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Triangle Singularities in the $\Lambda_b \to J/\psi K^- p$ Reaction

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In this work, we have studied the $\Lambda_b \to J/\psi K^- p$ decay via Λ^*-charmonium-proton intermediate states and discussed all possible triangle singularities. Using this process we have done a detailed analysis of the singularities of the triangle amplitude and derived a formula for an easy evaluation of the singularities. We have stressed that the χ_{c1} and the $\psi(2S)$ are the relatively most relevant states among all possible charmonia up to the $\psi(2S)$. Particularly the $\Lambda(1890)\chi_{c1}$ pair plays a very special role, since the threshold and triangle singularities merge. In the case o of $J^P = \frac{3}{2}^-$, $\frac{5}{2}^+$ for the narrow P_c, one needs P- and D-waves, respectively, in the $\chi_{c1}\,p$. This feature reduces the strength of the contribution and smoothens very much the peak. In this case the singularities cannot account for the observed narrow peak. On the other hand, for the case of $J^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$ with the latter one of the favored quantum numbers, where $\chi_{c1}\,p \to J/\psi\, p$ can proceed in an S-wave, the $\Lambda(1890)\,\chi_{c1}\,p$ triangle diagram could play an important role.

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1. Introduction

In 2015, the LHCb collaboration observed a narrow peak around 4450 MeV in the $\Lambda_b \to J/\psi K^- p$ decay in the $J/\psi p$ invariant mass spectrum [1, 2]. The possibility that these hidden charm pentaquark-like structures might be due to a triangle singularity was immediately noted [3, 4, 5].

Triangle singularities in physical processes were introduced by Landau [6] and stem from Feynman diagrams involving three intermediate particles when the three particles can be placed simultaneously on shell and the momenta of these particles are collinear (parallel or antiparallel) in the frame of an external decaying particle at rest. In one of the cases (we call it parallel), two of the particles in the loop will go in the same direction and might fuse into other external outgoing particle(s) [7], so that the rescattering process can even happen as a classical process. In this case, the decay amplitude has a singularity close to the physical region and, thus, can produce an enhancement. One particular classical case is given when the two on shell particles move in the same direction and with the same velocities. In the center-of-mass frame of the rescattering particles, these two particles would also be at rest and the triangle singularity is then located at the threshold.

In this talk, we invesigate possible triangle singularities in the $\Lambda_b \to J/\psi K^- p$ process involving various Λ^* and charmonium intermediate states. We search for many combinations of a Λ^* hyperon and a charmonium in the triangle diagram. However, only a few of them are important that we are going to discuss here. For detail see Ref. [8].

2. Detailed analysis of the triangle singularity

The triangle diagram for the $\Lambda_b \to J/\psi K^- p$ reaction is depicted in Fig. 1. In this process we assume that Λ_b decays first to a Λ^* and a charmonium state, the Λ^* decays into $K^- p$ and then the charmonium state and the p react to give the $J/\psi p$. Thus we have $J/\psi K^- p$ in the final state as in the experiment of [1].

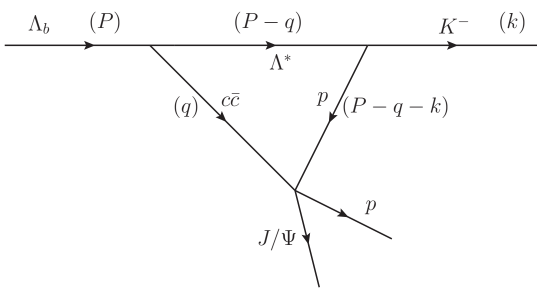


Figure 1: Triangle diagram for the $\Lambda_b \to J/\psi K^- p$ decay, where Λ^* stands for the different Λ^* considered in the analysis of [1] and $c\bar{c}$ stands for different charmonium states. In brackets, the momenta of the corresponding lines are given.

Let us first consider a triangle diagram in general which is shown in Fig. 2. To analyze the singularity structure, it is sufficient to calculate the following integral:

$$I(m_{23}) = \int d^3q \frac{1}{[P^0 - \omega_1(\vec{q}) - \omega_2(\vec{q}) + i\varepsilon]\left[E_{23} - \omega_2(\vec{q}) - \omega_3(\vec{k}+\vec{q}) + i\varepsilon\right]}$$

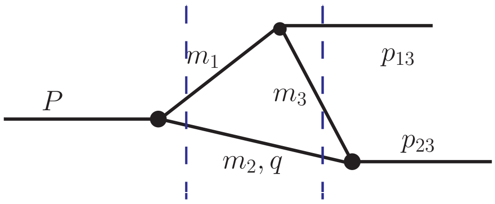


Figure 2: A triangle diagram showing the notations used in the general discussion of triangle singularities, where m_i's denote the masses of the intermediate particles, and P, p_{13}, p_{23} correspond to the four-momenta of the external particles. The two dashed vertical lines correspond to the two relevant cuts.

