## Are the $Y(4260)$ and $Y(4360)$ molecular states?

## M. Bayar*

Department of Physics, Kocaeli University, 41380 Izmit, Turkey
E-mail: melahat.bayar@kocaeli.edu.tr

## B. Durkaya

Department of Physics, Kocaeli University, 41380 Izmit, Turkey
E-mail: beyzadurkaya@gmail.com

We investigate the $Y(4260)$ and $Y(4360)$ within the framework of the Faddeev Equations under the fixed center approximation. We find a state of $I=1$ with masses around 4320 MeV and width about 25 MeV for the case of $\rho X(3700)$ and 4256 MeV for the case of $\bar{D}-D_{1}(2420)$ with similar width to that of the $\rho X(3700)$. Hence these states could be associated with the $Y(4260)$ and $Y(4360)$.

XVII International Conference on Hadron Spectroscopy and Structure - Hadron2017
25-29 September, 2017
University of Salamanca, Salamanca, Spain

[^0]
## 1. Introduction

In the last three decades, many new charmonium like states are discovered. Hence charmonium spectroscopy has been of great interest to both experimentalist and the theorists. These new states can not be explained within the framework of the quark model and should have a different structure, such as charmed hybrids, tetraquark, and molecular states. The observation of the $X(3872)$ [1], $Y(4260)[2,3,4], Y(4360)[5,6,7]$ and $Y(4140)$ [8] and so on have supported the existence of new types of hadronic states.

In this study we investigate the $Y(4260)$ and $Y(4360)$ mesons whether they are molecular states or not. For this reason we investigate the $\rho D \bar{D}$ three body system solving the Faddeev equations under the Fixed Center Approximation (FCA). This method is an effective tool when dealing with the three body systems. In this method a pair of particles that interact strongly among themselves make a cluster and this cluster structure is not varied so much by the interaction of the third particle.

In this talk we study the $\rho D \bar{D}$ three body system using the FCA to the Faddeev equations. There are two possible scattering cases for this three body system since the $\rho D$ and $D \bar{D}$ system leads to the formation of a dynamically generated states, $D_{1}(2420)$ and $X(3700)$, respectively. For the detail of this study see Ref. [9].

## 2. FORMALISM

We investigate the $Y(4260)$ and $Y(4360)$ states. For this aim, we study the $\rho D \bar{D}$ system within the fixed center approximation to Faddeev equation. We may choose $D \bar{D}$ or $\rho D$ as a cluster. The $X(3700)$ is the $D \bar{D}$ molecular state with isospin $I=0$ and $D_{1}(2420)$ is the $\rho D$ molecular state with isospin $I=1 / 2$. Hence there are two possibilty for three body states. One is the $\rho(D \bar{D})_{X(3700)}$ and the other one is the $\bar{D}(\rho D)_{D_{1}(2420)}$. To calculate the three body states one needs to study the $\rho D$, $\rho \bar{D}$ and $D \bar{D}$ two body unitarized amplitudes. These two body amplitudes are investigated within the framework of the chiral unitary approach.

Let us start with the $D \bar{D}$ two body amplitude. To study the $D \bar{D}$ unitarized amplitude with coupled channels, there are six coupled channels, $D \bar{D}, K \bar{K}, \pi \bar{\pi}, \eta \eta, \eta_{c} \eta, D_{s} \bar{D}_{s}$, in the $I=0$ case and five coupled channels, $D \bar{D}, K \bar{K}, \pi \bar{\pi}, \pi \eta, \eta_{c} \pi$, in the $I=1$ case. The Bethe-Salpeter equation is given as

