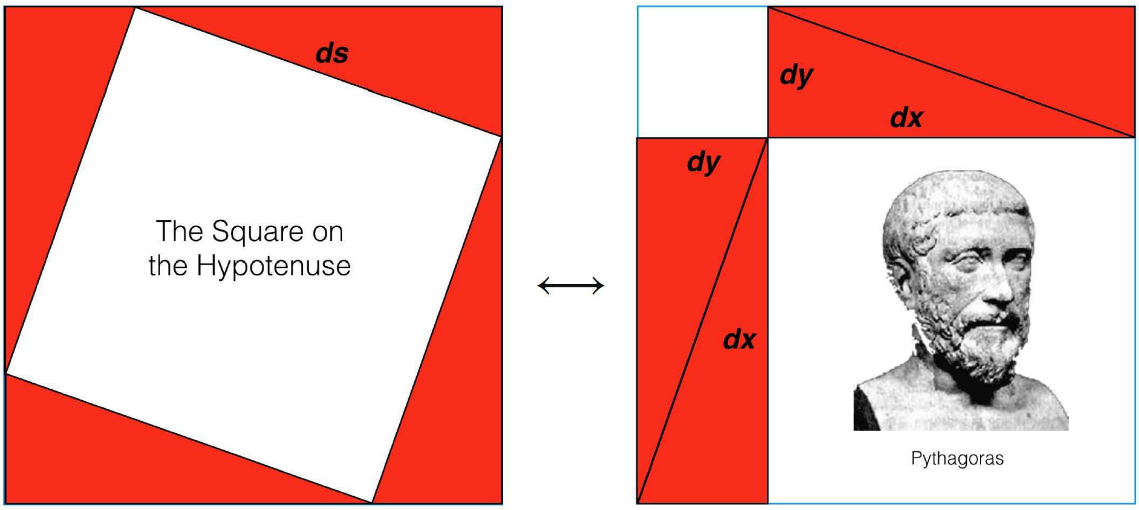
1. Introduction

Over those fifty thousand or so years since we human beings first acquired spoken language, our creation and development of physics has taught us an enormous amount about the universe, and also about our own role in that universe—but perhaps the most amazing thing that we have discovered is that the universe speaks our human-invented abstract language algebra. That fact has huge intellectual, and possibly even religious, implications: certainly no one anticipated that God might turn out to be a high-school algebra teacher, and some perhaps will not be pleased by that peculiar fact, but—it is better to learn it, and to use it, and to appreciate it—than not.

To that end, I provide a derivation of $E = mc^2$ from the slight extension of the ancient and famous Pythagorean theorem that is (Hermann Minkowski's 1908 restatement of) Einstein's famous 1905 theory of special relativity. Learn it, and you will become one with the universe!

2. Pythagoras

Pythagoras famously asserted that for a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides. And we can prove—at a glance—that Pythagoras was right simply by comparing these two drawings:

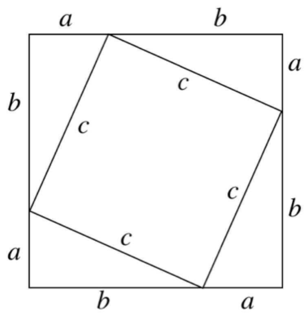


The two drawings are exactly the same size, so their areas are exactly the same. They include eight identical red right-angled triangles—four in each drawing. The remaining (white) areas in each of the two drawings must therefore be identical, which proves the Pythagorean theorem.

Notice that even though my figure includes identification of distances dx, dy, and ds, these symbols played no role in the proof. And notice also that the proof is a (non-mathematical) statement about the nature of physical space (e.g., a piece of paper; a computer screen)—that is, a statement about the nature of the actual universe in which you live.

In contrast, algebra was invented by us human beings—algebra is ideas in the human mind: and although algebra can be put down on paper by using arbitrary symbols that we invent, it is not external to the mind (in contrast to the above proof of the Pythagorean theorem, which is intrinsic to the actual universe). Algebra was invented precisely because it could successfully symbolically represent statements that were known by experience to be true regarding the external universe. Of course just as is the case with words (words are sometimes called fuzzy math) algebra can also make false statements.

Now it surely seems that there could be no actual addition to our knowledge of the universe if we were to use our human-invented algebra to generate a second (this time algebraic) proof of the Pythagorean theorem—which, nonetheless, and even so, we will now do. No, we are not masochistic: it will turn out that we will be very glad indeed that we engaged in this only seemingly redundant activity:



Our new diagram is seen to be identical to the simpler of the two diagrams that we used in our first proof of the Pythagorean theorem. But this time we have more extensively labeled our diagram with algebraic symbols: letters of our alphabet. Line segments that are the same length (as can be verified by simply cutting them out, and placing them on top of each other—that being experimental physics) have been assigned the same algebraic symbol.

We can then use the human-invented rules of algebra to determine the areas: for rectangles, the area is the product of the two sides; for squares, the area is, well, the "square" of one side. And for triangles, the area is "one half of the base $\times$ the altitude"—or, more transparently, exactly half of an $a \times b$ rectangle.

Now, I will describe two ways by which we might carry out the evaluation of the area of the biggest square (that is, the area of the entire diagram): and of course each of those two ways must give exactly the same answer!

One way to calculate the area, is to note that each side of the biggest square is $(a+b)$ in length, so that the area of the biggest square must be $(a+b)^2 = a^2 + 2ab + b^2$, where we have used our human-invented knowledge of simple algebra.

The second way to calculate the area is, to first calculate the total area of the four triangles: each triangle has area $\frac{1}{2}(a \times b)$ —and then—multiply by four, giving $2ab$—and, finally; to add in the area of the big square in the middle, which is of course c^2. Our second method therefore gives us $2ab + c^2$.

