PROCEEDINGS OF SCIENCE

Quark and Gluon Spin Dependent GPDs in a Flexible Spectator Model for Deeply Virtual Lepton Scattering Processes

Gary R. Goldstein[a*]
Tufts University
E-mail: gary.goldstein@tufts.edu

S. Liuti[b] **and O. J. Gonzalez Hernandez**[c]
b. University of Virginia
E-mail: sl4y@virginia.edu
c. Universita di Torino
E-mail: joghdr@gmail.com

Chiral Even and Odd Generalized Parton Distributions for valence quarks were obtained in a "flexible" spectator model based on the covariant scattering matrix approach. The parametrization of the GPDs was constrained by nucleon form factors, PDFs and some deeply virtual Compton scattering data. The model is extended to sea quarks and gluons. A broad range of measured and measurable electron, muon and neutrino processes, including cross sections and polarization asymmetries, are compared with existing data. Predictions are made for processes sensitive to the newly parameterized GPDs.

QCD Evolution 2017
22-26 May, 2017
Jefferson Lab, Newport News, VA - USA

*Speaker.

PoS(QCDEV2017)037

1. Introduction

Generalized Parton Distributions (GPDs), in two decades since their introduction [1], have played a major role in understanding the non-perturbative QCD momentum and angular momentum partonic substructure of the hadrons. Experimental efforts to measure them, indirectly through leptoproduction, have flourished. Theoretical constraints and models have also blossomed and phenomenology has sought to keep pace with models and data. Of the many models proposed, one set, that we have developed, has been particularly successful in connecting to existing data and in encouraging further experiments. In the following we will summarize the role of the GPDs in determining cross sections and polarization observables in exclusive leptoproduction of photons and mesons. In the course of that summary, we will sketch our own "flexible" model of GPDs. We have begun to extend the spectator model of the GPDs to include sea quark and gluon distributions. When assembled together and transformed to configuration space, these GPDs can provide a 3-dimensional picture of the spin and momentum structure of the nucleon [2]. We will indicate current developments, connections to forthcoming experiments and ongoing work.

Initially, with other collaborators, one of the authors [3] constructed a spectator model for the chiral even GPDs, $H^q(X,\zeta,t)$ and $E^q(X,\zeta,t)$. This work incorporated constraints and showed promise in confronting data. The present authors extended this model to include all four of the chiral even valence quark GPDs, in conjunction with Deeply Virtual Compton Scattering (DVCS), in a series of papers that developed the "flexible model", a "Reggeized diquark model" (**RxDq**) [4, 5].

We will start with a brief reminder of the relevant formalism. The model for the valence quark GPDs will be developed throughout and compared with a small sample of data. The chiral odd GPDs arise from applications of parity to the model and, in turn, will be compared with some representative data. The extension of all of these considerations to neutrino production of photons and mesons will be indicated next. Then new extensions of the flexible model for gluons and the quark "sea" will be shown. Preliminary results and some observable quantities will be indicated. Finally we describe continuing work that will contribute to further developments.

2. Valence Quark GPDs

For valence quarks, formally, there are 8 leading twist GPD's, four chiral even and four chiral odd, the latter corresponding to various combinations of quark transversity. The counting of the independent GPDs is in accord with the number of independent helicity amplitudes for spin 1/2 nucleons emitting spin 1/2 quarks and recombining further along the light cone. The helicity amplitude functions are linear combinations of the GPDs. There also are 8 leading twist GPDs for gluons, as well; that number corresponding to the number of on-shell gluon plus nucleon helicity amplitudes. Four of those conserve gluon helicity and four are helicity double flip. The latter correspond to on-shell gluon transversity, which is equivalent to gluon polarization perpendicular and parallel to the gluon-hadron momentum plane. Through evolution, there also must be 8 GPDs for each flavor of anti-quarks.

Deeply Virtual Compton Scattering and Deeply Virtual Meson Production can be described within QCD factorization, through the convolution of specific GPDs and hard scattering ampli-

PoS(QCDEV2017)037

tudes. There are four valence quark chiral-even GPDs, $H, E, \widetilde{H}, \widetilde{E}$ [6] and four additional chiral-odd GPDs, the latter for twist-two quark operators that flip quark helicity by one unit, $H_T, E_T, \widetilde{H}_T, \widetilde{E}_T$ [7, 8]. All GPDs depend on kinematic variables (x, ξ, t, Q^2), the parton's Light Cone (LC) momentum fraction, x, and the scale set by convoluting with DVCS or DVMS process' four-momentum transfer, Q^2, along with $t = \Delta^2$ where $\Delta = P - P'$ is the momentum transfer between the initial and final protons, and ξ, the skewness or the fraction of LC momentum transfer, $\xi = \Delta^+/(P^+ + P'^+)$. The GPDs that enter *observables*, bilinearly, are the Compton Form Factors (CFFs) - convolutions over x of GPDs with the struck quark propagator - "handbag" approximation.

