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Renormalization Issues of quasi-PDFs

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In these proceedings we present results for the renormalization of fermion bilinear operators which contain a Wilson line, to one-loop level in lattice perturbation theory. These operators are needed for the calculation of the so-called quasi-PDFs, recently proposed by X. Ji. Our calculations have been performed for a variety of formulations, including Wilson/clover fermions and a wide class of Symanzik improved gluon actions.

We focus on aspects related to the renormalization of the quasi-PDFs, which is a highly nontrivial component of their calculation. The extended nature of the Wilson-line operators results in additional divergences as compared to ultra-local currents. More precisely, there is a linear, as well as logarithmic divergence with the lattice spacing.

We demonstrate how certain operators mix in lattice regularization and we compute the finite mixing coefficients. These are necessary to disentangle individual matrix elements for each operator from lattice simulation data. Furthermore, based on our findings in the perturbative calculation, we develop a non-perturbative prescription to extract the multiplicative renormalization and mixing coefficients.

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1. Introduction

Parton distribution functions (PDFs) are important tools to study the the quark and gluon structure of hadrons. PDFs are light-cone correlation functions and, thus, they cannot be computed directly on a Euclidean lattice. Recently, a direct approach was proposed by X. Ji [1], according to which one may compute purely spatial matrix elements, the so called quasi-distribution functions (quasi-PDFs), which are accessible in Lattice QCD. For sufficiently large momenta, one can establish a connection with the physical PDFs through a matching procedure.

The extraction of the quasi-PDFs in lattice simulations involves computing matrix elements of gauge-invariant nonlocal operators, which are made up of a product of an anti-quark field at position x, a Dirac gamma structure, a path-ordered exponential of the gauge field (Wilson line) along a path joining points x and y, and a quark field at position y. The renormalization of Wilson loops was studied perturbatively, in dimensional regularization (DR) for smooth contours [2] and for contours containing singular points [3]. It was shown that smooth Wilson loops in DR are finite functions of the renormalized coupling, while the presence of cusps and self-intersections introduces logarithmically divergent multiplicative renormalization factors. It was also demonstrated that other regularization schemes are expected to lead to further renormalization factors which are linearly divergent with the dimensionful ultraviolet cutoff a.

The quasi-PDFs approach has been explored for the unpolarized, helicity and transversity cases using ensembles with pion masses at 310-375 MeV [4, 5, 6, 7]. Although these studies revealed promising results, the renormalization -a necessary ingredient- was missing, which was one of the major reasons that prohibited quantitative comparison with phenomenological data. Recently, we have completed a perturbative calculation on the renormalization of bilinear operators with a Wilson line that are related to the quasi-PDFs [8]. Among other things, this calculation revealed a finite mixing for certain operators, which appears in lattice regularization and in formulations that break chiral symmetry. These findings led to the development of a non-perturbative renormalization prescription, first proposed in Ref. [9] and refined in Ref. [10]. The prescription has recently been used for a computation of quasi-PDFs directly at the physical point [11].

The paper is organized as follows: In Section 2 we provide the theoretical setup with definitions for the lattice action, the fermion operators and the renormalization prescription. Section 3 contains our perturbative calculations in dimensional and lattice regularizations, as well as, the conversion to the $\overline{\text{MS}}$ scheme. Section 4 describes the non-perturbative renormalization scheme and presents some representative results for the helicity operator. Finally, in Section 4 we summarize our results.

