The $B_c \to J/\psi DK$ Weak Decay Testing the Molecular Nature of $D_{s0}^*(2317)^+$

M. Bayar
Department of Physics, Kocaeli University, 41380 Izmit, Turkey
E-mail: melahat.bayar@kocaeli.edu.tr

Z-F. Sun
Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC
Institutos de Investigación de Paterna, Aptdo. 22085, 46071 Valencia, Spain
E-mail: sunzf@lzu.edu.cn

P. Fernandez-Soler
Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC
Institutos de Investigación de Paterna, Aptdo. 22085, 46071 Valencia, Spain
E-mail: pedro.fernandez@ific.uv.es

E. Oset
Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC
Institutos de Investigación de Paterna, Aptdo. 22085, 46071 Valencia, Spain
E-mail: oset@ific.uv.es

In this talk, we investigate the molecular nature of the $D_{s0}^*(2317)^+$ resonance in the weak decay of the $B_c$ meson to $J/\psi DK$. In this process, the heavy meson $B_c$ first decays into the quark pair $c\bar{s}$ via weak interaction and then the quark pair hadronizes into final meson states. The $D_{s0}^*(2317)^+$ resonance is dynamically generated in the final state interaction. We describe this interaction using the chiral unitary approach. Finally, we compute the $KD$ invariant mass distribution of the decay $B_c \to J/\psi DK$ and we learn about the nature of the $D_{s0}^*(2317)^+$.
1. Introduction

The \( D_{s0}^{*}(2317)^+ \) resonance first was discovered as a narrow peak in the inclusive \( e^+e^- \rightarrow D_1^+ \pi^0X \) annihilation process by the BABAR Collaboration in 2003 [1, 2] and later confirmed by CLEO, BELLE and FOCUS [3, 4, 5]. The average mass of \( D_{s0}^{*}(2317)^+ \) listed in the Particle Data Group (PDG) is \( m_{D_{s0}^{*}} = 2317.7 \pm 0.6 \) MeV [6].

The \( B_c \) state was first discovered by the CDF collaboration in the \( B_c \rightarrow J/\psi l^+\bar{\nu}_l \) process at Fermilab [7]. Later, the D0 collaboration has seen the \( B_c \) states in the \( B^\pm \rightarrow J/\psi \pi^\pm \) process [8]. The LHCb collaboration also observed the \( B_c \) meson in proton-proton collisions [9, 10]. The mass of the \( B_c \) meson listed in the PDG is \( 6274.9 \pm 0.8 \) MeV [6].

There are many theoretical works for the \( D_{s0}^{*}(2317)^+ \) resonance. For instance, the \( D_{s0}^{*}(2317)^+ \) meson was studied in the framework of molecular state, four-quark state, the mixture of two-meson and four-quark state and \( KD \) mixing with \( c\bar{s} \) state. Since the mass of the \( D_{s0}^{*}(2317)^+ \) is about 50 MeV below the threshold of the \( KD \) system, the molecular nature interpretation was proposed.

There is also a result from the lattice QCD simulations [11] the \( KD \) scattering length, where was extrapolated to physical pion masses making use of the Unitarized Chiral Perturbation Theory formalism, and by means of the Weinberg compositeness condition [12, 13] the amount of \( KD \) content in the \( D_{s0}^{*}(2317)^+ \) was determined.

2. Formalism

In this work, we investigate the \( D_{s0}^{*}(2317)^+ \) resonance in the \( B_c \rightarrow J/\psi DK \) decay. The detailed analysis can be seen in Ref. [14]. The decay mechanisms that we take into account here are the \( B_c \) meson decay into \( J/\psi DK \) and also into \( J/\psi D_{s0}^{*}(2317)^- \). We show the leading mechanisms describing the weak process in Fig 1. First, in these transitions we assume that the matrix element is constant in a small range of the \( KD \) invariant mass close to the \( KD \) threshold. The next step consists of the hadronization of the \( c\bar{s} \) pair into two mesons which is shown in Fig. 2. The hadronization is done introducing a \( \bar{q}q \) pair with the quantum numbers of the vacuum, \( c\bar{s}: c\bar{s}(u\bar{u} + d\bar{d} + s\bar{s}) \). First we consider the \( q\bar{q} \) matrix \( M \) as

\[
M = \begin{pmatrix}
  u\bar{u} & u\bar{d} & u\bar{s} & u\bar{c} \\
  d\bar{u} & d\bar{d} & d\bar{s} & d\bar{c} \\
  s\bar{u} & s\bar{d} & s\bar{s} & s\bar{c} \\
  c\bar{u} & c\bar{d} & c\bar{s} & c\bar{c}
\end{pmatrix}
= \begin{pmatrix}
  u \\
  d \\
  s \\
  c
\end{pmatrix}
\begin{pmatrix}
  \bar{u} & \bar{d} & \bar{s} & \bar{c}
\end{pmatrix}, \tag{2.1}
\]