To be specific, the quark GPDs are defined (at leading twist) as the matrix elements of the following projection of the unintegrated quark-quark proton correlator, wherein $\bar{P}^+ = \frac{P^+ + P'^+}{2}$ (see Ref.[8] for a detailed overview),

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\overline{P}^+ z^-} \left. \langle P', \Lambda' \mid \overline{\psi}\left(-\frac{z}{2}\right) \Gamma \psi\left(\frac{z}{2}\right) \mid P, \Lambda\rangle \right|_{z^+=0, \mathbf{z}_T=0}, \tag{2.1}$$

where $\Gamma = \gamma^+, \gamma^+\gamma_5, i\sigma^{i+}\gamma_5 (i = 1,2)$, and the target's helicities are Λ, Λ'. For the two chiral-even cases

$$W^{[\gamma^+]}_{\Lambda',\Lambda}(x,\xi,t) = \frac{1}{2\overline{P}^+}\overline{U}(P',\Lambda')\left(\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}(-\Delta_\mu)}{2M}E(x,\xi,t)\right)U(P,\Lambda); \tag{2.2}$$

$$W^{[\gamma^+\gamma^5]}_{\Lambda',\Lambda}(x,\xi,t) = \frac{1}{2\overline{P}^+}\overline{U}(P',\Lambda')\left(\gamma^+\gamma^5 \tilde{H}(x,\xi,t) + \gamma^5 \frac{-\Delta^+}{2M}\tilde{E}(x,\xi,t)\right)U(P,\Lambda) \tag{2.3}$$

For the chiral-odd case, $\Gamma = i\sigma^{i+}\gamma_5$, $W^{\Gamma}_{\Lambda',\Lambda}$ is parametrized as [8],

$$\begin{aligned} W^{[i\sigma^{i+}\gamma_5]}_{\Lambda',\Lambda}(x,\xi,t) &= \frac{1}{2\overline{P}^+}\overline{U}(P',\Lambda')\left(i\sigma^{+i}H_T(x,\xi,t) + \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2M}E_T(x,\xi,t)\right. \\ &+ \left.\frac{P^+\Delta^i - \Delta^+ P^i}{M^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^+ P^i - P^+\gamma^i}{2M}\widetilde{E}_T(x,\xi,t)\right)U(P,\Lambda) \end{aligned} \tag{2.4}$$

The quark chiral even GPDs connect to pdf's in the forward limit

$$H^q(x,0,0) = q(x) = h_1^q(x) \ \text{ and } \ \widetilde{H}^q(x,0,0) = \Delta q(x) = q(x)^{\rightarrow}_{\Rightarrow} - q(x)^{\leftarrow}_{\Rightarrow} = g_1^q(x) \tag{2.5}$$

and the chiral even GPDs integrate to the nucleon form factors, which constrains the GPD t-dependence,

$$\int_0^1 H^q(X,\zeta,t) = F_1^q(t), \int_0^1 E^q(X,\zeta,t) = F_2^q(t), \int_0^1 \widetilde{H}^q(X,\zeta,t) = G_A^q(t), \int_0^1 \widetilde{E}^q(X,\zeta,t) = G_P^q(t). \tag{2.6}$$

where $F_1^q(t)$ and $F_2^q(t)$ are the Dirac and Pauli form factors for the quark q components in the nucleon. $G_A^q(t)$ and $G_P^q(t)$ are the axial and pseudoscalar form factors. Hence, the quark GPDs are constrained by well measured pdf's and EM form factors, but the gluon GPDs have less explicit connection to measured quantities.

The spin structures of GPDs that are directly related to spin dependent observables are most effectively expressed in term of helicity dependent amplitudes, developed extensively for the covariant description of two body scattering processes (see also Ref.[8]).

PoS(QCDEV2017)037

Our “flexible model" [4] for the chiral even sector satisfies the measured constraints. For the valence quarks we have a quark-diquark approximation to the light-front Fock states or covariant vertices. Using a spectral function for the diquark mass, we obtain small x Regge-like behavior. This model has successfully parameterized measurements of DVCS. Applying a parity transformation to one vertex, the chiral odd GPDs are obtained [9, 10], as applied to π^0 electroproduction. Some of the GPDs are shown in Fig. 1.