Our two results must be equal, so we see that $2ab + c^2 = a^2 + 2ab + b^2$. Subtracting $2ab$ from each side of our equation leaves us with $a^2 + b^2 = c^2$, which is the Pythagorean theorem: or $dx^2 + dy^2 = ds^2$ if we were to use the symbols that were present in our first diagram.

There seems to be NO gain from our second proof—it merely tells us what we already knew from inspection. So why did we bother with it at all? We bothered with it because we can use it to open powerful and totally new doors on our grasp of that universe in which all of us find ourselves immersed! That is, it will—but only if we are smart enough to find those doors! Sadly, accomplishing this took a very long time, simply because we humans are just NOT that all smart, as I will now demonstrate!

3. Einstein

Some decades before Einstein, the great mathematical physicist William Rowan Hamilton had a truly wonderful idea—which he blew completely! I warned you that we humans are not very smart. Hamilton speculated that time is a fourth dimension! He knew the Pythagorean theorem for three dimensions, $ds^2 = dx^2 + dy^2 + dz^2$, and so he wrote it down again, but this time for four dimensions:

$$ds^2 = dx^2 + dy^2 + dz^2 + dt^2$$

I have used the symbol dt for that fourth dimension simply because the word time starts with a t and time is what the man was after. But Hamilton was certainly smart enough to know that, as we have written it, this Pythagorean equation does NOT really include time: his additional dimension was really just the postulating of one more dimension of space—an additional dimension that clearly does not actually exist.

Hamilton agonized over this (he even wrote a poem about it)—and then gave up!

More than forty years passed.

Then in 1905 the young Albert Einstein made what turned out to be the most significant discovery about the universe that has ever been made: his theory of special relativity. He made the discovery by noticing that while physicists had one mathematical description for certain electrical/magnetic experiments if the equipment involved was stationary and so was the experimenter—they had a quite different mathematical description of exactly the same experiments if the experiments were, instead, inspected by an observer who was in motion. This, quite rightly, did not seem proper to him—and so Einstein worked until he found a single unified description. His new description required a novel and seemingly somewhat complex relationship between the three coordinates of space, and the one coordinate of time.

That sounds complicated, and indeed, as put forward by Einstein, it was. But then in 1908 Einstein's former teacher, Hermann Minkowski, made the greatest discovery ever in the history of the human race: that Einstein's new theory really was simply that in our universe,

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2$$

... and William Rowan Hamilton rolled over in his grave!

4. Newton: We will need his mv, and his $\frac{1}{2}mv^2$

Isaac Newton was well aware of the Pythagorean theorem (and moreover, he knew that light from the sun takes "seven or eight minutes" to travel the 93,000,000 miles to Earth—and so Newton fully realized the enormity of the speed of light). But, just like the later Hamilton, Newton failed to guess the trivial extension to the familiar Pythagorean theorem that successfully ropes in time as a fourth dimension: Newton instead postulated what he called "absolute time," having no connection with space.

If either Newton or Hamilton had given the ancient Pythagorean idea just the tiniest bit of imaginative exploration you would, today, likely never have heard of Albert Einstein.

Newton's most famous accomplishment—and a great one indeed—was his law of gravitation, and I will briefly consider gravitation in due course. But, Newton's even more important accomplishment was his identification, for masses that are in motion, of their momentum $p = mv$, and of their kinetic energy $E = \frac{1}{2}mv^2$, as being two quantities that are conserved. Those expressions—together with Newton's $F = ma$ —form the foundation of the classical mechanics that has facilitated our industrial civilization.

Now! We do want to keep our eye on the ball!

What we are trying to do in this paper, is simply to deduce that

$$\text{Pythagoras} \rightarrow \quad ds^2 = dx^2 + dy^2 + dz^2 - dt^2 \qquad \text{leads to} \qquad E = mc^2 \quad \leftarrow \text{Einstein}$$

But in order to succeed in doing that, we are going to need to spot Newton's two conserved quantities, mv and $\frac{1}{2}mv^2$, hiding inside the simple algebraic development, from Minkowski's extension of the Pythagorean theorem, that we are just about to embark on. And, in succeeding in doing this, we will also discover—with Albert Einstein—that Newton was wrong about the actual expressions for each of his two conserved quantities! Only a tiny bit wrong, or they wouldn't have functioned as superbly well as they did (and as they still do today for our engineers)—but, nonetheless: wrong.

So let us commence!

5. You, and your sister redhead

To understand the implications of $ds^2 = dx^2 + dy^2 + dz^2 - dt^2$ we will need to set up a coordinate system, and we must, somehow, introduce something that changes with time.

Suppose that you are stationary—and that you erect a rectangular coordinate system (x, y, z, t) —naturally, putting yourself at the origin. Next, suppose that you spot your twin sister redhead moving at constant velocity v —along your x axis. Redhead now erects her own (of course moving, with her, at v) rectangular coordinate system (x', y', z', t'), choosing her x' axis to lie atop your x axis; and, just as you did in your coordinate system, placing herself permanently at its origin.

Redhead snaps her fingers twice as she glides—and you and she each record the locations and the times of each of those two events—those two finger-snaps of hers.

It is on those **two events** that we will focus our attention.