2. Theoretical Setup

In the perturbative calculation we consider the clover fermion action [12]

$$\begin{aligned} S_F =& -\frac{a^3}{2}\sum_{x,f,\mu}\Big[\bar\psi_f(x)\left(r-\gamma_\mu\right)U_{x,x+a\mu}\,\psi_f(x+a\mu)+\bar\psi_f(x+a\mu)\left(r+\gamma_\mu\right)U_{x+a\mu,x}\,\psi_f(x)\Big] \\ &+a^4\sum_{x,f}\Big(\frac{4r}{a}+m_0^f\Big)\bar\psi_f(x)\psi_f(x)-\frac{a^5}{4}\sum_{x,f,\mu,\nu}c_{\rm SW}\,\bar\psi_f(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi_f(x)\,, \end{aligned} \tag{2.1}$$

PoS(QCDEV2017)020

where r is the Wilson parameter (henceforth set to 1), f is a flavor index, $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2$ and $F_{\mu\nu}$ is the standard clover discretization of the gluon field tensor. The Lagrangian masses , m_0^f , are set to zero, which is their critical value for a one-loop calculation. This choice may be used for mass-independent renormalization schemes and simplifies significantly the algebraic expressions. However, a special treatment of potential IR singularities is required.

For gluons we use a family of Symanzik improved actions [13]:

$$S_G = \frac{2}{g_0^2}\Big[c_0 \sum_{\text{plaq.}} \text{Re}\,\text{Tr}\{1 - U_{\text{plaq.}}\} + c_1 \sum_{\text{rect.}} \text{Re}\,\text{Tr}\{1 - U_{\text{rect.}}\}$$
$$+ c_2 \sum_{\text{chair}} \text{Re}\,\text{Tr}\{1 - U_{\text{chair}}\} + c_3 \sum_{\text{paral.}} \text{Re}\,\text{Tr}\{1 - U_{\text{paral.}}\}\Big]. \qquad (2.2)$$

with several values of the Symanzik coefficients c_i. Here we only present results for the Wilson, tree-level Symanzik-improved and Iwasaki actions.

In this work we study a complete set of bilinear operators:

$$\mathscr{O}_\Gamma \equiv \overline{\psi}(x)\,\Gamma\,\mathscr{P}\,e^{i\,g\,\int_0^z A_\mu(x+\zeta\hat{\mu})d\zeta}\,\psi(x+z\hat{\mu})\,, \qquad (2.3)$$

in which a Wilson line of length z inserted between the fermion fields to ensure gauge invariance. Although in the limit $z \to 0$, Eq. (2.3) gives the ultra-local fermion operators, our calculation of the Green's functions is for strictly $z \neq 0$. This is due to the fact that the appearance of contact terms beyond tree level renders the limit $z \to 0$ nonanalytic.

We consider only a straight Wilson line along any one of the four perpendicular directions, which we conventionally choose to be μ=1, (identified with the z direction). We perform our calculation for all independent combinations of Dirac matrices, Γ, that is:

$$\Gamma = \hat{1}, \quad \gamma^5, \quad \gamma^\nu, \quad \gamma^5\gamma^\nu, \quad \gamma^5\sigma^{\nu\rho}, \quad \sigma^{\nu\rho}. \qquad (2.4)$$

In the above, $\rho \neq \mu$ and we distinguish between the cases in which the index ν is in the same direction as the Wilson line ($\nu = \mu$), or perpendicular to the Wilson line ($\nu \neq \mu$). For convenience, the 16 possible choices of Γ are separated into 8 subgroups, defined in Ref. [8].

3. Perturbative Renormalization

The one-loop calculation involves the Feynman diagrams shown in Fig. 1, where the filled rectangle represents a nonlocal operator with a Wilson line of length z. These diagrams will appear in our calculations in both in Lattice (LR) and in Dimensional (DR) regularizations.

3.1 Dimensional Regularization

We perform a computation in D Euclidean dimensions, where $D = 4-2\varepsilon$ and ε is the regularizing parameter. The latter appears in the bare Green's functions as a Laurent series of the form $\sum_{i=-n}^{\infty} c_i\,\varepsilon^i$, where n is the order on perturbation theory. The renormalization must eliminate all poles in ε, before the limit $D \to 4$ can be taken. The most convenient renormalization scheme is the $\overline{\text{MS}}$, according to which the renormalization functions (RFs) are defined to only remove poles

