# The $a_{1}(1420)$ peak as the special decay mode of the $a_{1}(1260)$ 

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We study the mechanism of triangular singularity of the $a_{1}(1260)$ decay to $\pi^{+} f_{0}(980)$ within the chiral unitary approach. Our results provide a natural explanation of all the features observed in the COMPASS experiment. Therefore the $a_{1}(1420)$ cannot be accepted as a new resonance since conventional explanation for it has found. The present finding is rather remarkable example of how a decay mode of one resonance can peak at 200 MeV higher energy than the nominal resonance mass.

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## 1. Motivation

Recent COMPASS experiment reported the observation of a peak around 1420 MeV in the $f_{0}(980) \pi$ final state, with the pion in P-wave. Then, the $f_{0}(980)$ decays into $\pi^{+} \pi^{-}$in S-wave. This state was observed in the diffractive scattering of $190 \mathrm{GeV} \pi^{-}$beam on a proton target [1]. The quantum numbers of the final state correspond to a $I^{G}\left(J^{P C}\right)=1^{-}\left(1^{++}\right)$configuration, so it was claimed as a new $a_{1}(1420)$ resonance. Is this really a new resonance? Could this peak be viewed by conventional explanation?

In this talk, the calculated results of $a_{1}(1420)$ peak as the $\pi f_{0}(980)$ decay mode of the $a_{1}(1260)$ triangular mechanism within the chiral unitary approach [2] is presented. Surprisingly, we find a natural explanation of all the features observed in the COMPASS experiment.

Actually the triangle singularity was firstly studied by Landau [3] and it appears from processes involving a Feynman diagram which has a loop with three intermediate particles, when the three of them are placed on shell and the momenta of the particles are collinear. Theoretically, there are some recent works to study triangular singularity, such as describing the $\eta$ (1405) isospin forbidden decays [4] and recently being advocated [5] as an explanation for $\Lambda_{b}$ pentaquark state [6]. Here we would like to mention the paper [7], in which they challenged the claim of the COMPASS peak as a signal of a new resonance, providing a natural explanation of it based on the triangular singularity that unavoidably stems from the decay of the $a_{1}(1260)$ resonance into $K^{*} \bar{K}$ followed by $K^{*}$ decay to $\pi K$, with the pion emitted and the remaining $K \bar{K}$ merging into the $f_{0}(980)$ resonance. However, there is a very important difference between the work of [5] and those of [7]. The work of [5] provides a speculation and there is lack of the couplings needed and of strength of the singularity, while in the work of Ref. [7] the dynamics is well known, and then a prediction of the strength of the singularity can be made, but they do not know exactly everything.

The purpose of the present work is to provide an independent confirmation of the results and conclusions of [7] by studying the triangle singularity within the chiral unitary approach. The novelty should be emphasized. Firstly, we provide an independent confirmation, in which the strength is based on our previous theoretical findings. we can provide good grounds for strength of the singularity which was a guess made in Ref. [7]. Secondly, we offer a technically different derivation and we can provide an answer to questions which were left open in [1]. Thirdly and importantly, we can determine the interference of the $K^{*} K \bar{K}$ and $\rho \pi \pi$ loops which could not be resolved in [7]. Here we can do it, since in our picture the $a_{1}(1260)$ is dynamically generated from the $K^{*} \bar{K}$ and $\rho \pi$ channels [8] and the theory provides the coupling of the resonance to these channels with a well defined relative sign. We do use the chiral unitary approach. Yet, the final results are very similar in what concerns the position, width and relative strength of the peak, giving a boost to the idea raised in Ref. [7], and providing a natural description of the peak seen in COMPASS experiment[1].

## 2. Formalism

In the present case, the decay process observed in [1] can be evaluated by means of the following triangular mechanism, in which four diagrams contribute to the process and the two kaons
or the two pions rescatter, leading to the $\pi \pi$ pair in the final state via the production of the $f_{0}(980)$ resonance.