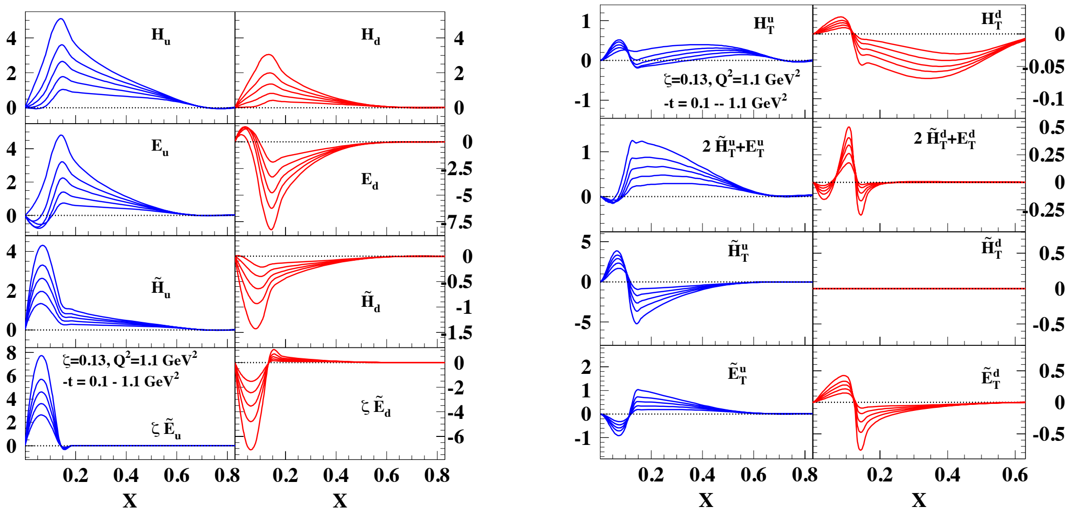

Figure 1: (Color online) The chiral-even (left panel) and chiral-odd GPDs (right panel) evaluated using the model described in the text plotted vs. X at $x_{Bj} = \zeta = 0.13$, $Q^2 = 2$ GeV^2. The range in $-t$ is: $0.1 \leq -t \leq 1.1\,\text{GeV}^2$. Curves with the largest absolute values correspond to the lowest t. Adapted from [4, 10]

The applications to a few observables in DVCS and π^0 electroproduction are shown in Fig. 2. Our more extensive results can be seen in the indicated references.

3. Gluon GPDs

The helicity conserving gluon distributions with t-channel even parity are defined :

$$
\begin{aligned}
F^g &= \frac{1}{\bar{P}^+}\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P',\Lambda'|G^{+\mu}(-\frac{1}{2}z)G_\mu{}^+(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0} \\
&= \frac{1}{2\bar{P}^+}\bar{U}(P',\Lambda')[H^g(x,\xi,t)\gamma^+ + E^g(x,\xi,t)\frac{i\sigma^{+\alpha}(-\Delta_\alpha)}{2M}]U(P,\Lambda)
\end{aligned}
\tag{3.1}
$$

and for t-channel odd parity

$$
\begin{aligned}
\tilde{F}^g &= \frac{-i}{\bar{P}^+}\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P',\Lambda'|G^{+\mu}(-\frac{1}{2}z)\tilde{G}_\mu{}^+(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0} \\
&= \frac{1}{2\bar{P}^+}\bar{U}(P',\Lambda')[\tilde{H}^g(x,\xi,t)\gamma^+\gamma_5 + E^g(x,\xi,t)\frac{\gamma_5(-\Delta^+)}{2M}]U(P,\Lambda),
\end{aligned}
\tag{3.2}
$$