$$
\begin{equation*}
T=(\hat{1}-V \hat{G})^{-1} V \tag{2.1}
\end{equation*}
$$

where $\hat{G}$ is the loop function of pseudoscalar-pseudoscalar mesons and $V$ is the potential.
For the $\rho D$ two body scattering, there are eight coupled channels, $\pi D^{*}, D \rho, K D_{s}^{*}, D_{s} K^{*}, \eta D^{*}$, $D \omega, \eta_{c} D^{*}, D J / \psi$ in $I=1 / 2$, and two coupled channels, $\pi D^{*}$ and $D \rho$, in $I=3 / 2$ case. The Bethe-Salpeter equation in coupled channels as follow

$$
\begin{equation*}
T=(\hat{1}+V \hat{G})^{-1}(-V) \vec{\varepsilon} \cdot \vec{\varepsilon}^{\prime} \tag{2.2}
\end{equation*}
$$

where $\vec{\varepsilon}\left(\vec{\varepsilon}^{\prime}\right)$ represents a polarization vector of the incoming (outgoing) vector mesons and $V$ is an interaction kernel.

Now let us solve the $\rho D \bar{D}$ system by using the Faddeev equations under the FCA. In this method the three body scattering amplitude $T$ can be obtained as a summation of the two partition functions $T_{1}$ and $T_{2}$. Diagrammatic representation of the $T_{1}$ and $T_{2}$ are depicted in Fig 1. $T_{1}\left(T_{2}\right)$ sums all the diagrams of the series of Fig. 1 which begin with the interaction of particle 3 with particle 1(2) of the cluster.


Figure 1: Diagrammatic representation of the fixed center approximation to Faddeev equations.

Then we find

$$
\begin{equation*}
T=T_{1}+T_{2}, \quad T_{1}=t_{1}+t_{1} G_{0} T_{2}, \quad T_{2}=t_{2}+t_{2} G_{0} T_{1} \tag{2.3}
\end{equation*}
$$

where the $G_{0}$ is the meson exchange propagator and given by

$$
\begin{equation*}
G_{0}=\frac{1}{2 m_{c l s}} \int \frac{d^{3} q}{(2 \pi)^{3}} F_{c l s}(q) \frac{1}{q^{0^{2}}-\vec{q}^{2}-m_{3}^{2}+i \varepsilon} \tag{2.4}
\end{equation*}
$$

where $m_{3}$ is the mass of the particle 3 , and $m_{c l s}$ the mass of the cluster. The variable $q^{0}$ is the energy carried by the particle 3 .
$F_{c l s}$ is the form factor in Eq. (2.4). The expression for the form factor reproduce as following

$$
\begin{align*}
F_{c l s}(q) & =\frac{1}{\mathscr{C}} \int_{|\vec{p}-\vec{q}|<k_{\max }} d^{3} p \frac{1}{2 \omega_{1}(\vec{p})} \frac{1}{2 \omega_{2}(\vec{p})} \frac{1}{m_{c l s}-\omega_{1}(\vec{p})-\omega_{2}(\vec{p})+i \varepsilon} \\
& \times\left(\frac{1}{2 \omega_{1}(\vec{p}-\vec{q})}\right)\left(\frac{1}{2 \omega_{2}(\vec{p}-\vec{q})}\right) \frac{1}{m_{c l s}-\omega_{1}(\vec{p}-\vec{q})-\omega_{2}(\vec{p}-\vec{q})+i \varepsilon}, \tag{2.5}
\end{align*}
$$

where the normalization $\mathscr{C}$ is given by

$$
\begin{equation*}
\mathscr{C}=\int_{p<k_{\max }} d^{3} p\left[\frac{1}{2 \omega_{1}(\vec{p})} \frac{1}{2 \omega_{2}(\vec{p})} \frac{1}{m_{c l s}-\omega_{1}(\vec{p})-\omega_{2}(\vec{p})+i \varepsilon}\right]^{2} \tag{2.6}
\end{equation*}
$$

where $\omega_{1}$ and $\omega_{2}$ are the energies of the particles 1,2 , and $k_{\max }$ is a cutoff that regularizes the integral of Eqs. (2.5) and (2.6).