PoS(QCDEV2017)020

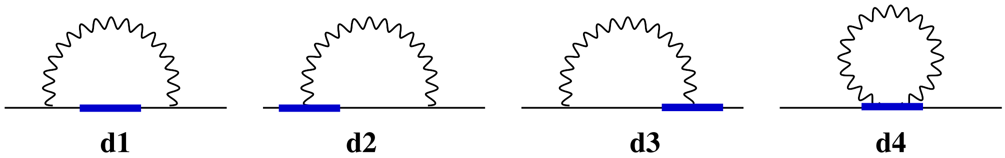

Figure 1: Feynman diagrams of the one-loop calculation of the Green's functions of $\mathcal{O}_\Gamma$. The straight (wavy) lines represent fermions (gluons). The operator insertion is denoted by a filled rectangle.

in ε. However, an RI-type scheme is more suitable for non-perturbative calculations, and an appropriate conversion factor is necessary in order to obtain the $\overline{\text{MS}}$. In this Section we present our results for the RFs in the $\overline{\text{MS}}$ scheme, and provide the conversion factor between an RI-type and $\overline{\text{MS}}$.

3.1.1 Renormalization Functions

The clover and gauge fixing parameters have been kept general throughout the calculation. This allows one to adopt the results for Wilson-type actions with and without a clover term. In addition, we were able to confirm the cancellation of the gauge dependence. Note that we define the gauge fixing parameter β, such that β=0(1) corresponds to the Feynman (Landau) gauge. We find that $1/\varepsilon$ terms arise only from diagrams d2, d3 and d4, giving a total contribution of

$$\langle \psi\, \mathcal{O}_\Gamma\, \bar{\psi}\rangle^{DR}\Big|_{1/\varepsilon} = g^2\, \lambda_\Gamma\, \Gamma\, e^{i q_\mu z}\,, \qquad \lambda_\Gamma = \frac{C_f}{16\pi^2}\,\frac{1}{\varepsilon}\,(4-\beta)\,, \qquad C_f \equiv \frac{N^2-1}{2N}. \tag{3.1}$$

To one-loop level in DR the pole parts are multiples of the tree-level values, $\Gamma e^{i q_\mu z}$, which indicates no mixing between operators of equal or lower dimension. Another feature of the $\mathcal{O}(g^2)/\varepsilon$ contributions is that they are operator independent, in terms of both the Dirac structure and the length of the Wilson line, z. Eq. (3.1) is combined with the renormalization function of the fermion field, Z_ψ, [14], leading to a gauge independent renormalization function for the operators of Eq. (2.3):

$$Z_\Gamma^{DR,\overline{\text{MS}}} = 1 + \frac{3}{\varepsilon}\,\frac{g^2\, C_f}{16\pi^2}\,. \tag{3.2}$$

More details on the extraction of $Z_\Gamma^{DR,\overline{\text{MS}}}$ and discussion of the DR results appears in Ref. [8].

3.1.2 Conversion to $\overline{\text{MS}}$ scheme

Continuum perturbation theory is particularly useful for the computation of the conversion factors between different renormalization schemes. With the perspective of developing a non-perturbative renormalization for the RFs, we employ an RI$'$ scheme that is used for the ultra-local fermion bilinear operators:

$$Z_q^{-1}\, Z_{\mathcal{O}}(z)\,\frac{1}{12}\text{Tr}\left[\mathcal{V}(p,z)\left(\mathcal{V}^{\text{Born}}(p,z)\right)^{-1}\right]\Big|_{p^2=\bar{\mu}_0^2} = 1\,, \tag{3.3}$$

where Z_q is the renormalization function of the quark field obtained via

$$Z_q = \frac{1}{12}\text{Tr}\left[(S(p))^{-1}\, S^{\text{Born}}(p)\right]\Big|_{p^2=\bar{\mu}_0^2}\,. \tag{3.4}$$

PoS(QCDEV2017)020

S is the bare quark propagator and S^{tree} is its tree-level value; the one-loop computation of S can be found, e.g., in Ref. [15].