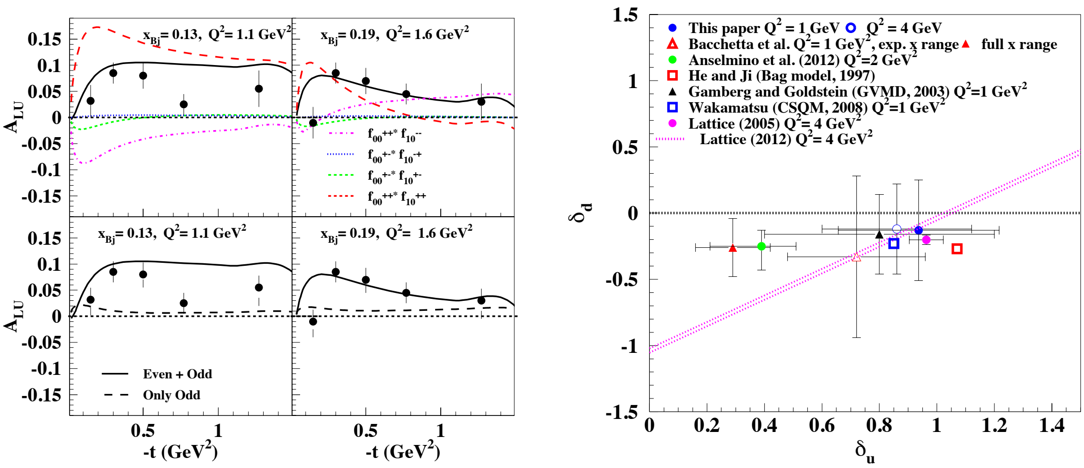

Figure 2: (Color online) **Left** Beam spin asymmetry, A_{LU}, in $e+p \to e'+\pi^0+p$ plotted vs. $-t$ for two different kinematics: $Q^2 = 1.1$ GeV^2, $x_{Bj} = 0.13$ (left), $Q^2 = 1.6$ GeV^2, $x_{Bj} = 0.19$ (right). Experimental data from Ref.[12]. Different helicity amplitude combinations contributing to A_{LU} are shown. The full curve describes the result obtained including all combinations. Lower panels show results including both the chiral-even and odd GPDs (full curve) compared to only the chiral-odd contribution (dashes). Right (b): Tensor charge values for the d quark, δ_d plotted vs. the u quark, δ_u, as obtained from our analysis of exclusive deeply virtual processes, from the other experimental extractions existing to date: Pavia group [13] (Q^2= 1 GeV^2, flexible set), and Torino group [14], and from different models. The thin band is the lattice QCD result for the isovector component [15] (Q^2= 4 GeV^2). For our model we also show the effect of PQCD evolution from Q^2=1 GeV^2 to Q^2= 4 GeV^2. Adapted from Ref. [16].

summing over transverse indices $j = 1,2$, and using the dual gluon field strength $\tilde{G}^{\mu\nu}(x) = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}(x)$.[1] The transverse polarization components enter here because we are considering leading order (twist 2) on-shell (in light cone quantization, Ref.[8]). The longitudinal polarization (helicity 0) enters at twist 3.

There are also contributions involving the transverse helicity flip ($\pm 1 \to \mp 1$), which can be thought of as gluon states of transversity, or equivalently, linear polarization states. These gluon transversity distributions are defined as

$$F_T^g = -\frac{1}{\bar{P}^+}\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P',\Lambda'|\mathbf{S}G^{+j}(-\frac{1}{2}z)G^{+k}(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0}$$
$$= \mathbf{S}\frac{1}{2\bar{P}^+}\frac{\bar{P}^+\Delta^j - \Delta^+\bar{P}^j}{2M\bar{P}^+}$$
$$\times \bar{U}(P',\Lambda')\Bigg[H_T^g(x,\xi,t)i\sigma^{+k} + \tilde{H}_T^g\frac{\bar{P}^+\Delta^k - \Delta^+\bar{P}^k}{M^2}$$
$$+E_T^g(x,\xi,t)\frac{\gamma^+\Delta^k - \Delta^+\gamma^k}{2M} + \tilde{E}_T^g\frac{\gamma^+\bar{P}^k - \bar{P}^+\gamma^k}{M}\Bigg]U(P,\Lambda) \quad (3.3)$$

wherein $\mathbf{S}$ symmetrizes in (j,k) and removes the trace [8].

[1]With this convention H_g reduces to the pdf $xg(x,0,0)$.

The double helicity flip amplitudes **do not mix** through evolution with quark distributions, which makes gluon transversity unique and useful. In the definition of transversity [11] for on-shell gluons or photons, wherein there are no helicity 0 states, the transversity states are

$$\begin{aligned} |+1)_{trans} &= \{|+1\rangle + |-1\rangle\}/2 = |-1)_{trans} \\ |0)_{trans} &= \{|+1\rangle - |-1\rangle\}/\sqrt{2} \\ \text{helicity} \quad |\pm 1\rangle &= \{\mp\hat{x} - i\hat{y}\}/\sqrt{2} \\ \hat{x} &= -|0)_{trans} = P_{parallel} \\ \hat{y} &= i\sqrt{2}\,|+1)_{trans} = P_{normal} \end{aligned} \tag{3.4}$$

where the two-body scattering plane is the X-Z plane, with $\hat{y}$ along the normal to the scattering plane.