The arguments of the three body amplitude $T(s)$ and two body amplitudes $t_{j}\left(s_{j}\right)$ are not same where $s$ is the total invariant mass of the three body system and $s_{j}$ are the invariant masses of the two body systems. To evaluate the arguments $s_{j}$ of the two body amplitude, $t\left(\sqrt{s_{j}}\right)$, we share the binding energy among the three particles, proportionally to their masses. Hence the energy of the particles 1,2 and 3 are given by

$$
\begin{gather*}
E_{1}=\frac{\sqrt{s}}{\left(m_{c l s}+m_{3}\right)} \frac{m_{1} m_{c l s}}{\left(m_{1}+m_{2}\right)}  \tag{2.7}\\
E_{2}=\frac{\sqrt{s}}{\left(m_{c l s}+m_{3}\right)} \frac{m_{2} m_{c l s}}{\left(m_{1}+m_{2}\right)}  \tag{2.8}\\
E_{3}=m_{3} \frac{\sqrt{s}}{\left(m_{c l s}+m_{3}\right)} \tag{2.9}
\end{gather*}
$$

Thus the total energy of the two body system can be calculated as

$$
\begin{equation*}
s_{1(2)}=\left(p_{3}+p_{1(2)}\right)^{2}=\left(\frac{\sqrt{s}}{m_{c l s}+m_{3}}\right)^{2}\left(m_{3}+\frac{m_{1(2)} m_{c l s}}{m_{1}+m_{2}}\right)^{2}-\vec{P}_{2(1)}^{2} \tag{2.10}
\end{equation*}
$$

with approximate value of $\vec{P}_{2(1)}$

$$
\begin{equation*}
\frac{\vec{P}_{2(1)}^{2}}{2 m_{2(1)}} \simeq B_{2(1)} \equiv \frac{m_{2(1)} m_{c l s}}{\left(m_{1}+m_{2}\right)} \frac{\left(m_{c l s}+m_{3}-\sqrt{s}\right)}{\left(m_{c l s}+m_{3}\right)} \tag{2.11}
\end{equation*}
$$

where $B_{2(1)}$ is the binding energy of the particle 2 (1).
Note that the isospin of the cluster should be considered for the $t_{1}$ and $t_{2}$ amplitudes. For the case of $\rho(D \bar{D})_{X(3700)}$, the cluster of $X(3700)$ has isospin $I=0$. Therefore we have

$$
\begin{equation*}
|D \bar{D}\rangle^{I=0}=\frac{1}{\sqrt{2}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle-\frac{1}{\sqrt{2}}\left|-\frac{1}{2}, \frac{1}{2}\right\rangle \tag{2.12}
\end{equation*}
$$

with the nomenclature $\left|I_{z_{1}}, I_{z_{2}}\right\rangle$ for the $D \bar{D}$ system. We get

$$
\begin{align*}
\langle\rho D \bar{D}| t|\rho D \bar{D}\rangle & =\left\langle\rho^{+}(D \bar{D})^{I=0}\right|\left(\hat{t}_{\rho D}+\hat{t}_{\rho \bar{D}}\right)\left|\rho^{+}(D \bar{D})^{I=0}\right\rangle \\
& =\left(\frac{2}{3} t_{\rho D}^{I I=3 / 2}+\frac{1}{3} t_{\rho D}^{I=1 / 2}\right)+\left(\frac{2}{3} t_{\rho \bar{D}}^{I=3 / 2}+\frac{1}{3} t_{\rho \bar{D}}^{I=1 / 2}\right) \tag{2.13}
\end{align*}
$$

and for $\bar{D}(\rho D)_{D_{1}(2420)}$ system for total isospin $I=1$ case we obtain

$$
\begin{align*}
\langle\rho D \bar{D}| t|\rho D \bar{D}\rangle & =\left\langle\bar{D}(\rho D)^{I=1 / 2}\right|\left(\hat{t}_{D \rho}+\hat{t}_{D \bar{D}}\right)\left|\bar{D}(\rho D)^{I=1 / 2}\right\rangle \\
& =\left(\frac{8}{9} t_{D \rho}^{I=3 / 2}+\frac{1}{9} t_{D \rho}^{I=1 / 2}\right)+\left(\frac{2}{3} t_{D \bar{D}}^{I=1}+\frac{1}{3} t_{D \bar{D}}^{I=0}\right) . \tag{2.14}
\end{align*}
$$