Redhead finds it very easy indeed to note down those two times, because both events of course happen right where she is—and she's always at exactly at $x' = 0$ in her own moving coordinate system—so she just has to glance at her watch! But it is much more difficult for you—you must enlist two assistants, both of them located on the x axis: one assistant at each of the two points, x_1 and x_2, in your coordinate system, where redhead's two finger snaps occur. Your assistants can each give redhead a wink, if they like, as she sails by and they and she both jot down the time—everyone of course using their own clocks. Your two assistants, who—just like you—are stationary, write down (and later report back to you) that redhead's two finger snaps occurred at

$(x_1, 0, 0, t_1)$ and $(x_2, 0, 0, t_2)$ in your (and their) coordinate system, while redhead herself wrote down that they occurred in her coordinates system at $(0, 0, 0, t'_1)$ and at $(0, 0, 0, t'_2)$—perhaps redhead even handed each of your two assistants a note, reporting her times, as she sailed by!

The changes dx and dt (between the two finger snaps) that you calculate from your two assistants' notes are $dx = x_2 - x_1$ and $dt = t_2 - t_1$. The dx' and dt' that redhead (moving at v) finds are $dx' = x'_2 - x'_1 = 0 - 0 = 0$ (of course) and $dt' = t'_2 - t'_1 \equiv d\tau$.

$d\tau$ **will be key!** Also—redhead is forever at $x' = 0$ in her own moving frame—which is why I wrote that $dx' = 0$.

Now, for each of you, the two finger snaps occur right on your x axis, and so: all of $dy = y_2 - y_1 = 0 - 0 = 0$ and $dy' = y'_2 - y'_1 = 0 - 0 = 0$ and $dz = z_2 - z_1 = 0 - 0 = 0$ and $dz' = z'_2 - z'_1 = 0 - 0 = 0$.

Finally! I will write down Einstein's relativity claim (in the form that Einstein's theory had been, ever so elegantly, restated by Minkowski in 1908): the claim was that

$$\boxed{dx^2 + dy^2 + dz^2 - dt^2 = ds^2 = dx'^2 + dy'^2 + dz'^2 - dt'^2}$$

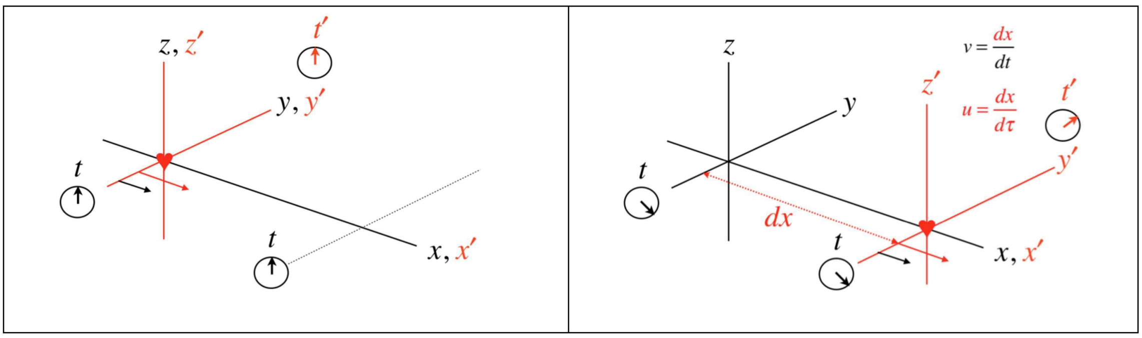
That is, the claim was that the square of the ds that separates (in what we will call spacetime) the two "finger-snap" events—for you and for redhead—*is shared*—that for our two finger snaps,

$$dx^2 + 0 + 0 - dt^2 = 0 + 0 + 0 - d\tau^2$$

Tidying (and rearranging) our equation, we now have the heart of relativity—that for someone who is moving with respect to you with velocity v,

$$\boxed{d\tau^2 = dt^2 - dx^2}$$

Einstein had found the value of $d\tau$ compared with dt ! Newton—it turns out quite incorrectly—had claimed time to be universal! Below are pictures of what happened: the first frame gives the situation at redhead's first finger-snap ♥ — the other, the situation at redhead's second finger snap ♥, after having moved by dx. Notice that Redhead has two velocities—v, and u.



Since $d\tau^2 = dt^2 - dx^2$, $d\tau$ is always less than dt —and thus velocity u is always greater than velocity v. That is why the red arrow is longer than the black arrow in our diagram.

I very deliberately went through the equations before showing you those two diagrams. It is often said that a picture is worth a thousand words—well, equations are worth infinitely more than pictures, if the pictures (such as the two above) involve time. For such pictures can be much more distorted than is a Mercator-projection map of the Earth. In contrast, equations involving time (we have discovered) can proclaim exact and verifiable truth—and the fundamental equation $d\tau^2 = dt^2 - dx^2$ that we have just deduced has been thoroughly experimentally tested, and it has been found to be true: time would actually slow down for your moving sister redhead. Because of that slowing down, the trip, as it is clocked by redhead, is recorded by her as taking less time than your (stationary) measurement of that same time interval: the black arrow in the diagram represents her speed according to your stationary clocks; while the red arrow, which is longer, represents her higher speed according to redhead's own wristwatch.

6. Experimental test

We can by experiment test whether it is true that time actually does slow down for that moving sister of yours: and we have tested it—with complete vindication for Einstein's idea! The "redhead" of our test is particles called muons that, every day, are created high in the Earth's atmosphere by the impact of extremely high-energy cosmic rays (originating far out in our galaxy) on the particles of the upper air, some tens of kilometers above the Earth's surface. Those newly created muons hurtle down towards Earth, where we are able to detect their arrival.