Figure 1: Feynman diagrams for the process $a_{1}^{+}(1260) \rightarrow \pi^{+} \pi^{+} \pi^{-}$, where the $a_{1}^{+}(1260)$ decays to $\bar{K}^{*} K$ followed by the $K^{*} \rightarrow \pi K$ [diagrams (a) and (b)] and to $\rho \pi$ followed by the $\rho \rightarrow \pi \pi$ [diagrams (c) and (d)]

The production mechanism is completely analogous to the one already used to evaluate the decay $f_{1}(1285) \rightarrow a_{0}(980) \pi$ and we follow the same procedure [9]. There are three vertices contributing to the diagrams.

1. The $a_{1} P V$ vertex

$$
\begin{equation*}
-i t_{1}=-i g_{i} C_{1} \varepsilon_{a_{1}}^{\mu} \varepsilon_{\mu} \tag{2.1}
\end{equation*}
$$

where $\varepsilon_{a_{1}}$ is the polarization vector of the $a_{1}(1260)$ and $\varepsilon$ the one of the $K^{*}$ for the diagrams (a) and (b) and of the $\rho$ for (c) and (d). The couplings $g_{i}$ of the $a_{1}(1260)$ to its building blocks, where $i=K^{*} \bar{K}, \rho \pi$, are obtained as the residue at the pole of the scattering amplitude in isospin $I=1$, which close to the pole can be written as

$$
\begin{equation*}
T_{i j} \simeq \frac{g_{i} g_{j}}{s-s_{P}} \tag{2.2}
\end{equation*}
$$

with $\sqrt{s_{P}}$ the position of the pole on the complex plane corresponding to the resonance. In Ref. [8], the authors get the following values for the couplings in isospin basis,

$$
\begin{equation*}
g_{K^{*} \bar{K}}=(1872-i 1486) \mathrm{MeV} \quad g_{\rho \pi}=(-3795+i 2330) \mathrm{MeV} \tag{2.3}
\end{equation*}
$$

corresponding to a pole in $\sqrt{s_{P}}=(1011+i 84) \mathrm{MeV}$.
2. The $P P V$ vertex

The structure of the vertex for the $P P V$ interaction can be evaluated by means of the hidden gauge symmetry Lagrangian [10, 11, 12, 13].

$$
\begin{equation*}
\mathscr{L}_{P P V}=-i g\left\langle V^{\mu}\left[P, \partial_{\mu} P\right]\right\rangle \tag{2.4}
\end{equation*}
$$

where the symbol $\left\rangle\right.$ stands for the trace in $S U(3)$ and $g=\frac{m_{V}}{2 f}$, with $f=93 \mathrm{MeV}$ the pion decay constant and $m_{V} \simeq m_{\rho}$. The matrices $P$ and $V$ contain the nonet of the pseudoscalar mesons and the one of the vectors respectively, and the resulting amplitude for the vertex is

$$
\begin{equation*}
-i t_{2}=-i g C_{2}(P-q-2 k)_{\mu} \varepsilon^{\mu} \tag{2.5}
\end{equation*}
$$

where the coefficients of $C_{1}$ and $C_{2}$ for different diagrams are listed in the Ref.[2, 8].
3. The third vertex

It corresponds to the mechanism for the production of the $\pi^{+} \pi^{-}$pair in the final state, after the rescattering of the $K \bar{K}$ or $\pi \pi$, that dynamically generates the $f_{0}(980)$ resonance as intermediate state [14],

$$
\begin{equation*}
-i t_{3}=-i t_{i f} \tag{2.6}
\end{equation*}
$$

where $t_{i f}$ is the element of the $5 \times 5$ scattering matrix $t$, we have $f=1,2,3,4,5$ for the channels $\pi^{+} \pi^{-}, \pi^{0} \pi^{0}, K^{+} K^{-}, K^{0} \bar{K}^{0}$ and $\eta \eta$ and $i=1,2,3,4$ for the diagrams (d), (c), (a) and (b), respectively. The $t$ matrix is obtained using the Bethe-Salpeter equation, with the tree level potentials given in Refs. [14, 15]. The loop functions for the intermediate states are regularized using the cutoff method and the peak of the $f_{0}(980)$ is well reproduced using a cutoff of 630 MeV .