$$= 2\pi \int_0^\infty dq \, \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\varepsilon} f(q)\,, \tag{2.1}$$

where $\omega_{1,2}(q) = \sqrt{m_{1,2}^2 + q^2}$, $\omega_3(\vec{q} + \vec{p}_{13}) = \sqrt{m_3^2 + (\vec{q} + \vec{p}_{13})^2}$, $E_{23} = P^0 - p_{13}^0$, and

$$f(q) = \int_{-1}^{1} dz \, \frac{1}{E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + k^2 + 2qkz + i\varepsilon}}\,, \tag{2.2}$$

where $k = |\vec{p}_{13}| = \sqrt{\lambda(M^2, m_{13}^2, m_{23}^2)}/(2M)$, with $M = \sqrt{P^2}$ and $m_{13,23} = \sqrt{p_{13,23}^2}$, and $q = |\vec{q}\,|$. We need to analyze the singularity structure of a double integration: one over q and one angular integration over z.

The cut crossing particles 1 and 2 provides a pole of the integrand of $I(m_{23})$ given by

$$P^0 - \omega_1(\vec{q}\,) - \omega_2(\vec{q}\,) + i\varepsilon = 0\,, \tag{2.3}$$

identifying $m_1 = m_{\Lambda^*}$ and $m_2 = m_{c\bar{c}}$. However, we have kept the $i\varepsilon$ here explicitly, which is important to determine the singularity locations in the complex-q plane. The solution is

$$q_{\text{on}+} = q_{\text{on}} + i\varepsilon\,, \qquad q_{\text{on}} = \frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_2^2)}\,. \tag{2.4}$$

The function $f(q)$ has endpoint singularities, which are logarithmic branch points, given when the denominator of the integrand vanishes for z taking the endpoint values ± 1, i.e., the solutions of

$$E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + k^2 \pm 2qk} + i\varepsilon = 0\,, \tag{2.5}$$

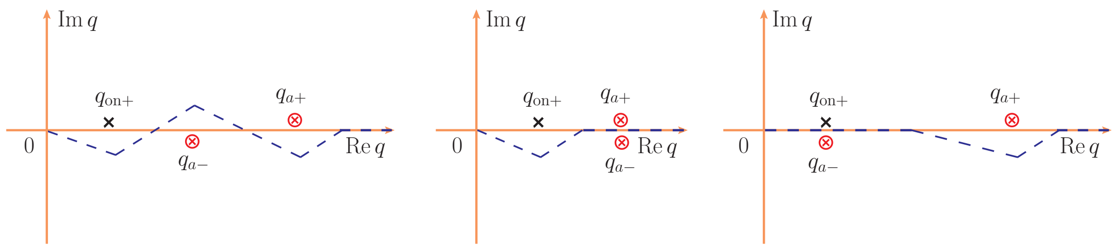
by identifying $m_2 = m_{c\bar{c}}$ and $m_3 = m_p$. The $+$ and $-$ signs correspond to $z = +1$ and -1, i.e., the situations for the momentum of particle 2 to be anti-parallel and parallel to the momentum of the (2,3) system in the frame with $\vec{P} = 0$, respectively.

Eq. (2.5) has two solutions for $z = -1$:

$$q_{a+} = \gamma(vE_2^* + p_2^*) + i\varepsilon\,, \qquad q_{a-} = \gamma(vE_2^* - p_2^*) - i\varepsilon\,, \tag{2.6}$$

The solutions for $z = 1$ are:

$$q_{b+} = \gamma(-vE_2^* + p_2^*) + i\varepsilon\,, \qquad q_{b-} = -\gamma(vE_2^* + p_2^*) - i\varepsilon\,. \tag{2.7}$$



(a) (b) (c)

Figure 3: Pertinent singularities of the integrand of $I(m_{23})$ when $\lim_{\varepsilon\to 0}(q_{a-})$ is positive. (a) is for the case without any pinching, (b) shows the case when the integration path is pinched between q_{a+} and q_{a-}, which gives the two-body threshold singularity, and (c) is for the case when the pinching happens between $q_{\rm on+}$ and q_{a-}, which gives the triangle singularity. The dashed lines correspond to possible integration paths.