Figure 1: Diagrams for the \( B_c^+ \) weak decay mechanism into a final configuration with a \( c\bar{s} \) state.
The \( B_c \rightarrow J/\psi DK \) Weak Decay Testing the Molecular Nature of \( D_{s0}^*(2317)^+ \)

M. Bayar

The hadronization of \( c \bar{s} \rightarrow c \bar{s}(u \bar{u} + d \bar{d} + s \bar{s}) \).

which has the property,

\[
M \cdot M = M \times (u \bar{u} + d \bar{d} + s \bar{s} + c \bar{c}) .
\]

If we write the matrix \( M \) in terms of mesons using the standard \( \eta - \eta' \) mixing, we have

\[
\phi = \begin{pmatrix}
\eta / \sqrt{3} + \pi^0 / \sqrt{2} + \eta'/6 \\
\pi^- / \sqrt{3} - \pi^0 / \sqrt{2} + \eta'/6 \\
K^- / \sqrt{3} - \eta / \sqrt{6} - \eta'/\sqrt{6} \\
D^0 \\
D^+
\end{pmatrix} , \quad (2.3)
\]

Then, in terms of meson fields we get

\[
(\phi \phi)_{43} = \eta^e D^+_s \bar{D}^0 K^0 + D^0 K^+ + K^0 D^+ - \frac{1}{\sqrt{3}} \eta D^+_s + \sqrt{\frac{2}{3}} D^+_s \eta'.
\]

Here, we neglect the contribution of \( \eta' \) and \( \eta^e \) because of their large mass compared with the \( K \) and \( \eta \) masses.

In a next step, the two mesons produced in the second process may interact with themselves in coupled channels, which is depicted in Fig. 3. The amplitude of the \( B_c^+ \rightarrow J/\psi D^+ K^0 \) decay is

\[
t(B_c^+ \rightarrow J/\psi D^+ K^0) = V_p h_1 + \sum_i t_{ij}^i G_i \eta_t_{i1}^i \). \quad (2.5)
\]

Here \( i = 1, 2, 3 \) which label the channels \( D^+ K^0, D^0 K^+ \) and \( D^+_s \eta \) respectively. \( t_{ij} \) is the scattering matrix element for the transition channel \( i \rightarrow j \). The unitarization of the amplitudes is done solving the on-shell version of the factorized Bethe-Salpeter equation in coupled channels:

\[
t = [1 - VG]^{-1} V, \quad (2.6)
\]
In Eq. (2.5), \( G_i \) is the loop function of two meson propagators
\[
G_i = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{(P - q)^2 - m_i^2 + i\epsilon} \frac{1}{q^2 - M_i^2 + i\epsilon}.
\] (2.7)
The loops are integrated using dimensional regularization, and regularized including a subtraction constant at some scale \( \mu \).

Since the process depicted in Fig. 1 is a \( 0^- \rightarrow 1^- 0^+ \) transition, the angular momentum between the \( J/\psi \) and the quark pair \( (c\bar{s}) \) is \( L = 1 \) due to the total angular momentum conservation. So \( V_p \) should have the form of
\[
V_p = \sqrt{3} A_{J/\psi} \cos \theta.
\] (2.8)

Thus, we can get the expression of \( d\Gamma/dM_{inv} \)
\[
\frac{d\Gamma}{dM_{inv}} = \frac{A^2}{(2\pi)^3} \frac{1}{4m_{B_i}^2} p^3_{J/\psi} \bar{p}_{DK} \sum_i \sum_j |\tilde{f}_{B_i^+ \rightarrow J/\psi D^+ K^0}|^2,
\] (2.9)
where \( M_{inv} \) is the invariant mass of the \( D^+ K^0 \) system, and \( \tilde{f}_{B_i^+ \rightarrow J/\psi D^+ K^0} \) is \( t_{B_i^+ \rightarrow J/\psi D^+ K^0}/V_p \). The value of \( A \) is chosen to normalize the invariant mass distribution and it will cancel in the ratios that we shall construct. In Eq. (2.9) \( p_{J/\psi} \) is the momentum of the \( J/\psi \) in the global CM frame and \( \bar{p}_{DK} \) is the kaon momentum in the \( D^+ K^0 \) rest frame.