Now, we have also created muons ourselves, in our laboratories—muons that not only don't hurtle, they are, comparatively speaking, barely moving at all—and we find that these (nearly stationary) muons decay into electrons in (on average) 2.2×10^{-6} seconds. Those highly-energetic newly-created upper-atmosphere muons head down towards the surface of the Earth at speeds about 0.995 of the speed of light—and light goes at 299,792,458 meters per second. Now ask yourself: how far can you go in 2.2×10^{-6} seconds, if you are moving at 0.995 times $299,792,458$ meters per second? Answer: a typical muon would only go $d = vt = 0.995 \times 299,792,458 \times 2.2 \times 10^{-6} = 656.2$ meters—that's less than one kilometer—before transforming into an electron! The muons simply could not make it to the Earth's surface, to be detected, if they decayed in the same time that the slow-moving muons in the laboratory are actually observed to decay.

But in fact, the muons that were created tens of kilometers above the Earth's surface really do make it down to the Earth's surface before they have changed into electrons. Time has slowed down for them! So! Case is closed! Einstein was right! Time really does slow down for those muons—and in that same way, time would slow down for our redhead.

7. Uh oh?

I have made a point of—hard and fast—bringing out the actual experimentally-established reality of time dilation, because what I am about to do next is to attack the whole notion of time dilation as (seemingly) crazy—and I can do that in simple and convincing fashion. For consider the idea of your switching roles with redhead: as we have presented the circumstances, we had you consider yourself to be stationary while you saw redhead moving at speed v. But of course from redhead's point of view, it is she who is stationary, and it is YOU who are moving (albeit in the opposite direction) at speed v. And, following our logic above, redhead would conclude, from our Pythagorean hypothesis, that it is for YOU that time slows down, and not for her!

Uh-oh indeed!

The answer is very simple: you are both right.

But how can you both be right?

Here's how:

In the scenario that it is twin redhead who is moving, if she were to stop, turn around, and come back to you at the same speed v, she would be younger than you when she arrives.

But if, instead, we look at the scene from redhead's point of view, and if, after you have left her (moving in the opposite direction at speed v), it is YOU who stop, YOU who turn around, and YOU who come back to HER at speed v, it will be YOU who will be the younger of the two.

It is all relative. That's why it is called relativity. But it is real; it actually happens—we KNOW that: please never forget those muons! Relativity is in fact the best-tested physics theory ever.

Also, remember that you are on Earth, while redhead is in a small spaceship. No way you are going to make the Earth scoot off in the other direction!

It is not the physical acts of stopping, and then restarting in the opposite direction, that produces these results, it is simply the adoption of the different reference frames that is responsible for what we find. The universe is a house of mirrors!

(Perhaps it is possible that William Rowan Hamilton, in his consideration of time as a possible fourth dimension, did try our minus sign—but that he—maybe—then rejected the idea because of the supposed problem that we have just discussed? —for Hamilton of course did not know about those muons.)

Relativity is nothing but extremely simple (if peculiar) geometry—it is not physics (and in particular, it is not any involvement of acceleration). It is simply that the geometry of spacetime is a geometry that our human brains, which were produced—just as were the brains of cows and sheep—by evolution, simply cannot correctly visualize (and that is why I really don't like diagrams), but that (thank God) we can nevertheless faultlessly handle, by using .... our human-invented algebraic equations! Bizarre, but true—that is the most amazing thing that I know!

Yes, OK, this is exciting—but wait: there's more!

8. Something huge: a limit on possible velocities: $v \leq 1$

We have successfully obtained (and, with muon decay, we have also experimentally verified) our key result: that for relative velocity v, it is a fact that $d\tau^2 = dt^2 - dx^2$. And now this will give us a new and truly astounding fact about our universe: that there is a limiting velocity that can never be exceeded—because we will now discover that there simply IS NO higher velocity (in much the same sense that you can't go north of the north pole). And this result will appear with the greatest of ease, by just our asking: for you and redhead, what is the value of dx?

Just glance at our two diagrams! Distance, of course, is velocity times time, and you record redhead as going a distance dx in your time dt with velocity v, so: $dx = v\,dt$, and therefore

$$d\tau^2 = dt^2 - dx^2 = dt^2 - v^2 dt^2 = (1 - v^2)\,dt^2$$

But $d\tau^2$ is necessarily positive (and that is true also for dt^2) and so v can never be greater than 1, because if it were, our equation would say that a positive number is equal to a negative number: which is not true. We have discovered that if Minkowski is right (and we know from a multitude of tests—it's not just those muons—that Minkowski and Einstein are indeed right) then there must be a limiting velocity: nothing can move faster than 1. One? Yes, **1** !

The speed of light is, of course, **1** light year per year. We have never found anything that moves faster than light does—and it's not for want of trying: we have built many particle accelerators and pumped huge energies into particles to force them to go faster and faster, and yet, when we measure their speed, the fastest speed that they ever achieve is close to, but always just the tiniest bit short of, 1 light year per year.

Let's put light speed into perspective by asking how fast your own car is really going if you drive it at, say, 60 miles per hour (our everyday—well, American—units for velocity). Let's work it out! The number 299,792.458 kilometers per second is the exact speed of light. Here's how we convert 60 *mph* to mother nature's units (which are light years per year):

$$60\,\frac{miles}{hour}\times\frac{hour}{3600\;s}\times\frac{1.609344\;km}{mile}\times\frac{s}{299,792.458\,km}=0.000,000,0894\,\frac{light\;years}{year}$$

So, your car goes 89.4 billionths of a light year per year. Notice that all units cancel in our equation: speed has no units. Except for cosmic rays, the fastest material speed we have measured is your own personal speed (as well as mine) at this very moment, as the sun and Earth move through the cosmic microwave background radiation left over from the big bang: we move at speed 0.00123—which is very fast, but which is still peanuts compared with 1, the speed of light.