The conversion factors between the RI$'$ and $\overline{\text{MS}}$ schemes is then given by:

$$\mathscr{C}^{\overline{\text{MS}},\text{RI}'} = (Z^{DR,\overline{\text{MS}}})^{-1} \cdot (Z^{DR,\text{RI}'}) = 1 + g^2\, z_\Gamma^{DR,\text{RI}'} - g^2 z_\Gamma^{DR,\overline{\text{MS}}} + \mathscr{O}(g^4)\,. \tag{3.5}$$

The quantities $z_\Gamma^{DR,\text{RI}'}$ ($z_\Gamma^{DR,\overline{\text{MS}}}$) are the 1-loop results of the RFs in the RI ($\overline{\text{MS}}$) scheme, which we computed for all operators shown in Eqs. (2.4). The conversion factor is the same for each of the following pairs of operators: Scalar and pseudoscalar, vector and axial, as well as for the tensor with and without γ^5, as we adopt the prescription in which γ_5 has only 4 dimensions. The general expression for $\mathscr{C}_\Gamma$ are shown in Ref. [8] for general gauge fixing parameter. They are expressed compactly in terms of the quantities $F_1(\bar{q},z)$ - $F_5(\bar{q},z)$ and $G_1(\bar{q},z)$ - $G_5(\bar{q},z)$, which are integrals over modified Bessel functions of the second kind, K_n. These integrals are presented in Appendix A of Ref. [8]. Here we only present the conversion factor for the unpolarized and helicity cases, with the index μ in the same direction as the Wilson line ($V_1(A_1)$):

$$\begin{aligned}
\mathscr{C}_{V_1(A_1)} = 1 - \frac{g^2 C_f}{16\pi^2}\Bigg(&-7 - 4\gamma_E + \log(16) + \frac{4|z|\left(\left(\bar{q}^2+\bar{q}_\mu^2\right)F_5 - \bar{q}^2 F_4\right)}{\bar{q}} \\
&+4F_2 - 3\log\left(\frac{\bar{\mu}^2}{\bar{q}^2}\right) - (\beta+2)\log(\bar{q}^2 z^2) \\
&+\beta\Big[3 - 2\gamma_E + \log(4) - \frac{2\bar{q}_\mu^2|z|}{\bar{q}}F_4 - 2F_1 - 2\left(\bar{q}^2+\bar{q}_\mu^2\right)G_3 \\
&+z^2\left(\bar{q}_\mu^2(F_3 - F_1 + F_2) + \bar{q}^2\frac{F_1 - F_2}{2}\right)\Big] \\
&+i\Big\{4\bar{q}_\mu(2z(F_1 - F_2 - F_3) + G_1) \\
&+\beta\bar{q}_\mu\left[\bar{q}(z|z|F_5 + 2(G_4 - 2G_5)) - 2G_1 + 2G_2\right]\Big\}\Bigg)\,,
\end{aligned} \tag{3.6}$$

where $q \equiv \sqrt{q^2}$. The conversion factors depend on the dimensionless quantities $z\bar{q}$ and $\bar{q}/\bar{\mu}$, and the RI$'$ ($\bar{q}$) and $\overline{\text{MS}}$ ($\bar{\mu}$) scales have been kept free. The conversion factors as defined in Eq. (3.5) are to be multiplied by the $Z^{\overline{\text{MS}}}$ to give $Z^{\text{RI}'}$. In non-perturbative calculations one may obtain $Z^{\overline{\text{MS}}}$ by multiplying $Z^{\text{RI}'}$ with $\mathscr{C}_\Gamma(g^2 \to -g^2)$ which is valid to one-loop level.