Our approach to modeling and parameterizing the valence quark GPDs [4] has a natural generalization to the gluon and sea quark GPDs [17]. To begin with, in our spectator model for gluon GPDs the nucleon decomposes into a gluon and a color octet baryon, so that the overall color is a singlet (projected from the $8 \otimes 8 = 1 \oplus 8 \oplus 8' \oplus 10 \oplus 27$). The color octet baryon has the same flavor as the nucleon and is a Fermion (with color⊗flavor⊗spin being antisymmetric under quark label exchanges), which we take to be spin 1/2, (8, 1/2) for simplicity. The (8, 3/2) spin 3/2 spectator can contribute to double gluon helicity flip as readily as the spin 1/2 spectator.

Evolving with Q^2 also requires a sea quark contribution, which we take in a spectator picture with $N \to \bar{u} \oplus (uuud)$ or $\bar{d} \oplus (uudd)$. They are also "normalized" by fitting parameters to the phenomenologically determined sea quark pdf's. These contributions are of interest also, particularly in applications to exclusive neutrino photon production.

As with the quark GPDs, the 8 gluon GPDs can be written in terms of helicity amplitudes, $A^{g*}_{\Lambda',\Lambda_{g'};\Lambda,\Lambda_g}$ (Ref. [8]). We note here only the gluon double helicity flip amplitudes,

$$\begin{aligned} A_{++,+-} &= e^{2i\phi}\sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\left(\tilde{H}^g_T + (1-\xi)\frac{E^g_T + \tilde{E}^g_T}{2}\right) \\ A_{-+,--} &= e^{2i\phi}\sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\left(\tilde{H}^g_T + (1+\xi)\frac{E^g_T - \tilde{E}^g_T}{2}\right) \\ A_{++,--} &= +e^{i\phi}(1-\xi^2)\frac{\sqrt{t_0-t}}{2M}\left(H^g_T + \frac{t_0-t}{M^2}\tilde{H}^g_T - \frac{\xi^2}{1-\xi^2}E^g_T + \frac{\xi}{1-\xi^2}\tilde{E}^g_T\right) \\ A_{-+,+-} &= -e^{3i\phi}(1-\xi^2)\frac{\sqrt{t_0-t}^3}{8M^3}\tilde{H}^g_T, \end{aligned} \tag{3.5}$$

similar to the quark helicity flip amplitudes.

In a forthcoming publication [17] we will present our explicit model for the gluon GPDs, generalizing from the Regge-diquark spectator model, the "flexible model". We will address some questions that are unique to gluon distributions: how are the t and skewness ξ dependences normalized? How is the small x behavior accounted for? Our spectator model takes direct, point-like "vertex functions" $\mathscr{G}_{\Lambda_X;\Lambda_g,\Lambda}(x,\vec{k}_T^2)$ for $N(\Lambda) \to g(\Lambda_g) + X(\Lambda_X)$ and their conjugate to construct the s-channel and u-channel gluon+nucleon helicity amplitudes for both the helicity conserving and helicity double flip amplitudes. One simplification that results from this spectator picture is that

$$\widetilde{H}^g_T = 0,\ (1-X)A_{-+,--} = (1-X')A_{++,+-},\ \widetilde{E}^g_T = 0, \tag{3.6}$$

as in the initial determination of the double flip GPDs ref. [7]. This simplification should have interesting consequences for various spin asymmetries. The model for the gluon GPDs answers the questions raised above regarding observables for different processes. Graphs of a few of the resulting unpolarized $H_g(X,\zeta,t)$'s are shown in Fig. 3.

