## Tensor resonances in $\eta \pi$ using COMPASS data

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We present results on the extraction of tensor resonances in the $\eta \pi$ system in a joint analysis between the JPAC and COMPASS collaborations. We use the $S$-matrix principles, unitarity and analyticity, to constrain the reaction model. We find two $J^{P C}=2^{++}$resonance poles, the $a_{2}$ and $a_{2}^{\prime}$.

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## 1. Introduction

Hadron spectroscopy provides insight into the understanding of Quantum Chromodynamics (QCD). The constituent quark model has been historically successful in categorizing the hadrons [1], but as many experiments are gathering unprecedented high-statistics, high-precision data [2, 3, 4], there has been a renewed interest in understanding the QCD spectrum. Exotic states such as hybrids, which involves active gluons in a $q \bar{q}$ system [5, 6], are forbidden by the constituent quark model, but are in principle allowed by QCD. The $\eta \pi^{(1)}$ system is one of the golden modes for studying hybrid mesons as the system can be found in a spin-exotic $J^{P C}=1^{-+}$state. In the era of precision spectroscopic experiments, collaboration between theorist and experimental is vital in determining hadronic resonance properties. $S$-matrix principles, such as unitarity and analyticity [7,8], need to be implemented in reaction models to ensure that the resonance physics that is extracted satisfies fundamental principles. In understanding non-quark model states such as the $1^{-+}$, it is important to understand conventional quark model states, and ensure our understanding of reaction models can reliably describe well-known resonances. The $a_{2}(1320)$ is a well-established state [9] and is known to couple to $\eta \pi$. In constructing analytic models to describe $\eta \pi$ in $P$ wave, testing the model on the $D$-wave will give a first indication that the model satisfies $S$-matrix principles. We present here the first results of the JPAC and COMPASS collaborations on an analysis of $D$-wave resonances in the $\eta \pi$ system [10]. This work uses the previous $\eta \pi$ partial wave analysis in [2]. We find two resonances, corresponding to the $a_{2}$ and it's first excited state the $a_{2}^{\prime}$, as pole singularities in the complex energy plane. For details of the analysis, we refer to [10].

## 2. Amplitude Model

We consider the reaction $\pi p \rightarrow \eta \pi p$, with a $190 \mathrm{GeV} / c \pi$-beam (see Fig. 1). At these high energies, the process is dominated by pomeron $(\mathbb{P})$ exchange [2], and we assume factorization of the reaction into a "top" vertex $(\pi \mathbb{P} \rightarrow \eta \pi)$ and a "bottom" vertex $(p \rightarrow \mathbb{P} p)$. We are interested in meson resonances of the top vertex and the $2^{++}$data we fit is integrated over the momentum transfer, so we not have information about the $t$-dependence of the pomeron exchange. Therefore we assume the pomeron to have be spin- 1 and have an effective momentum transfer $t_{\text {eff }}=-0.5$ $\mathrm{GeV}^{2}$. Let $a(s)$ be the partial wave amplitude of the $2^{++}$channel, with $s$ being the invariant mass squared of the $\eta \pi$ system and using $t_{e f f}$ as the momentum transfer. We fit to the partial wave intensity distribution in [2] to our amplitude model with $I(s)=\mathscr{N} p|a(s)|^{2}$, where $\mathscr{N}$ is a normalization factor and $p=\lambda^{1 / 2}\left(s, m_{\eta}^{2}, m_{\pi}^{2}\right) / 2 \sqrt{s}$ is the break-up momentum of the $\eta \pi$ final state, with $\lambda(x, y, z)=x^{2}+y^{2}+z^{2}-2(x y+y z+x z)$ being the Källen triangle function.