9. Finding Newton hiding inside Pythagoras

Probing into $d\tau^2 = dt^2 - dx^2$ we will now easily locate Newton's mv and $\frac{1}{2}mv^2$

First step: $1 = (\frac{dt}{d\tau})^2 - (\frac{dx}{d\tau})^2$ To you, redhead's velocity is $v = \frac{dx}{dt}$

But to redhead herself, her velocity is $u = \frac{dx}{d\tau}$ giving us $u = \frac{dx}{dt}\cdot\frac{dt}{d\tau} = v\,\frac{dt}{d\tau} \equiv v\gamma$

So $1 = (\frac{dt}{d\tau})^2 - v^2(\frac{dt}{d\tau})^2 = (\frac{dt}{d\tau})^2(1-v^2)$ and so $\frac{dt}{d\tau} = \gamma = \sqrt{\frac{1}{1-v^2}}$ and so

$$\boxed{u = \gamma\, v} \quad \text{and also} \quad \boxed{1 = \gamma^2 - u^2}$$

Redhead's velocity $u = \gamma\, v$ can be faster than light: if you were to supply redhead with enough energy, she could cross our galaxy in 30 years or less! (But—you would fry redhead, with hits from the thin interstellar gas, if you really did send her off traveling at so close to the speed of light). You—and your descendants—watching her travel at almost $v = 1$, would, in contrast to her own experience, record that redhead's trip took about 100,000 years.)

Our last equation before the boxes can be reorganized to read: $d\tau = dt\sqrt{1-v^2}$ —which predicts drastic consequences for photons of light—because photons move at $v = 1$! So, for photons, $d\tau = 0$. Thus, the time that a photon measures, from its coming into existence—say on the Sun—until that photon's ceasing to exist—for example, by depositing its energy in your retina—is zero!

Photons.never.exist! Just ask any photon!

The key to our smoking out of Newton's two conserved quantities that are tucked away inside $d\tau^2 = dt^2 - dx^2$ is $1 = \gamma^2 - u^2$. But to get there, we are going to have to appeal to Isaac Newton—not to Newton the great physicist, but to Newton the great mathematician.

We first note that when $v = 0$, $\gamma = \frac{dt}{d\tau} = \sqrt{\frac{1}{1-v^2}} = 1$. And we can easily see that as v grows from 0, γ grows as well—at first very slowly, but when v gets greater than about 0.5, the growth greatly quickens, and when $v = 1$, obviously γ becomes infinite. In our daily life, of course, γ departs but little from one—indeed, in daily life, γ is almost exactly one: recall that your 60 *mph* car goes only 89.4 billionths of a light year per year. In fact all of our human bustle is excruciatingly slow!

We are going to need an approximation to γ for those daily-life small values of v if we hope, as we do, to locate, in our equation, Newton's (slightly-incorrect) classical mechanics. Well, Newton himself, bless him, discovered—and he proved—a powerful general procedure for obtaining exactly such an approximation. We will just use his result—we won't prove it, but we'll see also that we don't actually need to prove it, either. Application of Newton's procedure gives

$$\gamma = \frac{dt}{d\tau} = \sqrt{\frac{1}{1-v^2}} \quad (= 1 + \frac{1}{2}v^2 + \frac{3}{8}v^4 + \cdots \; Newton)$$

What we will do is simply check whether this is a really good approximation for the crucial case of relatively small values of v : your pocket calculator will tell you that for the speed $v = 0.1$ (that is, for v = one tenth the speed of light) the exact value of γ is 1.005037815259212... while you can also easily (this time mentally) check that γ = 1.005 if you use (with $v = 0.1$) just the first two terms of Newton's approximation. So indeed, our approximation is—by test—an excellent one, even for speeds far faster than those we normally encounter in daily life.

(Using Newton's series expansion is the only piece of mathematics in this entire paper that is not just simple, and utterly elementary, high school algebra. And in any case, we have just tested Newton's mathematics pragmatically, and found out that it works—which is all that actually matters for the physics.)

Our very first step in this section was to note that $1 = \frac{dt}{d\tau}^2 - \frac{dx}{d\tau}^2 = \gamma^2 - u^2$.

So we are able, now, using Newton's series expansion, to state that, for material objects in motion, it is a fact that $1 = (1 + \frac{1}{2}v^2 + \frac{3}{8}v^4 + \cdots)^2 - (\gamma v)^2$

We are done! To find Newton's two conserved quantities hiding inside our expression, all we need to do, now, is to multiply our equation through by the square of redhead's mass m (or, depending on what interests you, we can have m represent redhead's mass plus the mass of her spaceship) giving us

$$\boxed{m^2 = (m + \frac{1}{2}mv^2 + \frac{3}{8}mv^4 + \cdots)^2 - (\gamma mv)^2}$$

I hope you have spotted Newton's kinetic energy $E = \frac{1}{2}mv^2$ and momentum $p = mv$?

Our last equation is a triumph for Albert Einstein, for it interconnects Newton's expressions for energy and momentum, and, remarkably, it also reveals both of them to have been—as I warned you!—not quite correct! Newton had said that energy was $\frac{1}{2}mv^2$—Einstein reveals energy, actually, to be:

$$E = m + \frac{1}{2}mv^2 + \frac{3}{8}mv^4 + \cdots$$

Newton had said momentum was mv—Einstein reveals momentum, actually, to be:

$$p = \gamma mv = \frac{mv}{\sqrt{1-v^2}}$$

So why do we trust the young Einstein on this, rather than sticking with the great Newton? Remember those muons! It is experiment that renders the ultimate judgement.