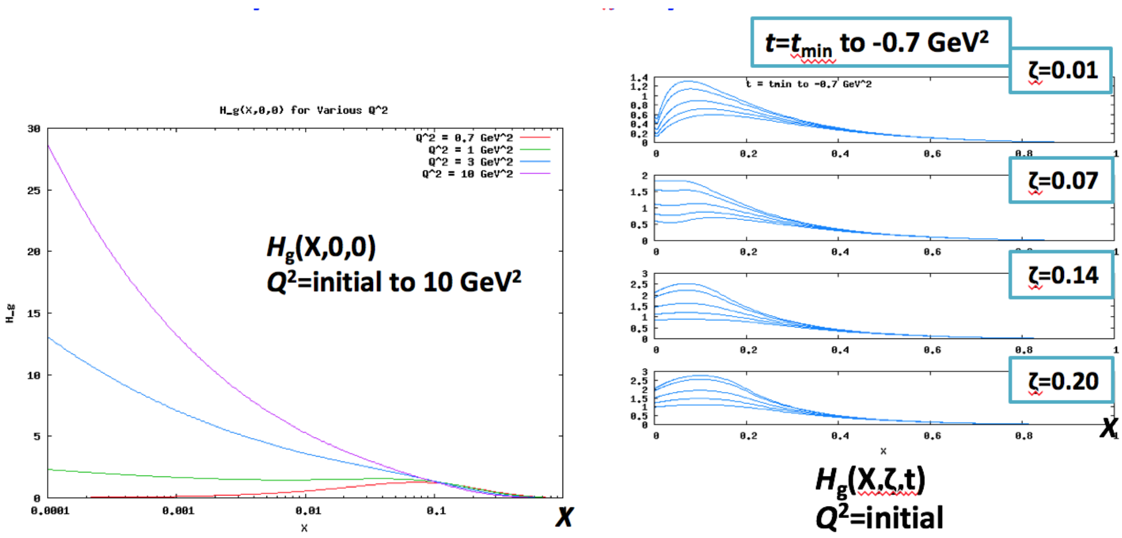

Figure 3: color online $H_g(X,0,0)$ and $H_g(X,\zeta,t)$ for several values of ζ and t. Adapted from Ref. [17].

In exclusive electroproduction of photons via high virtuality photon exchange (i.e. at large Q^2), or DVCS, the model restricts the gluon distributions. To measure the transversity GPD's of the gluons in DVCS or DVMP (with neutral vector mesons), electroproduction requires order α_s quark loop amplitudes (see Fig. 4). These appear as azimuthal modulations of the differential cross section at 4th order in sines and cosines. The gluon transversity GPDs can be separated from the helicity conserving gluon GPDs in the interference cross section for Bethe-Heitler and DVCS phase modulations. The multiply differential cross section for DVCS is

$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T|^2\,, \tag{3.7}$$

where α is the electromagnetic fine structure constant, $\gamma = 2Mx_{Bj}/Q$, $s=(k+p)^2$, M is the mass of the target, and T is a coherent superposition of the DVCS and Bethe-Heitler amplitudes,

$$T(k,p,k',q',p') = T_{DVCS}(k,p,k',q',p') + T_{BH}(k,p,k',q',p'), \tag{3.8}$$

yielding,

$$|T|^2 = |T_{\rm BH} + T_{\rm DVCS}|^2 = |T_{\rm BH}|^2 + |T_{\rm DVCS}|^2 + \mathscr{I}\,. \tag{3.9}$$

$$\mathscr{I} \;=\; T^*_{BH}T_{DVCS} + T^*_{DVCS}T_{BH}. \tag{3.10}$$

The transversity gluons appear in the $\cos(3\phi)$ modulation due to the double flip of the gluon helicity combined with the single flip of the BH amplitude. The product of the amplitudes gives rise to a term in the cross section modulation (involving the convolution of the GPD with the quark loop that couples to the photons) [8, 18, 19]

$$\frac{\sqrt{t_0-t}^3}{8M^3}\left[H_T^g F_2 - E_T^g F_1 - 2\tilde{H}_T^g\left(F_1 + \frac{t}{4M^2}F_2\right)\right]\cos 3\phi. \tag{3.11}$$

4. Observables and data

DVCS accesses Chiral Even GPDs through various cross sections and asymmetries. The GPDs, or their corresponding Compton Form Factors, enter as bilinears in pure DVCS or pure Bethe-Heitler in these observables, or linearly via Bethe-Heitler $\otimes$ DVCS interference. On the other hand, DVπ^0S accesses 2 Chiral Even + 4 Chiral Odd GPDs that enter bilinearly via $d\sigma/d\Omega$ & polarization asymmetries. For the π^0 case, the result of experimental observations that $d\sigma_T > d\sigma_L$ is that the chiral odd GPDs dominate, even though these GPDs enter the overall amplitudes through twist 3 π^0 distribution amplitudes.