3.2 Lattice Regularization

The main goal of the perturbative calculation is to evaluate the lattice-regularized bare Green's functions $\langle\psi\,\mathscr{O}_{\Gamma_1}\,\bar{\psi}\rangle^{LR}$ that would allow us to understand the renormalization pattern of Wilson line operators; this is by far a more complicated calculation, as compared to dimensional regularization. The main task is to write lattice expressions in terms of continuum integrals, plus additional terms which are lattice integrals, independent of external momentum q. We compute the one-loop Feynman diagrams Fig. 1 in LR, and we find that diagram 1 gives the same contribution as in DR. This is due to the fact that the latter is finite as $\varepsilon \to 0$, and thus the limit $a \to 0$ can be taken with no lattice corrections. Once the loop momentum p is rescaled within the Brillouin zone, $p \to p/a$, lattice divergences manifest themselves as IR divergences in the external momentum $q \to 0$.

PoS(QCDEV2017)020

3.2.1 Multiplicative Renormalization and Mixing

We want to extract the RFs in the $\overline{\text{MS}}$ scheme directly without an intermediate RI-type scheme. This is feasible by taking the difference of the Green's functions in DR and LR, which is necessarily polynomial in the external momentum (of degree 0 in this case). This leads to a prescription for defining $Z_{\mathcal{O}}^{LR,\overline{\text{MS}}}$ without any intermediate schemes. As reported in Ref. [2], in regularizations other than DR there might be a linear divergence associated to the Wilson line. Such a divergence we find in LR, and to one-loop it is proportional to $|z|/a$ arising from the tadpole diagram [8]. We also find the at one-loop, the linear divergence is the same for all operator insertions. In a resummation of all orders in perturbation theory, the powers of $|z|/a$ are expected to combine into an exponential of the form [2]:

$$\Lambda_\Gamma = e^{-c|z|/a}\,\tilde{\Lambda}_\Gamma\,, \tag{3.7}$$

where c is the strength of the divergence and $\tilde{\Lambda}_\Gamma$ is related to $\Lambda_\Gamma^{\overline{\text{MS}}}$ by an additional renormalization factor which is at most logarithmically divergent with a. Based on arguments from heavy quark effective theory, additional non-perturbative contributions may appear in the exponent [16].

To one loop, we find the following form for the difference between the bare lattice Green's functions and the $\overline{\text{MS}}$-renormalized ones:

$$\langle\psi\,\mathcal{O}_\Gamma\,\bar{\psi}\rangle^{DR,\overline{\text{MS}}} - \langle\psi\,\mathcal{O}_\Gamma\,\bar{\psi}\rangle^{LR} = \frac{g^2 C_f}{16\pi^2}\,e^{iq_\mu z}\Bigg[\Gamma\Big(\alpha_1+\alpha_2\,\beta+\alpha_3\,\frac{|z|}{a}+\log\big(a^2\bar{\mu}^2\big)\,(4-\beta)\Big)$$
$$+\big(\Gamma\cdot\gamma_\mu+\gamma_\mu\cdot\Gamma\big)\,\Big(\alpha_4+\alpha_5\,c_{\rm SW}\Big)\Bigg]\,. \tag{3.8}$$

The presence of the term $\big(\Gamma\cdot\gamma_\mu+\gamma_\mu\cdot\Gamma\big)$ indicates mixing between operators of equal dimension, which is finite and appears in the lattice regularization. In particular, this affects formulations that break chiral symmetry. The combination of Dirac matrices in the mixing term vanishes for certain choices of the Dirac structure Γ in the operator. This is true for the operators P, V_ν ($\nu\neq\mu$), A_μ, $T_{\mu\nu}$ ($\nu\neq\mu$), and as a consequence, only a multiplicative renormalization is required. The mixing was identified for the first time in Ref. [8] and has significant impact in the non-perturbative calculation of the unpolarized quasi-PDFs, as there is a mixing with a twist-3 scalar operator [17]. Such a mixing must be eliminated using a proper renormalization prescription, ideally non-perturbatively [10].

