PROCEEDINGS OF SCIENCE

BChPT$\times 1/N_c$: masses and currents

Ishara. P. Fernando and José L. Goity*
Department of Physics, Hampton University, Hampton, VA 23668, USA, and
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA.
E-mail: ishara@jlab.org, goity@jlab.org

A summary of the implementation of the combined BChPT$\times 1/N_c$ expansion for three flavors is presented, along with its applications to the octet and decuplet baryon masses, $SU(3)$ charges and axial couplings.

XVII International Conference on Hadron Spectroscopy and Structure - Hadron2017
25-29 September, 2017
University of Salamanca, Salamanca, Spain

*Speaker.

1. Introduction

The fact that in principle QCD admits an expansion in $1/N_c$ should be reflected in effective field theories (EFTs). The many indications that general consequences of the $1/N_c$ expansion seem to be manifested at $N_c = 3$, as shown by phenomenology and also by lattice QCD (LQCD), emphasizes the need of that implementation. The consequences in the case of baryons are very significant, as it was realized long ago [1–4], in particular due to the emergence of spin-flavor symmetry at large N_c. In the case of BChPT, spin-flavor symmetry requires the inclusion of spins 1/2 through $N_c/2$ baryons, and gives constraints on the low energy constants (LECs), leading to a combined chiral and $1/N_c$ expansion. Following the lead of Ref. [5] and subsequent works [6–9], *BChPT* $\times 1/N_c$ in the case of flavor $SU(3)$ was implemented in detail at one-loop [10], work that is reported here. The approach is based on *HBChPT* and considers the 'tHooft large N_c expansion, i.e., fixed number of flavors. The non-commutativity between the chiral and $1/N_c$ expansion [1] requires linking them: the natural choice that leaves non-analytic contributions unchanged is the power counting $\mathscr{O}(p) = \mathscr{O}(1/N_c) \equiv \mathscr{O}(\xi)$: the theory is then consistently expanded in powers of ξ. In [10] the calculation of masses, $SU(3)$ breaking in the vector charges, and the axial couplings were carried out to one-loop (generic $\mathscr{O}(\xi^3)$), with the respective renormalization. In addition, the results were provided for generic N_c, so that possible future LQCD calculations at $N_c > 3$ will help study the N_c dependencies, which in the effective theory also reside in the LEC's, as these themselves admit a $1/N_c$ expansion.

The first good lead is given by the transition axial coupling (see definition in [10]) $g_A^{\Delta N} = 1.235 \pm 0.011$ (obtained from the Δ decay width), which at large N_c should be equal to $g_A^N = 1.267 \pm 0.004$. This indicates a small violation of the spin-flavor symmetry, which in this case is a violation suppressed by $\mathscr{O}(1/N_c^2)$ [3,4]. Other relations, as discussed below, serve to confirm the remarkably good validity of spin-symmetry, and thus support the need for $1/N_c$ consistency in the EFT.

2. Baryon masses

The LO Lagrangian is determined in terms of three LECs in addition to F_π:

$$\mathscr{L}_{\mathbf{B}}^{(1)} = \mathbf{B}^\dagger \left(iD_0 + \mathring{g}_A u^{ia} G^{ia} - \frac{C_{\rm HF}}{N_c} \hat{S}^2 + \frac{c_1}{2\Lambda}\, \hat{\chi}_+ \right) \mathbf{B}, \tag{2.1}$$

where $\mathbf{B}$ is the baryon spin-flavor multiplet field, $\hat{S}^2$ is the square of the spin operator, and G^{ia} are spin-flavor generators of $SU(6)$. $\mathring{g}_A$ is identified with $6/5 g_A^N$ at LO, and C_{HF} gives the octet-decuplet mass splitting. The quark mass effects are in the term c_1 (see [10] for details).

To one loop, the baryon self-energy is calculated to $\mathscr{O}(\xi^3)$ (NNLO). The following observations are made: i) the wave function renormalization factor has a piece that is $\mathscr{O}(N_c)$ which is a spin-flavor singlet, which is essential for the mechanism of eliminating N_c power counting violations in individual loop diagrams involving operators such as currents or amplitudes such as π-N scattering. ii) baryon masses receive UV divergent pieces driven by contributions of baryons of a different spin in the loop which are spin-flavor singlet $\mathscr{O}(p^2 N_c^0)$, and spin-flavor non-singlet $\mathscr{O}(p^2/N_c)$, and

[1] The non-commutativity has its origin in the appearance of the small mass difference $m_\Delta - m_N = \mathscr{O}(1/N_c)$.