We also investigate the production of the resonance \( D^*_{s0}(2317)^+ \) under the assumption that it is dynamically generated from the \( DK \) and \( \eta D_s \) channels. The amplitude for the production of the resonance \( R \) (in this case the \( D^*_{s0}(2317)^+ \)) is given by
\[
t(B_c^+ \rightarrow J/\psi R) = V_p \sum_i h_i G_i |\psi|_{pole},
\] (2.10)
where \( i \) sums over \( K^+ D^0, K^0 D^+, \eta D_s \). The width for the production of the resonance \( R \), irrelevant of which decay channel it has, is given by
\[
\Gamma(B_c^+ \rightarrow J/\psi R) = \frac{A^2}{8\pi m_{B_c^+}^2} |\tilde{f}(B_c^+ \rightarrow J/\psi D^*_{s0}(2317)^+)|^2 p^3_{J/\psi} |\psi|_{pole}.
\] (2.11)
It is then interesting to study the ratio
\[
\frac{d\tilde{\Gamma}}{dM_{inv}} = M_R^2 \frac{(d\Gamma/dM_{inv})/p^3_{J/\psi} \bar{p}_{DK}}{\Gamma(B_c^+ \rightarrow J/\psi R)/p^3_{J/\psi} |\psi|_{pole}}
\]
\[
= \frac{M_R^2}{4\pi^2} \frac{|\tilde{f}(B_c^+ \rightarrow J/\psi D^+ K^0)|^2}{|\tilde{f}(B_c^+ \rightarrow J/\psi D^*_{s0}(2317)^+)|^2}
\]
\[
= \frac{M_R^2}{4\pi^2} \frac{|h_{D^+ K^0} + \sum_i h_i G_i |\psi|_{pole}}{|\sum_i h_i G_i |\psi|_{pole}}.
\] (2.12)
where the factor \( M_R^2 \) is put in the formula for convenience in order to have a dimensionless quantity.
3. Results

We show the result for the differential decay width for the reaction of $B_c^- \rightarrow J/\psi D^+ K^0$ in Fig. 4. There the line shape of the differential decay width and the phase space have been normalized to unity over the range of the $DK$ invariant mass in the figure.

**Figure 4:** Differential decay width for the reaction $B_c^- \rightarrow J/\psi D^+ K^0$. The solid curve corresponds to $(\alpha(\mu), \mu) = (-1.265, 1.50 \text{ GeV})$. The dash dot curve is the phase space.

In Fig. 5 we plot $\frac{df}{dM_{inv}}$ of Eq. (2.12). We see a fall down of the distribution as a function of the $K^+ D^0$ invariant mass. This is a clear indication of the presence of a resonance below threshold since we have divided the original invariant mass distribution by the phase space. Hence, essentially we are plotting $|\Gamma(B_c^- \rightarrow J/\psi D^+ K^0)|^2$, which peaks at the mass of the $D_{s0}^*(2317)^+$ and we are seeing the tail of the resonance.

As a summary we have investigated the $B_c^- \rightarrow J/\psi D^+ K^0$ where $B_c^- \rightarrow J/\psi$ and the quark pair $c\bar{s}$ via weak interaction; then the quark pair $c\bar{s}$ hadronizes into $D^+ K^0$, $D^0 K^+$ or $D_s^+ \eta$ components which can interact among themselves generating the $D_{s0}^*(2317)^+$ resonance.
We have calculated the differential decay width of the reaction $B_c^+ \rightarrow J/\psi D^+ K^0$. One can appreciate that the shape of the distribution peaks closer to the $DK$ threshold than the phase space, indicating the coupling of $DK$ to a resonance below threshold (the $D^{*}_{s0}(2317)^+$ in this case). We also evaluated the rate of production of the $D^{*}_{s0}(2317)^+$ resonance and then constructed the ratio of $d\Gamma/dM_{inv}(B_c^+ \rightarrow J/\psi D^+ K^0)$ to the width for $D^{*}_{s0}(2317)^+$ production, where the unknown factor $V_p$ of our theory cancels. The new normalized distribution obtained is then a prediction of the theory, only tied to the fact that the $D^{*}_{s0}(2317)^+$ is dynamically generated from the $DK$ and $\eta D_s$ channels.

Acknowledgments

This work is partly supported by the Spanish Ministerio de Economia y Competitividad and European FEDER funds under the contract number FIS2011-28853-C02-01 and FIS2011-28853-C02-02, and the Generalitat Valenciana in the program Prometeo II-2014/068. We acknowledge the support of the European Community-Research Infrastructure Integrating Activity Study of Strongly Interacting Matter (acronym HadronPhysics3, Grant Agreement n. 283286) under the Seventh Framework Programme of EU.

References