In Eq. (3.8) all coefficients α_i depend on the Symanzik parameters, except for α_2 which has a numerical value $\alpha_2 = 5.792$. This value was expected, as all gauge dependence must disappear in the $\overline{\text{MS}}$ scheme for gauge invariant operators: Indeed, this term will cancel against a similar term in $Z_\psi^{LR,\overline{\text{MS}}}$ (see e.g., Ref. [18]) The mixing coefficients may be obtained perturbatively by constructing an appropriate 2×2 mixing matrix [8],

$$\begin{pmatrix}\mathcal{O}^R_{\Gamma_1}\\ \mathcal{O}^R_{\Gamma_2}\end{pmatrix} = \begin{pmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{pmatrix}^{-1}\begin{pmatrix}\mathcal{O}_{\Gamma_1}\\ \mathcal{O}_{\Gamma_2}\end{pmatrix}. \tag{3.9}$$

and the multiplicative RFs (Z_{ii}) as well as the mixing coefficients (Z_{ij}) are extracted from

$$\langle\psi^R\,\mathcal{O}^R_{\Gamma_i}\,\bar{\psi}^R\rangle_{\rm amp} = Z_\psi\sum_{j=1}^{2}(Z^{-1})_{ij}\,\langle\psi\,\mathcal{O}_{\Gamma_j}\,\bar{\psi}\rangle_{\rm amp}\,,\qquad \psi = Z_\psi^{1/2}\,\psi^R\,, \tag{3.10}$$

PoS(QCDEV2017)020

where the renormalization matrix Z and the fermion field renormalization Z_ψ have the following perturbative expansion:

$$Z_{ij} = \delta_{ij} + g^2 z_{ij} + \mathscr{O}(g^4)\,, \qquad Z_\psi = 1 + g^2 z_\psi + \mathscr{O}(g^4)\,. \tag{3.11}$$

The condition for extracting $Z_{11}^{LR,\overline{\rm MS}}$ and $Z_{12}^{LR,\overline{\rm MS}}$ is the requirement that renormalized Green's functions be regularization independent:

$$\langle \psi^R\, \mathscr{O}^R_{\Gamma_i}\, \bar\psi^R\rangle^{DR,\overline{\rm MS}}_{\rm amp} = \langle \psi^R\, \mathscr{O}^R_{\Gamma_i}\, \bar\psi^R\rangle^{LR,\overline{\rm MS}}_{\rm amp}\Big|_{a\to 0}\,, \tag{3.12}$$

leading to

$$\begin{aligned}\langle \psi^R\, \mathscr{O}^R_{\Gamma_1}\, \bar\psi^R\rangle^{DR,\overline{\rm MS}} - \langle \psi\, \mathscr{O}_{\Gamma_1}\, \bar\psi\rangle^{LR} &= g^2 \left(z_\psi^{LR,\overline{\rm MS}} - z_{11}^{LR,\overline{\rm MS}}\right)\langle \psi\, \mathscr{O}_{\Gamma_1}\, \bar\psi\rangle^{\rm tree} \\ &- g^2\, z_{12}^{LR,\overline{\rm MS}} \langle \psi\, \mathscr{O}_{\Gamma_2}\, \bar\psi\rangle^{\rm tree} + \mathscr{O}(g^4)\,. \end{aligned}\tag{3.13}$$

The above equation can be combined with Eq. (3.8) for the extraction of the multiplicative renormalization and mixing coefficients in the $\overline{\rm MS}$-scheme and LR. To one-loop level, the diagonal elements of the mixing matrix (multiplicative renormalization) are the same for all operators under study, and through Eq. (3.13) one obtains:

$$Z_\Gamma^{LR,\overline{\rm MS}} = 1 + \frac{g^2\, C_f}{16\,\pi^2}\left(e_1 + e_2\,\frac{|z|}{a} + e_3\, c_{\rm SW} + e_4\, c_{\rm SW}^2 - 3\log\left(a^2\bar\mu^2\right)\right), \tag{3.14}$$

where the coefficients $e_1 - e_4$ are given in Table 1, for the Wilson, tree-level Symanzik and Iwasaki improved actions.