We look for resonances in the elastic $\eta \pi$ amplitude, which we denote as $f(s)$. The model satisfies elastic unitarity of the $\eta \pi \rightarrow \eta \pi$ system, and the inelastic unitarity condition of the production process $\pi \mathbb{P} \rightarrow \eta \pi$, given as

$$
\begin{align*}
& \operatorname{Im} \widehat{a}(s)=\rho(s) \widehat{f^{*}}(s) \widehat{a}(s)  \tag{2.1}\\
& \operatorname{Im} \widehat{f}(s)=\rho(s)|\widehat{f}(s)|^{2}, \tag{2.2}
\end{align*}
$$

where $\widehat{a}$ and $\widehat{f}$ are reduced amplitudes that have the kinematic threshold factors removed, and $\rho=2 p^{5} / \sqrt{s}$ is the intermediate $\eta \pi$ phase space factor that includes the $D$-wave threshold behavior.


Figure 1: Diagram for $\pi p \rightarrow \eta \pi p$ with $\mathbb{P}$-exchange.

In our model, we assume only the $\eta \pi$ system as the intermediate state in the unitarity conditions. The effects of coupled channels (dominated by the $\rho \pi$ system) have a small effect on the pole positions, and are included as a systematic uncertainty estimate for the resonance parameters.

The amplitudes are subject to the N-over-D method, which we effectively write as

$$
\begin{equation*}
\widehat{a}(s)=\frac{n(s)}{D(s)}, \tag{2.3}
\end{equation*}
$$

where $n(s)$ contains the production process physics, and $D(s)$ contains the resonance physics of the $\eta \pi$ rescattering. We take $D(s)$ as

$$
\begin{equation*}
D(s)=c_{0}-c_{1} s-\frac{c_{2}}{c_{3}-s}-\frac{s}{\pi} \int_{s_{t h}}^{\infty} d s^{\prime} \frac{\rho\left(s^{\prime}\right) N\left(s^{\prime}\right)}{s^{\prime}\left(s^{\prime}-s\right)}, \tag{2.4}
\end{equation*}
$$

where $\rho(s) N(s)=g \lambda^{5 / 2}\left(s, m_{\eta}^{2}, m_{\pi}^{2}\right) /\left(s+s_{R}\right)^{7}$ is our model for the $\eta \pi$ exchange mechanism, with $s_{R}$ being responsible for the finite range of strong interactions, and is taken to be $s_{R}=1.5 \mathrm{GeV}^{2}$. Here, $c_{0}, c_{1}, c_{2}, c_{3}$, and $g$ are fit parameters. The terms $c_{0}-c_{1} s$ and $c_{2} /\left(c_{3}-s\right)$ are referred to as CDD poles [11], and associated with resonance parameters. In this analysis, we fix $c_{0}=(1.23)^{2}$. We write the production term as and

$$
\begin{equation*}
n(s)=\frac{1}{c_{3}-s} \sum_{j} a_{j} T_{j}(\omega(s)) . \tag{2.5}
\end{equation*}
$$

where the factor $1 /\left(c_{3}-s\right)$ cancels the zero of the second CDD pole in $D(s), T_{j}$ are Chebyshev polynomials, and $\omega(s)=s /(s+\Lambda)$, which is our approximation for the left-hand singularities of the production process. The $a_{j}$ are fit parameters, and we take $\Lambda=1 \mathrm{GeV}^{2}$.

## 3. Fit Results

We fit the model Eq. (2.3) to the $J^{P C}=2^{++}$intensity distribution given in [2] using $I(s)=$ $\mathscr{N} p|a(s)|^{2}$. We use a $\chi^{2}$ minimization with the 56 data points for the $D$-wave, which range from the $\eta \pi$ threshold to 3 GeV . The absolute normalization is not given in [2], so we fix the normalization $\mathscr{N}=10^{6}$ so that the production parameters $a_{j}$ are of $\mathscr{O}(1)$. We fit the parameters $a_{j}$ for a specified
number of production terms, and $c_{1}, c_{2}, c_{3}$, and $g$. We estimate statistical uncertainties for the parameters and observables by performing a bootstrap analysis using $10^{5}$ pseudo datasets.