In Fig. 3, the integrand has three relevant singularities: a pole $q_{\rm on+}$ and two logarithmic branch points $q_{a\pm}$. In Fig. 3 (a), all of them are located at different positions and one can deform the integration path freely as long as it does not hit any singularity of the integrand. In such a kinematic region, $I(m_{23})$ is analytic. In Fig. 3 (b), when $m_{23} = m_2 + m_3$ or $p_2^* = 0$, the integration path is pinched between q_{a-} and q_{a+} we have the normal two-body threshold singularity. In Fig. 3 (c), the integration path is pinched between q_{a-} and $q_{\rm on+}$ and one gets the triangle singularity or anomalous threshold which is a logarithmic branch point. Therefore, the condition for a triangle singularity to emerge is given mathematically by

$$\lim_{\varepsilon\to 0}(q_{\rm on+} - q_{a-}) = 0\,. \tag{2.8}$$

This is only possible when all three intermediate particles are on shell and meanwhile $z = -1$.

Note that the singularity is in the physical region only when the process can happen classically: all the intermediate states are on shell, and the particle 3 emitted from the decay of the particle 1 moves along the same direction as the particle 2 with a largespeed than the particle 2 and can catch up with it to rescatter.

For given m_2, m_3 and invariant masses for external particles, one can also work out the range of m_1 where the triangle singularity shows up, as well as the range of the triangle singularity in m_{23}. For $q_{\rm on}$ and q_{a-} (taking $\varepsilon = 0$) taking values in their physical regions, one needs to have $m_1 \le M - m_2$ and $m_{23} \ge m_2 + m_3$.

$$m_1^2 \in \left[\frac{M^2 m_3 + m_{13}^2 m_2}{m_2 + m_3} - m_2 m_3\,,\ (M - m_2)^2\right], \tag{2.9}$$

$I(m_{23})$ has a triangle singularity, and it is within the range

$$m_{23}^2 \in \left[(m_2 + m_3)^2,\ \frac{M m_3^2 - m_{13}^2 m_2}{M - m_2} + M m_2\right]. \tag{2.10}$$

3. Results

We study the $\Lambda_b \to K^- J/\psi\, p$ from triangle diagrams with a Λ^* hyperon, a charmonium and a proton as the intermediate states. We take $\Lambda(1405)$, $\Lambda(1520)$, $\Lambda(1600)$, $\Lambda(1670)$, $\Lambda(1690)$,

$c\bar{c}$	Most relevant range of M_{Λ^*} (MeV)	Range of triangle singularity (MeV)
η_c	[2226, 2639]	[3919, 4283]
J/ψ	[2151, 2523]	[4035, 4366]
χ_{c0}	[1949, 2205]	[4353, 4588]
χ_{c1}	[1887, 2109]	[4449, 4654]
χ_{c2}	[1858, 2063]	[4494, 4686]
h_{c1}	[1878, 2094]	[4464, 4664]
$\eta_c(2S)$	[1806, 1983]	[4575, 4741]
$\psi(2S)$	[1774, 1933]	[4624, 4775]

Table 1: For each charmonium, the triangle singularity produces prominent effects if the Λ^* mass takes a value within the the range given in the second column and the singularity range is shown in the last column correspondingly.

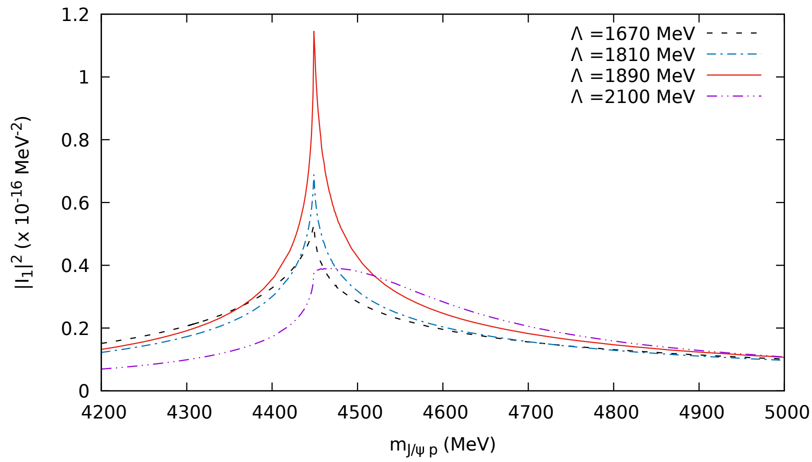


Figure 4: The value of $|I_1|$ for $\Lambda^*\chi_{c1}$ with a width $\Gamma = 100$ MeV for the hyperon.