In looking at Einstein's expression for the energy E, keep in mind that v is a tiny number, and so v^2 is super-tiny, so v^4 is utterly negligible compared with v^2 —therefore you can almost always ignore that third term in the expression for E. (The third term, and all subsequent terms, only kick in when you have particles that are moving close to the speed of light. In such instances, the presence of those additional terms is why the kinetic energy of such particles can be increased indefinitely—even though the actual speed of the particles no longer increases at all.) Particularly notice that the ratio of the first term m to the second term is $(\frac{m}{\frac{1}{2}mv^2}) = \frac{2}{v^2} =$ *always* a very large number indeed, since v is so very, very, small! So the first term—the particle's mass m —always dominates the energy, unless (as is the case for the photon) $m = 0$ — in such cases our present discussion is not applicable, because v is in such cases never small compared with 1.

10. $\mathbf{E = mc^2}$

If we apply Einstein's $E = m + \frac{1}{2}mv^2 + \frac{3}{8}mv^4 + \cdots$ to a stationary body, $v = 0$ and so we get $E = m$. So far so good! But, for political purposes just concluding that $E = m$ won't cut the mustard! The clamor of the mob, of course, is not for $E = m$, but for $E = mc^2$! And so far, we haven't even mentioned c! Could we get away with just stating, quite correctly, that $c = 1$?

No! Sigh! We must jam the conventional $c \equiv 299,792,458$ meters per second into our equation.

Sure it will be ugly—but politically—hey, it must be done! So, here goes—instead of

$$m^2 = (m + \frac{1}{2}mv^2 + \frac{3}{8}mv^4 + \cdots)^2 - (\gamma mv)^2$$

we will now, holding our noses, write

$$m^2 = [m + \frac{1}{2}m(\frac{v}{c})^2 + \frac{3}{8}m(\frac{v}{c})^4 + \cdots]^2 - [\gamma m(\frac{v}{c})]^2$$

Notice that I have inserted a c only where there was a v, and nowhere else. Before I committed this sacrilege, v ranged from 0 to 1—which is what had emerged both naturally and beautifully from Minkowski's great discovery! And remember that that was our truly great, and truly extraordinary discovery, the biggest discovery in the history of physics: that there is a mathematical limit, 1, to velocity! But now we do the opposite of putting lipstick on a pig; we hide and we disguise our great discovery and we deliberately make it look nasty, weird, and arbitrary: because meters correspond to scratches on a platinum bar in Paris; while seconds correspond to a certain arbitrarily chosen convenient atomic transition: taken together they make light travel, not at 1, which it really does, but at 299,792,458 meters per second—and we call that c, just as if we were saying something—which we really are not!

Shame on the obfuscating physicists!

Next, we polish up this travesty: first, we multiply the scarred equation by c^2 :

$$m^2c^2 = \frac{c^4}{c^2}[m + \frac{1}{2}m(\frac{v}{c})^2 + 3/8m(\frac{v}{c})^4 + \cdots]^2 - c^2[\gamma m(\frac{v}{c})]^2$$

, and now we tidy it up:

$$\boxed{m^2c^2 = \frac{1}{c^2}(mc^2 + \frac{1}{2}mv^2 + \frac{3}{8}m(\frac{v^4}{c^2}) + \cdots)^2 - (\gamma mv)^2 \equiv \frac{E^2}{c^2} - p^2}$$

If we now consider the important special case that $v = 0$, then $p = mv = 0$ and we get, at long last, $m^2c^2 = \frac{E^2}{c^2} - 0$ or $E = mc^2$.

Bit of an anticlimax, that!

11. The bomb

Your students do also need to know what our equation $E = m$ actually means.

In 1905 Einstein published his discovery of relativity—and then, in a two-page paper later that same year, Einstein published his famous deduction that $E = mc^2$. (Yes—Einstein included c^2.) He ended that most famous of all his papers by pointing to radium salts as perhaps providing a test of his theory. Well, just forty years later, the World War II application of Einstein's discovery led to the destruction of Hiroshima and Nagasaki through the military application of one single experimental fact: that if a ^{235}U atomic nucleus is struck by a neutron, n, the result is fragments: Krypton and Barium nuclei, which emerge with enormous energy $E = mc^2$ (plus, also emerging, three more neutrons—which can go on to hit other Uranium nuclei). Here it is in detail:

$$n + {}^{235}U \rightarrow {}^{236}U \rightarrow {}^{92}Kr + {}^{141}Ba + \langle E = [m_U - (m_{Kr} + m_{Ba})] \times c^2\rangle + (3n \text{ out} \rightarrow)$$

Those "neutrons-out" can create a chain reaction, making possible the atomic bomb. (That the sum of the masses of the Kr and Ba nuclei is less than the mass of a Uranium nucleus, was simply a well-established, experimentally-measured, fact.)

12. And now—more important matters than relativity

Much more important than $E = mc^2$ — and more important, even, than our utterly astonishing discovery that there is in our universe a limiting velocity **1** that cannot be exceeded—is how it is that we have made these remarkable discoveries!

For consider: from only **a**) the Pythagorean theorem (ancient, and provable by inspection), **b**) our expansion of that theorem (in an unintuitive way—that minus sign) to include a number to represent time, and then most critically, **c**) the use of human-invented algebra to manipulate the theorem, and, finally, **d**) multiplication by a number m representing mass)—we have successfully predicted the atomic bomb.

Please read that again! How the hell could that conceivably be possible? For goodness sakes, if we could have tipped off Pythagoras, Carthage could have been nuked!

Wigner (1960) [1] said "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve." (Nor, I might add—in too many cases—even appreciate!)

And it does not end there: let me add to these wonders, the recent detection of gravitational waves from the merging of two black holes more than a billion years ago! How did we humans accomplish that great discovery? Well, just one hundred years before that incredible 2016 discovery (a discovery made using gravitational wave detectors located in Louisiana and Washington) Albert Einstein had extended his relativity theory to include non-constant motion: that is, to include acceleration—which is to say, to include force ($F = ma$).