In preliminary studies, we found that seven terms in the production amplitude ( $a_{0}, \ldots a_{6}$ ) resulted in a production amplitude that was smooth, however between three and seven terms, little deviation was found for the resonance parameters, and we include their effects as systematics (see Ref. [10]). We find for the two CDD poles, and six terms in the production expansion in $n(s)$, a $\chi^{2} /$ d.o.f $=1.9$ (see Fig. 2).


Figure 2: Intensity distribution and fits to the $J^{P C}=2^{++}$wave. Red lines are fit results, and the data is taken from Ref. [2]. The error bands correspond to the $3 \sigma(99.7 \%)$ confidence level. Figure taken from Ref. [10].

## 4. Resonance Poles

Resonance are associated with poles of partial wave amplitudes on unphysical Riemann sheets of the complex energy-plane [8]. We are interesting in extracting the resonance pole positions For our model, which amounts to continuing the denominator function $D(s)$ to the second sheet, and searching for zeroes, i.e. when $D_{\mathrm{II}}\left(s_{p}\right)=0$ for the pole positions $s_{p}$, where $D_{\mathrm{II}}(s)=D(s)+$ $2 i \rho(s) N(s)$. We define the mass and width of the resonance pole as $m=\operatorname{Re} \sqrt{s_{p}}$ and $\Gamma=-2 \operatorname{Im} \sqrt{s_{p}}$ respectively.

Fig. 3 shows many poles in the complex $s$-plane, three of which have $\operatorname{Re} s>0$, and two for $\operatorname{Re} s<0$. We want to understand the nature of all these singularities, primarily in identifying if these poles correspond to QCD resonances. We see two poles (corresponding to the $a_{2}$ and $a_{2}^{\prime}$ resonances) migrate to the real axis as the coupling is turned off, which is expected for the two CDD poles we chose. The other poles all migrate to $s=-1.5 \mathrm{GeV}^{2}$, which is the location of the lefthand singularity in our model. This shows clearly the origin of the poles. Fig. 3 shows snapshots at different values of $g$ as $g \rightarrow 0$. The poles that tend to $s=-1.5 \mathrm{GeV}^{2}$ are purely model effects, as dynamically generated poles are expected to migrate off to infinity as the coupling vanishes.

The two $a_{2}$ poles were found to have mass and width (see Fig. 4)

$$
\begin{array}{ll}
m\left(a_{2}\right)=(1307 \pm 1 \pm 6) \mathrm{MeV}, & m\left(a_{2}^{\prime}\right)=(1720 \pm 10 \pm 60) \mathrm{MeV} \\
\Gamma\left(a_{2}\right)=(112 \pm 1 \pm 8) \mathrm{MeV}, & \Gamma\left(a_{2}^{\prime}\right)=(280 \pm 10 \pm 70) \mathrm{MeV}
\end{array}
$$

where the first uncertainty is statistical (from the bootstrap analysis) and the second one systematic. The systematic uncertainty is obtained adding in quadrature the different systematic effects related to the fit model.


Figure 3: The migration of poles as a function of $g$ as $g \rightarrow 0$.


Figure 4: Pole positions for the $a_{2}$ and $a_{2}^{\prime}$, with six production terms and with $s_{R}$ varied from 1.0 $\mathrm{GeV}^{2}$ to $2.5 \mathrm{GeV}^{2}$. Poles are shown with $2 \sigma(95.5 \%)$ confidence level contours from uncertainties computed using $10^{5}$ bootstrap fits. Figure taken from Ref. [10]

## 5. Summary

We have shown that amplitudes constrained by $S$-matrix principles can describe the peripheral production process $\pi p \rightarrow \eta \pi p$ in the $2^{++}$sector. Resonance parameters were extracted and two resonances were found, corresponding to the $a_{2}$ and the first excited state, the $a_{2}^{\prime}$. This analysis serves as a template for further analyses, such as the full analysis of the $\eta^{(1)} \pi$ system (including the exotic $P$-wave), the $3 \pi$ system in $\pi^{-} p \rightarrow \pi^{-} \pi^{-} \pi^{+} p$, where the first sector under study is $2^{-+}$ $[12,13]$, and resonance production at GlueX [14].

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