non-analytic pieces $\mathcal{O}(\xi^2\ \&\ \xi^3)$. The important mass relations, namely Gell-Mann-Okubo (GMO) and Equal Spacing (ES) are unchanged and exactly satisfied at NNLO tree level and arbitrary N_c, and one can thus calculate the deviations only in terms of the LO LECs $\mathring{g}_A/F_\pi$ ($F_\pi = \mathcal{O}(\sqrt{N_c})$) and C_{HF} and the π and K masses. In particular, the deviation of the GMO relation turns out to be $\mathcal{O}(1/N_c)$ in the strict large N_c limit, and it is mildly dependent on C_{HF}. Numerically is given by: $\Delta_{GMO} \sim (\mathring{g}_A/F_\pi)^2 \times 1.68 \times 10^5$ MeV^3, which has to match the physical value ~ 29 MeV (upon correcting by EM effects). It turns out that about 43% of the contribution is from the octet baryons in the loop: the decuplet is thus very important!. If the theory works, one can use Δ_{GMO} to obtain the value of $\mathring{g}_A/F_\pi$ which controls the GB-baryon couplings in the loop calculations. One should note that only 43% of Δ_{GMO} is due to the **8** baryons in the loop. Δ_{GMO} gives then $\mathring{g}_A/F_\pi$ significantly smaller than the corresponding physical ratio g_A^N/F_π, which is also required for a consistent fit to the axial couplings (see next). This sort of consistency between fits to Δ_{GMO} and axial couplings turns out to be entirely absent in ordinary *BChPT* with only **8** baryons.

Spin-symmetry also gives at tree level the relation involving octet and decuplet hyperons (Gürsey-Radicati): $m_{\Xi^*} - m_{\Sigma^*} - (m_\Xi - m_\Sigma) = 0$, whose deviation is affected by one LEC (h_2 in the $\mathcal{O}(\xi^3)$ Lagrangian [10]) and a non-analytic contribution, all UV finite; the small deviation of this relation is also $\mathcal{O}(1/N_c)$ and small in practice.

LQCD results for baryon masses can be used to fix LECs and test the theory. Using the results of Ref. [11] (obtained with m_s kept approximately fixed and $m_u = m_d$ such that 210 MeV $< M_\pi <$ 430 MeV), one finds that in the range $M_\pi < 310$ MeV they can be fitted along with the physical masses with natural size LECs; in part this is possible due to the still rather large error bars of the LQCD results, which, as evident from the trend in the results, has difficulties with the hyperfine **8** − **10** hyperfine mass splitting as M_π decreases. Fig. 1 illustrates the results obtained in [10]).

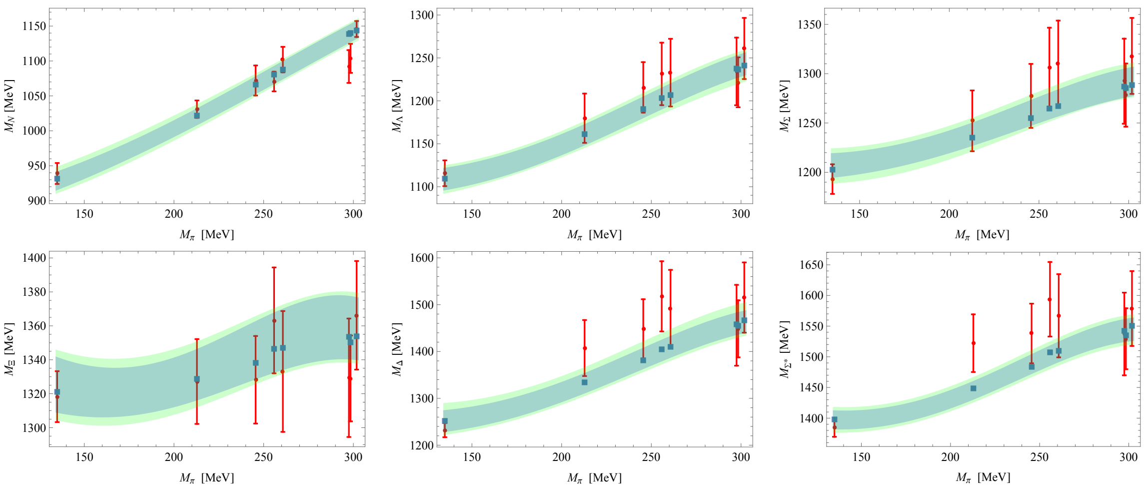

Figure 1: Fits to physical and LQCD [11] masses. $\mathring{g}_A/F_\pi$ fixed using Δ_{GMO}. The bands correspond to the 67% and 95% confidence intervals. The squares are the theoretical values for the values of M_π and M_K of the corresponding data point.

The Hellman-Feynman study of the σ-terms using the physical and LQCD masses will be presented elsewhere [12].