Unlike special relativity, general relativity is intellectually easy (if extremely mathy) and involves, apart from a factor of 8π that pops up—for no known reason!—no independent intellectual surprises: obviously if spacetime is curved, planets will move on curved paths, as they are observed to do. The only question is, does it work: does it reproduce or even improve on Newton's law of gravitation? Well, it did, and it does. I could easily lead the reader through the details, and it is an exciting story, but that would be a distraction from the main issue, which is: how can it be that such simple (indeed, almost trivial) algebra can possibly lead to a vastly better grasp by us as to how this great universe surrounding us really works? Especially considering that it is we ourselves who invented the algebra? For there certainly was no algebra, at least in this neck of the universe, in the time of the dinosaurs!

Some insight into what is going on is provided by quantum mechanics.

13. Quantum mechanics

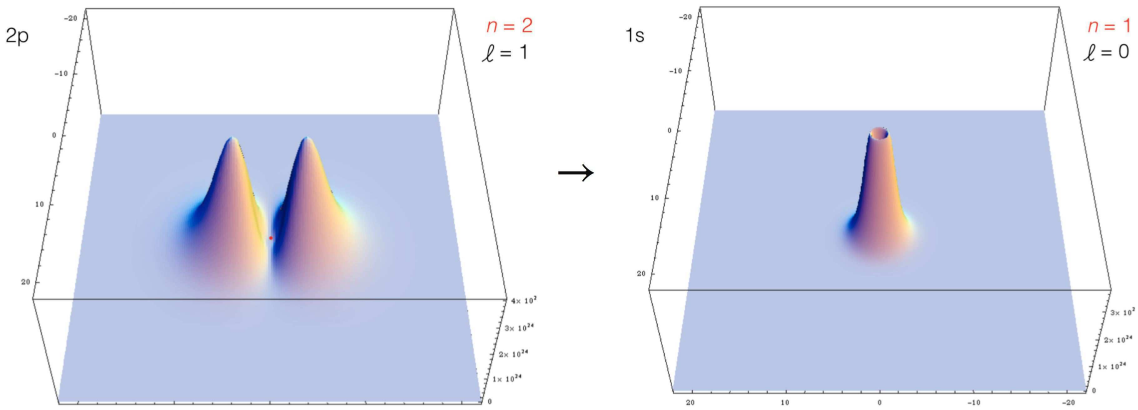
In 2005 in the magazine Nature I published an essay entitled "The mental universe," [2] in which I said, "The 1925 discovery of quantum mechanics solved the problem of the Universe's nature. Bright physicists were again led to believe the unbelievable—this time, that the Universe is mental." The bright physicists that I had in mind were Sir James Jeans and, above all, Arthur Stanley Eddington—whose 1928 book "The Nature of the Physical World" had made me aware, even as a teenager, that with the universe, all is not what it seems. As a professor for decades teaching physics at the Johns Hopkins University, I finally was able to deduce for myself that Eddington's view was correct.

I first got on top of special relativity—as the reader herself now is. Then general relativity: I had the good luck to be a fortran programmer when Mathematica appeared, and so I could automate the horrendous algebra involved, with the result that I was the one who found the value of the Kretschmann scalar K for the most general possible black hole—K being a number specifying the amount of the curvature of spacetime—my result appeared in Astrophysical Journal, 535, 350, 2000 [3]— please see also https://en.wikipedia.org/wiki/Kretschmann scalar).

And, finally, I cracked quantum mechanics—in "Quantum mechanics made transparent," American Journal of Physics, 58, 1087, 1990 [4]—and see also M. Shapiro 2008, J. Phys. A, 41, 17 [5]. I'm glad to say that the quantum mechanics result turned out to be boring: if observations have the character of numbers (and they do) and if simple symmetries are present in the world (and so they are) then quantum mechanics is inevitable. There is really no mystery to quantum mechanics at all! What is mysterious is the existence of our minds!

14. The Universe does not exist

Using the Socratic method of questioning, I expect that anyone who (somehow) already knew quantum mechanics could have induced Isaac Newton to discover quantum mechanics, with no experiments needed: for the fact is, the world could not be otherwise. And quantum mechanics further forces on us the realization that there can be no actual universe out there. How so? Until you make a measurement, an electron has no position in a hydrogen atom, which is really saying that there are no electrons—there are just measurements that we falsely attribute to a purely mythical "particle." Let me rub your nose in that: consider a hydrogen atom whose sole electron has been bumped up to the $n = 2$ energy level and supplied with one unit of angular momentum. After a tiny fraction of a second, that electron drops down to the lowest, $n = 1$, level (in which it has no angular momentum) with the emission of a Lyman alpha "photon" that carries away the angular momentum and the excess energy. Here are the "before" and "after" pictures:



The above figure is a movie: a movie in which, despite the fact that the electron in the first panel has angular momentum (i.e., "is orbiting"), nothing at all happens, apart from the abrupt change (to the second panel) when the Lα "photon" has departed. In each of the two frames there is only one electron: what is plotted is the square of the quantum mechanical wave function, which gives the probability that if you were to pin down the electron, you would find it at that particular spot in the hydrogen atom. It isn't that the electron's location is not known—it is that the electron does not have a location until a measurement is made. So!—so much for the idea of "particles" ! Nonexistent ! You will of course be particularly struck by the two lobes in the first panel. (The red dot in that panel shows the proton—tiny only because protons are much more massive than are electrons.)

And hey—you should see that *one* electron, in still more energetic states!