Now come to amplitude for $a_{1}^{+}(1260) \rightarrow \bar{K}^{*} K$ and $\rho \pi$,

$$
\begin{gather*}
t_{\bar{K}^{*} K}=-g_{\bar{K}^{*} K} g \frac{1}{\sqrt{2}} \overrightarrow{\varepsilon_{a_{1}}} \cdot \vec{k}\left(2 I_{1}+I_{2}\right)\left(t_{31}+t_{41}\right)=\tilde{t}_{\bar{K}^{*} K} \overrightarrow{\varepsilon_{a_{1}}} \cdot \vec{k},  \tag{2.7}\\
t_{\rho \pi}=g_{\rho \pi} g \overrightarrow{\varepsilon_{a_{1}}} \cdot \vec{k}\left(2 I_{1}^{\prime}+I_{2}^{\prime}\right)\left(t_{11}+\sqrt{2} t_{21}\right)=\tilde{t}_{\rho \pi} \overrightarrow{\varepsilon_{a_{1}}} \cdot \vec{k}
\end{gather*}
$$

The total amplitude for the decay given by

$$
\begin{equation*}
t_{\mathrm{tot}}=t_{\bar{K}^{*} K}+t_{\rho \pi} \tag{2.8}
\end{equation*}
$$

Here we should mention that the loop integrals $I_{1}$ and $I_{2}$ and $I_{1}^{\prime}$ and $I_{2}^{\prime}$ which are given in Ref. [2] and more discussions can be found in Ref. [16].

In our chiral unitary approach, the formalism differs technically from the Feynman parametrization, analytically in the integration of $d q^{0}$ while $d^{3} q$ integration numerically, here we only give

$$
\begin{align*}
I_{1}= & -\int \frac{d^{3} q}{(2 \pi)^{3}} \frac{1}{8 \omega(q) \omega^{\prime}(q) \omega^{*}(q)} \frac{1}{k^{0}-\omega^{\prime}(q)-\omega^{*}(q)+i \frac{\Gamma_{K^{*}}}{2}} \frac{1}{P^{0}-\omega^{*}(q)-\omega(q)+i \frac{\Gamma_{K^{*}}}{2}} \\
& \times \frac{2 P^{0} \omega(q)+2 k^{0} \omega^{\prime}(q)-2\left(\omega(q)+\omega^{\prime}(q)\right)\left(\omega(q)+\omega^{\prime}(q)+\omega^{*}(q)\right)}{\left(P^{0}-\omega(q)-\omega^{\prime}(q)-k^{0}+i \varepsilon\right)\left(P^{0}+\omega(q)+\omega^{\prime}(q)-k^{0}-i \varepsilon\right)} \tag{2.9}
\end{align*}
$$

where $\omega(q)=\sqrt{\vec{q}^{2}+m_{K}^{2}}, \omega^{\prime}(q)=\sqrt{(\vec{q}+\vec{k})^{2}+m_{K}^{2}}, \omega^{*}(q)=\sqrt{\vec{q}^{2}+m_{K^{*}}^{2}}$ are the energies of the kaons and of the $K^{*}$ in the triangular loop. We included the finite width of the $K^{*}$ in its propagator, $\Gamma_{K^{*}}$, that we take equal to 48 MeV .