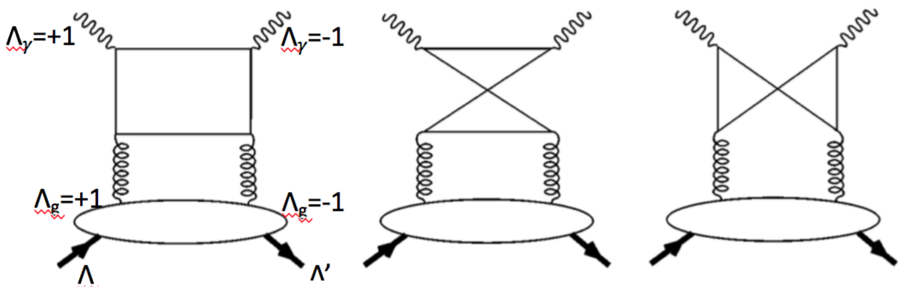

Figure 4: (Color online) Gluon transversity contributions to electroproduction DVCS. See Ref.[7].

In Ref.[9], after showing how DVπ^0P can be described in terms of chiral-odd GPDs, we estimated all of their contributions to the various observables with particular attention to the ones which were sensitive to the values of the tensor charge. The connection of the correlator, Eq.(2.1), with the helicity amplitudes for π^0 electroproduction proceeds by introducing a factorized form [9, 10],

$$f_{\Lambda_\gamma 0}^{\Lambda\Lambda'}(\zeta,t) = \sum_{\lambda,\lambda'} g_{\Lambda_\gamma 0}^{\lambda\lambda'}(X,\zeta,t,Q^2)\otimes A_{\Lambda'\lambda',\Lambda\lambda}(X,\zeta,t), \tag{4.1}$$

where the helicities of the virtual photon and the initial proton are, Λ_γ, Λ, and the helicities of the produced pion and final proton are 0, and Λ', respectively. This describes a factorization into a "hard part", $g_{\Lambda_\gamma 0}^{\lambda\lambda'}$ for the partonic subprocess $\gamma^* + q \to \pi^0 + q$, and a "soft part" given by the quark-proton helicity amplitudes, $A_{\Lambda',\lambda';\Lambda,\lambda}$ that contain the GPDs. We assumed this can be factorized, within the assumptions of the model, which goes beyond the proven leading twist factorization with chiral even GPDs [20]. The expressions for the chiral-odd quark-nucleon helicity amplitudes in terms of GPDs [8] are of the form

$$A_{++,--} = \sqrt{1-\xi^2}\left[H_T + \frac{t_0-t}{4M^2}\widetilde{H}_T - \frac{\xi^2}{1-\xi^2}E_T + \frac{\xi}{1-\xi^2}\widetilde{E}_T\right], \ldots \tag{4.2}$$

where we use the symmetric notation for the kinematic variables. Analogous forms have been written for the chiral even and the remaining chiral odd sectors [8].

The fitting procedure of GPDs is quite complicated owing to its many different steps. We described the "recursive procedure" that we used for the chiral even GPDs, the flexible model, in Ref. [4]. The main points are that the parametrization is constrained by EM form factors, pdf's, sum rules and some DVCS relations. A more detailed description of the other transversity functions including the first moment of $h_1^\perp \equiv 2\widetilde{H}_T^q + E_T^q$ is given in [10]. In that reference we show a large set of results that are compared with data from Jefferson Lab, Hall B [21]. The various GPDs enter the helicity amplitudes and those, in turn, determine all the cross section terms for π^0 electroproduction. Note that the azimuthally independent unpolarized cross section, $F_{UU,T} + \varepsilon F_{UU,L}$ is determined primarily by H_T, and thus provides a determination of the pdf h_1 and the tensor charge δq. The transverse and longitudinal cross sections have been separated experimentally at small t [22]. Some preliminary data were shown and compared favorably with our predictions (see Fig.4(Right) in Ref. [23]).

DVCS observables that depend on the gluon GPDs are generally suppressed by one power of α_s, since the photons connect to the gluons through quark loops. These gluon contributions will be difficult to access, being in the helicity amplitudes along with the dominant quark GPDs. The one case that directly exposes gluon contributions is through double helicity flip of the photons, which connects to the transversity gluon distributions of Eq. 3.11. To observe that effect requires precise measurements of azimuthal asymmetries, up to $\cos 3\phi$ terms. Alternatively, electroproduction of vector mesons or lepton pairs will enable this. This phenomenology is being studied [17].

5. Conclusions & Outlook

We have extended all the distributions that can be accessed with our "flexible spectator" model, focusing on chiral even and odd quark GPDs, the latter entering the transversity parton distributions in the nucleon that were accessed through deeply virtual exclusive π^0 meson production. This represents a consistent quantitative step with respect to our previous work [9]. It should be noted that, in particular, H_T and the combination $2\widetilde{H}_T + E_T$, now are separated. A similar, somewhat simplified approach was taken also in Ref.[24] - we differ in the importance attached to the skewedness dependence of E_T, $\tilde{E}_T$.