15. Conclusion

Special relativity, once understood, should, ideally, supplant the naïve Newtonian picture of spacetime that we all have had ingrained in us as we have grow up. But special relativity is difficult indeed to actually grasp, and many eminent physicists, despite their great proficiency with the mathematics, have failed to come to grips with what it is that the mathematics is actually telling us about the structure of even flat spacetime. For example, one eminent authority on relativity wrote that "the inertial mass of a particle increases with v from a minimum of m_0 at $v = 0$ to infinity as $v \to c$," and then went on, "We should not be too surprised at this, since there must be some process in nature to prevent particles from being accelerated beyond the speed of light." That physicist's initial misunderstanding came from confusion of the two velocities u and v —but what is far more shocking is the complete lack of understanding by that physicist of the fact—a fact that I have therefore heavily emphasized above—that, from algebraic geometry, there simply are no velocities that are greater than c.

Remember: ANY velocity $v \leq 1$!

Also, the mass of a fundamental particle does not increase—at all—ever.

Lewis Carroll Epstein noted that "Algebra is a wonderful invention. It enables fools to do physics without understanding." In the present paper one fool has at least valiantly striven to understand.

I conclude that the universe is an illusion: the universe is all in my mind. A dream—but a dream that is subject to conservation laws—that is, a dream having, for unknown reasons, simple symmetries present which result in those conservation laws being enforced. The fact that it is symmetries that are responsible for conservation laws was discovered by Eugene Wigner, applying the work of the great mathematician Emmy Noether.

I know from experience that in my own dreams other people sometimes appear and sometimes do things that surprise me! Perhaps in some way those people are as real as you are! But those people are evanescent, while you are not—well, er, yes: you are not ... yes indeed! ... but, you 'are not' just for some period of, well, time! You've learned a lot about time from this paper, haven't you?

Live, and get your work done!

References

[1] Wigner, E. P. 1960, The unreasonable effectiveness of mathematics in the natural sciences, Communications on Pure and Applied Mathematics, 13, 1

[2] Henry, R. C. 2005, The mental universe, Nature, 436, 29

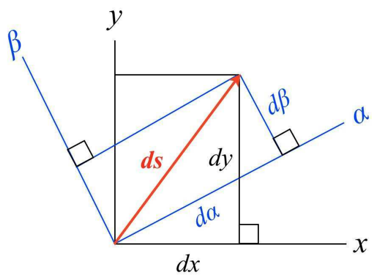
[3] Henry, R. C. 2000, Kretschmann scalar for a Kerr-Newman black hole, ApJ, 535, 350

[4] Henry, R. C. 1990, Quantum mechanics made transparent, Am. J. of Physics, 58, 1087

[5] Shapiro, M. 2008, Derivation of the relativistic "proper-time" quantum evolution equations from canonical invariance, J. Phys. A, 41, 17

16. The Appendix - Deriving $E = mc^2$

Here is an example of invariance under a two-dimensional rotation:



$$d\alpha^2 + d\beta^2 = ds^2 = dx^2 + dy^2$$

How Einstein's Theory of Relativity gives us $E = mc^2$

Suppose that you see your sister moving along your x-axis at a constant speed v , and that during her straight-line motion, your sister snaps her fingers twice, $d\tau$ seconds apart

For you, the two finger-snaps are separated in space by dx, and are separated in time by dt

the Theory: Pythagoras, Einstein, and Minkowski assert that $d\tau$ and dt are related by

Your sister $\rightarrow \quad d\alpha^2 + d\beta^2 + d\gamma^2 - d\tau^2 = ds^2 = dx^2 + dy^2 + dz^2 - dt^2 \quad \leftarrow$ **You**

$$d\tau^2 - 0^2 - 0^2 - 0^2 = dt^2 - dx^2 - 0^2 - 0^2$$

$d\tau^2 = dt^2 - dx^2 \quad \leftarrow$ **so time slows down for your sister!**

$d\tau^2 = dt^2 - dx^2$ **but** $dx = v\,dt$ **so** $d\tau^2 = (1 - v^2)\,dt^2$ **and so** $(\frac{dx}{dt} =)\, v \leq 1\,!$

$$1 = (\tfrac{dt}{d\tau})^2 - (\tfrac{dx}{d\tau})^2$$

$(\gamma \equiv)\ \frac{dt}{d\tau} = \sqrt{\frac{1}{1-v^2}}\quad (= 1 + \frac{1}{2}v^2 + \frac{3}{8}v^4 + \cdots$ **from Newton**$) \qquad \frac{dx}{d\tau} = \frac{dx}{dt}\cdot\frac{dt}{d\tau} = v\frac{dt}{d\tau} = v\gamma$

$$1 = (1 + \tfrac{1}{2}v^2 + \tfrac{3}{8}v^4 + \cdots)^2 - (\gamma v)^2$$

$1 = [1 + \frac{1}{2}(\frac{v}{c})^2 + \frac{3}{8}(\frac{v}{c})^4 + \cdots]^2 - [\gamma(\frac{v}{c})]^2 \qquad$ **where** $c = 1$ **or (by convention) 299,792,458** m/s

$m^2c^2 = \frac{1}{c^2}(mc^2 + \frac{1}{2}mv^2 + \cdots)^2 - (\gamma m v)^2 \qquad \leftarrow$ **We notice Newton's** $E = \frac{1}{2}mv^2$ **and** $p = mv\,!$

$m^2c^2 = \frac{E^2}{c^2} - p^2$ **and so actually** $E = mc^2 + \frac{1}{2}mv^2 + \cdots$ **and** $p = \gamma m v$

and if we consider the case $v = 0$, we obtain the most famous equation in human history:

$$E = mc^2$$

$$n + {}^{235}U \rightarrow {}^{236}U \rightarrow {}^{92}Kr + {}^{141}Ba + \langle E = [m_U - (m_{Kr} + m_{Ba})] \times c^2\rangle + (3n \text{ out} \rightarrow)$$