## 3. Discussion of the results

The invariant mass distribution for the process is given by

$$
\begin{equation*}
\frac{d \Gamma}{d M_{\mathrm{inv}}}=\frac{1}{(2 \pi)^{3}} \frac{1}{4 m_{a_{1}}^{2}} \frac{1}{3}|\vec{k}|^{3} p_{\pi}|\tilde{t}|^{2}, \tag{3.1}
\end{equation*}
$$

where $M_{\mathrm{inv}}$ is the invariant mass of the final $\pi^{+} \pi^{-}$pair, $p_{\pi}$ is the momentum of the pion of the interacting pion pair in the $\pi^{+} \pi^{-}$rest frame and $|\vec{k}|$ the momentum of the spectator $\pi^{+}$in the $a_{1}(1260)$ rest frame. The results of invariant mass distribution for $a_{1}^{+}(1260) \rightarrow \pi^{+} \pi^{+} \pi^{-}$decay are plotted in Figure 2, in which the widths of the $K^{*}$ and $\rho$ are removed in order to show the effect of singularity. From Figure 2 we can see clearly that the relative sign between the two mechanisms and their interference, that is the $\bar{K}^{*} K$ contribution is dominant and $\rho \pi$ contribution is small. This is consistent with the estimate [7]. If we change the mass of $a_{1}$, we can see the variation of the strength at the peak of the distribution, peaking around 1420 MeV .


Figure 2: $d \Gamma / d M_{\mathrm{inv}}$ for $a_{1}^{+}(1260) \rightarrow \pi^{+} \pi^{+} \pi^{-}$decay as a function of $M_{\mathrm{inv}}$ with nominal resonance mass 1230 MeV of $a_{1}$ (left); considering all the contributions but for different values of the mass of the $a_{1}$ (right)


Figure 3: Decay widths of $a_{1}^{+}$(1260) for different processes as a function of $m_{a_{1}}$ (left); cross section as a function of center of mass energy for the decay of $a_{1}^{+}(1260) \rightarrow \pi^{+} \rho^{0}$ (center); and $a_{1}^{+}(1260) \rightarrow \pi^{+} f_{0}(980)$ (right). Here the widths of the $K^{*}$ and $\rho$ are all kept.

Integrating the invariant mass distribution, we can evaluate the width for the decay $a_{1}^{+}(1260) \rightarrow$ $\pi^{+} f_{0}(980)$. In order to relate our findings to the results of Ref. [7], we evaluate the width of the decay of the $a_{1}(1260)$ to its dominant decay channel $\rho \pi$. All the results are given in Figure 3 (left),
from which we can see that when the widths are kept, the effect of the singularity is softened, and the shape of the distributions and relative weight are very similar with those in Ref. [7].

The cross sections can be obtained by multiplying the decay widths and the propagator

$$
\begin{equation*}
\frac{d \sigma_{\pi^{+} X}}{d s} \propto \frac{1}{\left(s-m_{a 1}^{* 2}\right)^{2}+\left(m_{a 1}^{*} \Gamma_{a_{1}}\right)^{2}} \Gamma_{\pi^{+} X}(s) \tag{3.2}
\end{equation*}
$$

where $\sqrt{s}=m_{a_{1}}$ is the center of mass energy of the decay, $m_{a_{1}}^{*}=1230 \mathrm{MeV}$ the nominal mass of the $a_{1}(1260)$ and $\Gamma_{a_{1}}$ its width, that we take equal to 350 MeV . The results of cross section are plotted in Figure 3. For $a_{1}^{+}(1260) \rightarrow \pi^{+} \rho^{0}$, it can be seen the shape of the $a_{1}^{+}(1260)$ resonance, peaking around 1230 MeV and with its standard width. However, for $a_{1}^{+}(1260) \rightarrow \pi^{+} f_{0}(980)$, we see a peak around 1420 MeV and a with of about 150 MeV . The relative strength agrees with COMPASS data [1] and similar with those of Ref. [7]. And more interestingly, we find that the peak appears only in the peculiar reaction due to the triangular singularity.

In summary, the triangular singularity of the $a_{1}^{+}(1260)$ decay to $\pi^{+} f_{0}(980)$ is studied within chiral unitary approach, the results provide a natural description of the peak seen in COMPASS experiment, so the $a_{1}(1420)$ cannot be accepted as a new resonance. This is quite a unique finding and rather remarkable, in the sense that it produces a peak for a decay mode of the resonance at an energy about 200 MeV higher than the nominal mass of the resonance.

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