$\Lambda(1800)$, $\Lambda(1810)$, $\Lambda(1820)$, $\Lambda(1830)$, $\Lambda(1890)$, $\Lambda(2100)$, $\Lambda(2110)$, $\Lambda(2350)$, $\Lambda(2585)$ as Λ^*, coupled to K^-p states, and $\eta_c(1S)$, J/ψ, $\chi_{cJ}(1P)$ $(J = 0,1,2)$, $h_c(1P)$, $\eta_c(2S)$, and $\psi(2S)$ as charmonium. The mass range allowed for the Λ^* particles can be seen in Table 1, for a certain charmonium state, in order to have a triangle singularity.

In Fig. 4, We show the contribution to $|I_1|^2$ (for $|I_1|$ see Eq. (2) Ref. [8]) from a selected choice of the Λ^* states. We can see that all of them peak around $m_{23} = 4450$ MeV, which is the $\chi_{c1}\,p$ threshold. The largest strength, with the sharpest shape, comes from the $\Lambda(1890)$ where the threshold and the triangle singularities merge.

The cusp structure in the curve for the $\Lambda(1670)$ comes from the threshold singularity (see in the second column of Table 1). The peak of the $\Lambda(1810)$ is sharper, even if the Λ^* mass is outside the range of the triangle singularity (see Table 1), but the most relevant factor in the structure is the threshold singularity.

The $\Lambda(2100)$ is inside the range of the triangle singularities as seen in Table 1. The structure of I_1 for this Λ^* state shows a bump, in addition to the normal threshold cusp, around that energy,

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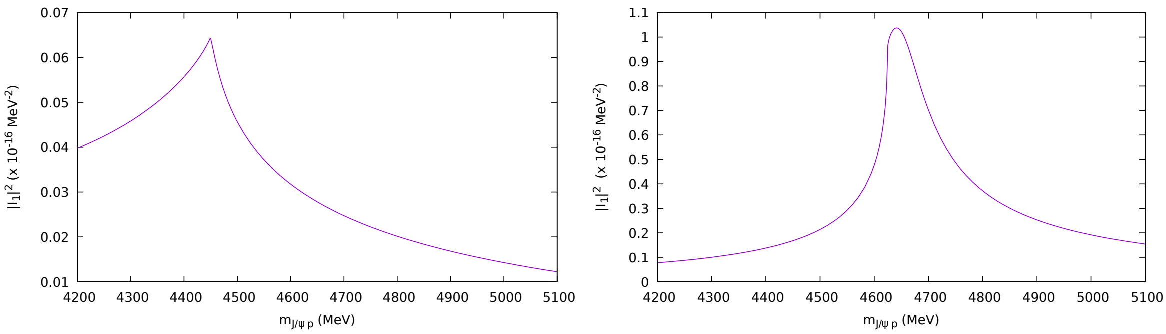

Figure 5: The value of $|I|^2$ for P-wave $\Lambda(1890)\chi_{c1}$ (left) and S-wave $\Lambda(1890)\psi(2S)$ (right). A constant width of $\Gamma = 100$ MeV is used for the $\Lambda(1890)$.

as a consequence of the smearing of the triangle singularity by the width of the Λ^*, as discussed before.

In Fig. 5, we depict the $|I_2|^2$ (for $|I_2|$ see Eq. (25) Ref. [8]) for the $c\bar{c} = \chi_{c1}$ which requires a P-wave in the $\chi_{c1}\, p$ system and $c\bar{c} = \psi(2S)$ which requires an S-wave in both the $\psi(2S)\, p$ and $J/\psi\, p$ channels. As we see in Fig. 5, the amplitude for P-wave $\Lambda(1890)\chi_{c1}$ is very much suppressed the S-wave case. This is natural. Because the singularity appears when the $\chi_{c1}\, p$ on shell and at threshold where the P-wave factor vanishes. The S-wave structure is very much peaked and a narrow, while the P-wave has a background below the peak accumulating more strength than the peak. In Fig. 5, right, we show $|I|^2$ for the case of $c\bar{c} = \psi(2S)$ and we see a peak around 4630 MeV. This could lead to $\frac{3}{2}^-$, but in the experimental data the $J/\psi p$ invariant mass distribution in this region is flat. We, thus, conclude that if the narrow $J/\psi\, p$ has $3/2^-$ the triangle singularities due to $\Lambda^*\, c\bar{c}\, p$ intermediate states cannot play an important role in the decay $\Lambda_b \to K^- J/\psi\, p$.

In the case of $1/2^+, 3/2^+$, the $\chi_{c1}\, p$ amplitude proceeds via an S-wave and we would have the situation shown as the solid curve in Fig. 4. The peak is narrow enough and located at the right position. Hence the $\Lambda(1890)\, \chi_{c1}\, p$ triangle diagram could play an important role.

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