As expected, $Z_\Gamma^{LR,\overline{\rm MS}}$ is gauge independent, and the cancelation of the gauge dependence was numerically confirmed up to $\mathscr{O}(10^{-5})$. Similar to $Z_\Gamma^{LR,\overline{\rm MS}}$, the nonvanishing mixing coefficients are operator independent and have the general form:

$$Z_{12}^{LR,\overline{\rm MS}} = Z_{21}^{LR,\overline{\rm MS}} = 0 + \frac{g^2\, C_f}{16\,\pi^2}\,(e_5 + e_6\, c_{\rm SW})\,, \tag{3.15}$$

where $Z_{ij}^{LR,\overline{\rm MS}}$ $(i \neq j)$ is nonzero only for the operator pairs: $\{S, V_1\}$, $\{A_2, T_{34}\}$, $\{A_3, T_{42}\}$, $\{A_4, T_{23}\}$ (for notation see Ref. [8]). The values of the coefficients e_5 and e_6 for Wilson, tree-level Symanzik and Iwasaki gluons are also shown in Table 1.

The strength of the mixing depends on the value of $c_{\rm SW}$, and one may compare the ratio $-e_5/e_6$ with the value of $c_{\rm SW}$ used in numerical simulations. This will allow to estimate how severe is the mixing for the particular ensemble.

Action	e_1	e_2	e_3	e_4	e_5	e_6
Wilson	24.3063	-19.9548	-2.24887	-1.39727	14.4499	-8.28467
TL Symanzik	19.8442	-17.2937	-2.01543	-1.24220	12.7558	-7.67356
Iwasaki	12.5576	-12.9781	-1.60101	-0.97321	9.93653	-6.52764

Table 1: Numerical values of the coefficients e_1 - e_4 of the multiplicative renormalization functions and e_5 - e_6 of the mixing coefficients for Wilson, tree-level (TL) Symanzik and Iwasaki gluon actions.

PoS(QCDEV2017)020

4. Non-Perturbative Renormalization

Understanding the renormalization and mixing pattern from the perturbative calculation led to the development of a proper non-perturbative prescription for the cases with (unpolarized) and without (helicity and transversity) mixing. Here we only consider γ_μ parallel to the direction of the Wilson line. However, choosing γ_μ in the temporal direction is ideal for the unpolarized as it eliminates any mixing.

The non-perturbative RFs will renormalize nucleon matrix elements of fermion operators with a straight Wilson line, denoted by $h_\Gamma(P_3,z)$. The nucleon is boosted with momentum P_3 that is traditionally taken in the same direction as the Wilson line. The quasi-PDFs can be computed from the Fourier transform of the following local matrix elements:

$$h_\Gamma(P_3,z) = \langle N|\bar{\psi}(0,z)\,\Gamma\, W_3(z)\,\psi(0,0)|N\rangle, \tag{4.1}$$

where $|N\rangle$ is a nucleon state with spatial momentum P_3 along the 3-direction and $W_3(z)$ is a Wilson line of length z in the same direction. For demonstration purposes we focus on the renormalization of the helicity operator. A prescription on how to eliminate the mixing for the unpolarized case is provided in Ref. [10]. The non-perturbative prescription in the RI$'$-scheme and in the absence of mixing, is given by Eqs. (3.3) - (3.4). The RI$'$ scale $\bar{\mu}_0$ is chosen to be democratic in order to suppress discretization effects, and scales of the form (n_t,n,n,n) have small discretization effects [19]. Using renormalization scales leading to a small value for such a ratio has been successful for the local fermion operators [20, 21]. Note that the vertex functions $\mathcal{V}(p,z)$ have the same linear divergence as the nucleon matrix elements, which allows one to extract both the linear and logarithmic divergence at once, through the renormalization condition of Eq. (3.3).