3. Axial couplings

Axial couplings are a particularly important test of $BChPT \times 1/N_c$. The reason is that they explicitly manifest the role of spin-flavor symmetry through cancellations of the N_c violating terms among the loop contributions. Those cancellations require the presence of the baryons of different spin in the loop (the large loop effects found in ordinary *BChPT* (with only **8** baryons) are in fact due to N_c power counting violating terms). The definitions of the axial couplings, and the details of the renormalization are given in [10]. Here we summarize the key points: i) axial couplings receive corrections $\mathcal{O}(1/N_c)$ which are spin-flavor singlet, ii) the quark mass dependent contributions are all $\mathcal{O}(1/N_c)$ or higher, iii) in the strict large N_c limit, the cancellation of N_c power counting violating terms is very drastic: it actually eliminates also the contributions $\mathcal{O}(N_c^0)$ to the axial couplings, terms which are not excluded by the counting. These facts seem to be precisely what is needed in order to explain the very mild quark mass dependency of the axial couplings found in LQCD calculations. Such calculations for the **8** and **10** baryons have only recently being initiated [13], where the axial couplings of the neutral axial currents $A_\mu^a = \frac{1}{2}\bar{q}\lambda^a\gamma_\mu\gamma_5 q$ $(a = 3, 8)$ were calculated. These results show the small quark mass dependency for all baryons. One finds that a LO fit, which only fits $\mathring{g}_A$ gives already a good approximation ($\chi^2_{\rm dof} \sim 4$). At one loop $\mathring{g}_A$ cannot be fitted, and instead it is necessary to give $\mathring{g}_A/F_\pi$ in order to calculate the loop contributions. Since the LQCD axial-couplings used in the study show the well known issue of being systematically too small, that input cannot be extracted from Δ_{GMO} at present; nonetheless, using a somewhat smaller value than the one determined that way, one finds consistent fits to the LQCD results with natural size LECs. Results are illustrated in Fig 2. The LQCD results lead to a significantly smaller g_A^N than the physical one, an issue that is slightly exacerbated by fitting to the rest of the axial couplings. The most important fact is that $BChPT \times 1/N_c$ very naturally describes the LQCD results, and that the mentioned cancellation of loop contributions are key to describing the small quark mass dependencies of the couplings. Continuous progress in LQCD calculations of axial couplings, including **8** − **10** transition ones, will be useful for further testing the EFT.

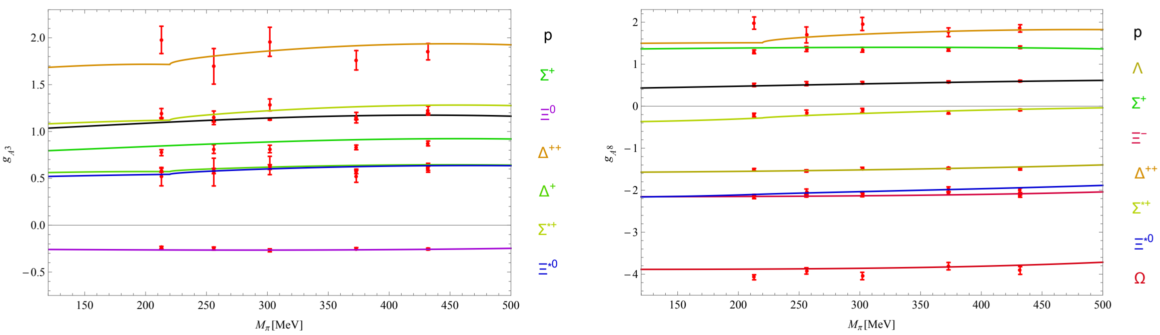

Figure 2: Fits to the axial couplings obtained in LQCD, Tables IV and V of Ref. [13]. The couplings have been redefined as explained in [10].

4. Flavor $SU(3)$ charges

Effects of $SU(3)$ breaking on the $SU(3)$ vector-current charges are suppressed according to

Charge	$\frac{f_1}{f_1^{SU(3)}}$	$\frac{f_1}{f_1^{SU(3)}}-1$			
		[14]	[18]	[17]	[19]
		HBChPT$\times 1/N_c$	HBChPT with 8 and 10	HBChPT only 8	RBChPT with 8 and 10
Λp	0.952	−0.048	−0.080	−0.097	−0.031
$\Sigma^- n$	0.966	−0.034	−0.024	0.008	−0.022
$\Xi^-\Lambda$	0.953	−0.047	−0.063	−0.063	−0.029
$\Xi^-\Sigma^0$	0.962	−0.038	−0.076	−0.094	−0.030

Table 1: Results from different versions of *BChPT* for the corrections to the $|\Delta S|=1$ $SU(3)$ charges.