We see the results of our extended approach for some of the many measured and measurable observables. What is especially gratifying is that certain asymmetries constrain the GPDs well enough to separately determine H_T, and consequently transversity through the limit $H_T(x,0,0)$, and the combination $2\widetilde{H}_T + (1\pm\xi)E_T$. (See Ref. [10] for details.)

We sketched the extension of the model to the gluon distributions, which is work in progress [17] and suggested an experimental means to indirectly measure some GTMDs.

6. Acknowledgements

G.R.G. is grateful to the organizers of QCD-Evolution 2017 and Jefferson Lab for a productive meeting. Work of S.L. supported by U.S. D.O.E. grant DE-SC0016286. We thank our collaborators, A. Courtoy, A. Rajan and J. Poage.

References

[1] D. Müller, *et al.*, Fortschr. Phys. **42**, 101 (1994); X-d. Ji, Phys. Rev. Lett. **78**, 610 (1997); A.V. Radyushkin, Phys. Lett. **B 380**, 417 (1996).

[2] M. Burkardt, IJMPA**18**, 173 (2003).

[3] S. Ahmad, H. Honkanen, S. Liuti *et al.*; *ibid* Eur. Phys. J. **C63**, 407-421 (2009); *ibid* Phys. Rev. **D75**, 094003 (2007).

[4] G. R. Goldstein, J. O. Hernandez and S. Liuti, Phys. Rev. D **84**, 034007 (2011).

[5] J. O. Gonzalez-Hernandez, S. Liuti, G. R. Goldstein and K. Kathuria, Phys. Rev. C **88**, 065206 (2013).

[6] X. D. Ji, Phys. Rev. D **55**, 7114 (1997)

[7] P. Hoodbhoy and X. Ji, Phys. Rev. D 58, 054006 (1998).

[8] M. Diehl, Eur. Phys. Jour. C **19**, 485 (2001); *ibid*, Phys. Rept. **388**, 41 (2003).

[9] S. Ahmad, G. R. Goldstein and S. Liuti, Phys. Rev. D **79**, 054014 (2009).

[10] G. R. Goldstein, J. O. Hernandez and S. Liuti, Phys. Rev. D **91**, 114013 (2015); *ibid*, J. Phys. G: Nucl. Part. Phys. **39**, 115001. (2012).

[11] G.R. Goldstein and M. J. Moravcsik, *Ann. Phys.* (N.Y.) **98**, 128 (1976); **142**, 219 (1982); **195**, 213 (1989).

[12] R. De Masi, *et al.*, Phys. Rev. C **77**, 042201 (2008).

[13] A. Bacchetta, A. Courtoy and M. Radici, JHEP **1303**, 119 (2013).

[14] M. Anselmino, M. Boglione, U. D'Alesio, S. Melis, F. Murgia and A. Prokudin, Phys. Rev. D **87**, 094019 (2013).

[15] J. R. Green, *et al.*, Phys. Rev. D **86**, 114509 (2012).

[16] G. R. Goldstein, J. .O. Gonzalez Hernandez and S. Liuti, arXiv:1401.0438 [hep-ph].

[17] J. Poage, J. O. Gonzalez-Hernandez, S. Liuti, G. R. Goldstein, work in progress; J. Poage, Ph.D. dissertation, Tufts University (2017).

[18] M. Diehl, T. Gousset, B. Pire, and J. P. Ralston, Phys. Lett. B **411**, 193 (1997); ăX-D. Ji and J. Osborne, Phys. Rev. D **58**, 094018 (1998); P. Kroll, M. Schurmann, and P. A. M. Guichon, Nucl. Phys. **A598**, 435 (1996).

[19] A.V. Belitsky, D. Mueller and A. Kirchner, Nucl. Phys. B **629**, 323 (2002).

[20] J.C. Collins, L. Frankfurt, and M. Strikman, Phys. Rev. D **56**, 2982 (1997).

[21] I. Bedlinskiy, *et al.*, Phys.Rev.Lett. 109, 112001(2012).

[22] F. Sabatie, "DVCS & π^0 electroproduction at Hall A, Jefferson Lab", presentation at CIPANP2015, Vail, Colorado (May 2015).

[23] G. R. Goldstein and S. Liuti, PoS QCDEV2015, 051 (2015).

[24] S. V. Goloskokov, P. Kroll, Eur. Phys. J. **A47**, 112 (2011).

PoS(QCDEV2017)037