## 3. Results

We calculate the scattering amplitude $T$ and associate the peaks in the modulus squared $|T|^{2}$ to bound states or resonances. The results are depicted in Figs. 2 and 3.

We show the result for the $\rho(D \bar{D})_{X(3700)}$ scattering with total isospin $I=1$ in Fig. 2. There is a clear peak around 4320 MeV and a width about 25 MeV . The threshold of the the $X(3700)$ and $\rho$ mesons is around 4475 MeV . Thus the peak is about 160 MeV below the $X(3700)$ and $\rho$ mesons threshold.


Figure 2: Modulus squared of the $\rho(D \bar{D})_{X(3700)}$ scattering amplitude with total isospin $I=1$.

In Fig. 3 we show the result of the modulus squared of the $\bar{D}(\rho D)_{D_{1}(2420)}$ three body scattering amplitude with total isospin $I=1$. There is a peak around 4256 MeV with a width $\Gamma \sim 25-30$ MeV . The peak is about 40 MeV below the $D_{1}(2420)$ and $D$ mesons threshold. The strength of the peak of $\left|T_{\rho-X(3700)}^{I=1}\right|^{2}$ is about four times bigger than the one of $\left|T_{\bar{D}-D_{1}(2420)}^{I=1}\right|^{2}$ and we see that $\bar{D}-D_{1}(2420)$ is more bound than $\rho-X(3700)$ component one.

We expect that a real state would be an admixture of both the $\rho(D \bar{D})_{X(3700)}$ and the $\bar{D}(\rho D)_{D_{1}(2420)}$ states. This new state could be correspond to a $c \bar{c}$ meson state of $I^{G}\left(J^{P C}\right)=?^{?}\left(1^{--}\right)$, the $Y(4360)$ [10]. The width of this state is about 74 MeV which is bigger than that we obtain (about 25 MeV ). This state would also be associated to the $Y(4260)$ with the quantum numbers $I^{G}\left(J^{P C}\right)=?^{?}\left(1^{--}\right)$ and with a width about 55 MeV [10]. Even though the widths of the $Y(4360)$ and the $Y(4260)$ are larger than that of the $\rho-X(3700)$ and the $\bar{D}-D_{1}(2420)$ states within the uncertainties of the FCA to the Faddeev equations these states could be correspond to the $Y(4360)$ or the $Y(4260)$.

## Acknowledgments

This work is supported by TUBITAK under the project No. 113F411.


Figure 3: Modulus squared of the $\bar{D}(\rho D)_{D_{1}(2420)}$ scattering amplitude with total isospin $I=1$.

## References

[1] S. K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. 91 (2003) 262001.
[2] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 95 (2005) 142001.
[3] Q. He et al. [CLEO Collaboration], Phys. Rev. D 74 (2006) 091104.
[4] C. Z. Yuan et al. [Belle Collaboration], Phys. Rev. Lett. 99 (2007) 182004.
[5] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 98 (2007) 212001.
[6] X. L. Wang et al. [Belle Collaboration], Phys. Rev. Lett. 99 (2007) 142002.
[7] Z. Q. Liu, X. S. Qin and C. Z. Yuan, Phys. Rev. D 78 (2008) 014032.
[8] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 102 (2009) 242002.
[9] B. Durkaya and M. Bayar, Phys. Rev. D 92, no. 3, 036006 (2015).
[10] C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40, no. 10, 100001 (2016).


[^0]:    *Speaker.