the Ademollo-Gatto theorem (AGT). At tree level, those effects start at $\mathcal{O}(p^4)=\mathcal{O}(\xi^4)$ in the EFT, and therefore at $\mathcal{O}(\xi^3)$ they are given by calculable non-analytic loop contributions. The detailed study in *BChPT* $\times 1/N_c$ was presented in [14], and in the strict ξ-expansion in [10]. A summary of results is given in Table 1. Although hyperon leptonic decays at the present level of accuracy cannot provide a sensitive test of the predicted corrections, there is in principle a test based in LQCD. Calculations of $|\Delta S|=1$ $SU(3)$ charges in LQCD are still in early stages; Refs. [15, 16] illustrate the present status, where it is obtained [16]:

$$\left(\frac{f_1}{f_1^{SU(3)}}\right)_{\Sigma N}=0.9571(60),\qquad \left(\frac{f_1}{f_1^{SU(3)}}\right)_{\Xi\Sigma}=0.9755(39). \tag{4.1}$$

The *BChPT* predictions depend on the value of the LO axial coupling, and thus it has an uncertainty that we estimate to be of the order of 10-20% or so. Within such an error estimate, the LQCD results nicely agree with the *BChPT* $\times 1/N_c$ prediction and to a slightly lesser extent with the other calculations that include **10** baryons, while it disagrees with the ordinary *BChPT* one [17]. As in the case of axial couplings, further progress in the LQCD calculations of $SU(3)$ breaking effects in the charges will be very useful for a finer test of the EFT.

5. Summary

There is clear evidence that the baryon decuplet plays a key role in *BChPT*. On one hand it is required for consistency with N_c power counting, and also by consistency with both phenomenology and LQCD results. An important observation is that calculations in *BChPT* $\times 1/N_c$ at one loop can describe well the various observables discussed in this contribution, something that is not possible with ordinary *BChPT*. Additional applications, such as in the case of magnetic moments [20, 21], and others currently in progress further support that evidence. The important future tasks are to extend the applications to observables, those in particular that can be calculated in LQCD, in order to test the theory and also derive predictions. One such an application to σ terms will be presented soon [12].

6. Acknowledgements

This work was supported by DOE Contract No. DE-AC05-06OR23177 under which JSA operates the Thomas Jefferson National Accelerator Facility, and by the National Science Foundation through grants PHY-1307413 and PHY-1613951.

References

[1] J-L. Gervais and B. Sakita. *Phys. Rev. Lett.*, 52:87, 1984.

[2] J-L. Gervais and B. Sakita. *Phys. Rev.*, D30:1795, 1984.

[3] R. F. Dashen and A. V. Manohar. *Phys.Lett.*, B315:425–430, 1993.

[4] R. F. Dashen and A. V. Manohar. *Phys.Lett.*, B315:438–440, 1993.

[5] E. E. Jenkins. *Phys. Rev.*, D53:2625–2644, 1996.

[6] R. Flores-Mendieta, E. E. Jenkins, and A. V. Manohar. *Phys. Rev.*, D58:094028, 1998.

[7] R. Flores-Mendieta and C. P. Hofmann. *Phys. Rev.*, D74:094001, 2006.

[8] A. Calle Cordon and J. L. Goity. *Phys. Rev.*, D87(1):016019, 2013.

[9] A. Calle Cordon and J. L. Goity. *PoS*, CD12:062, 2013.

[10] I. P. Fernando and J. L. Goity, 2017. arXiv:1712.01672.

[11] C. Alexandrou, V. Drach, K. Jansen, C. Kallidonis, and G. Koutsou. *Phys. Rev.*, D90(7):074501, 2014.

[12] J. M. Alarcón, I. P. Fernando, and J. L. Goity. work in preparation.

[13] C. Alexandrou, K. Hadjiyiannakou, and C. Kallidonis. *Phys. Rev.*, D94(3):034502, 2016.

[14] R. Flores-Mendieta and J. L. Goity. *Phys. Rev.*, D90(11):114008, 2014.

[15] S. Sasaki. *Phys.Rev.*, D86:114502, 2012.

[16] S. Sasaki. *Phys. Rev.*, D96(7):074509, 2017.

[17] A. Lacour, B. Kubis, and U-G. Meissner. *JHEP*, 0710:083, 2007.

[18] G. Villadoro. *Phys.Rev.*, D74:014018, 2006.

[19] L.S. Geng, J. Martin Camalich, and M.J. Vicente Vacas. *Phys.Rev.*, D79:094022, 2009.

[20] R. Flores-Mendieta. *Phys. Rev.*, D80:094014, 2009.

[21] G. Ahuatzin, R. Flores-Mendieta, and M. A. Hernandez-Ruiz. *Phys. Rev.*, D89(3):034012, 2014.