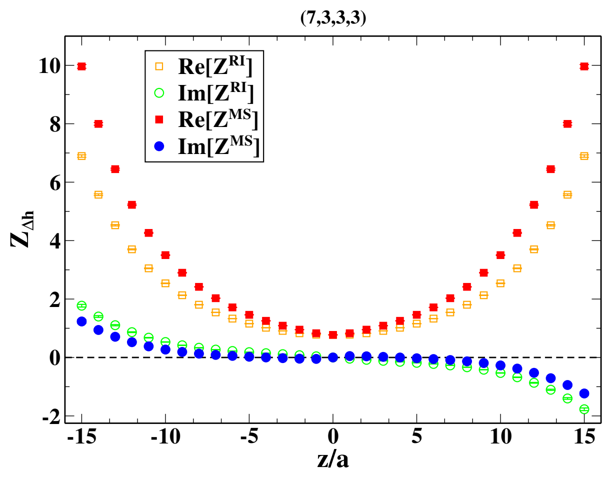

Figure 2: The z-dependent renormalization function for the matrix element $\Delta h(P_3,z)$ with $aP_3 = \frac{6\pi}{L}$. The RI$'$ scale is $a\bar{\mu}_0$ are: $\frac{2\pi}{32}\left(\frac{7}{2}+\frac{1}{4},3,3,3\right)$, while the $\overline{\rm MS}$ scale is set to 2GeV. Open (filled) symbols correspond to the RI$'$ ($\overline{\rm MS}$) estimates.

In Fig. 2, we plot the real and imaginary part of the helicity RFs, $Z_{\Delta h}$, that renormalizes the bare matrix element $\Delta h(P_3,z)$. We extract the RI$'$ (open symbols) RFs directly from lattice data and we convert to the $\overline{\rm MS}$ (filled symbols) using the one-loop conversion factor we computed perturbatively. The imaginary part of $Z_{\Delta h}^{\overline{\rm MS}}$ is reduced compared to its counterpart in $Z_{\Delta h}^{\rm RI'}$. This

PoS(QCDEV2017)020

is not a surprise, as the perturbative Z-factor in DR and in the $\overline{\text{MS}}$ scheme is real to all orders in perturbation theory (extracted from the poles). Therefore, it is expected that the imaginary part of the non-perturbative estimates should be highly suppressed. The behavior of our non-perturbative estimates is encouraging, as the imaginary part is very close to zero for $|z|$ up to $\sim 10a$.

In Ref. [10] we perform a careful analysis on the systematic uncertainties related to the lattice artifacts and the truncation of the conversion factor. We find an effect of 3-5% in the real part of the RFs, and up to 100% in the imaginary part. Despite the large uncertainty in the imaginary part, its value is less that 15% of the real part, which leads to a smaller influence in the renormalized matrix elements.

5. Summary

We presented an one-loop perturbative calculation of the renormalization functions for fermion operators including a straight Wilson line. We have demonstrated two important features of the quasi-PDFs in lattice regularization: finite mixing and a linear divergence with respect to the regulator. We extracted the Green's functions both in dimensional (DR) and lattice (LR) regularizations, which allows one to extract the LR renormalization functions in the $\overline{\text{MS}}$-scheme directly. Another important aspect of this work is the conversion factor between an RI$'$-type scheme and $\overline{\text{MS}}$, which is needed to bring non-perturbative estimates of the RFs to the $\overline{\text{MS}}$-scheme.

We demonstrate for the first time [8] that certain Wilson line operators mix in the lattice regularization. This affects the numerical simulations of the nucleon matrix elements for the unpolarized quasi-PDFs, which mix with a twist-3 [17] scalar operator. Such a mixing is not present for the unpolarized case if one uses a Dirac structure perpendicular to the Wilson line direction. Using the renormalization pattern revealed in this work we developed an appropriate non-perturbative renormalization prescription for the unpolarized, helicity and transversity quasi-PDFs, explaining how one can extract both the multiplicative renormalization and mixing coefficients [10].

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PoS(QCDEV2017)020

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PoS(QCDEV2017)020