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Eta-mesic nuclei

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In this contribution we report on theoretical studies of η nuclear quasi-bound states in few- and many-body systems performed recently by the Jerusalem-Prague Collaboration [1, 2, 3, 4, 5]. Underlying energy-dependent ηN interactions are derived from coupled-channel models that incorporate the $N^*(1535)$ resonance. The role of self-consistent treatment of the strong energy dependence of subthreshold ηN amplitudes is discussed. Quite large downward energy shift together with rapid decrease of the ηN amplitudes below threshold result in relatively small binding energies and widths of the calculated η nuclear bound states. We argue that the subthreshold behavior of ηN scattering amplitudes is crucial to conclude whether η nuclear states exist, in which nuclei the η meson could be bound and if the corresponding widths are small enough to allow detection of these η nuclear states in experiment.

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1. Energy and model dependence of ηN scattering amplitudes

Calculations of η nuclear quasi-bound states presented in this contribution are based on the ηN scattering amplitudes derived from coupled-channel models that incorporate the $N^*(1535)$ resonance. The amplitudes near threshold are both attractive and strongly energy dependent, as illustrated in Fig. 1 for three selected meson-baryon interaction models, GW [6], CS [7], and GR [8]. Moreover, the ηN scattering amplitudes are highly model dependent; they differ considerably from each other below as well as above the ηN threshold (except common value $\mathrm{Im}F_{\eta N} \approx 0.2-0.3$ fm at threshold). This suggests that the predictions for the η nuclear states would be model dependent and that the strong energy dependence of the ηN scattering amplitudes has to be treated self-consistently.

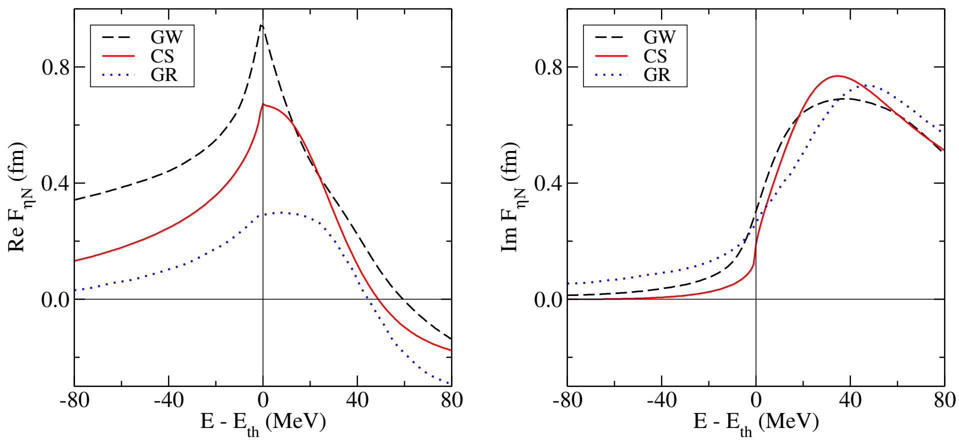

Figure 1: Real (left panel) and imaginary (right panel) parts of the free ηN c.m. scattering amplitude $F_{\eta N}(\sqrt{s})$ as a function of energy in three meson–baryon interaction models: dashed, GW [6]; solid, CS [7]; dotted, GR [8]. The vertical line denotes the ηN threshold.

The crucial point is that in the nuclear medium the energy argument $\sqrt{s}$ is given by

$$\sqrt{s} = \sqrt{(\sqrt{s_{\mathrm{th}}} - B_\eta - B_N)^2 - (\vec{p}_\eta + \vec{p}_N)^2} \leq \sqrt{s_{\mathrm{th}}}, \tag{1.1}$$

where $\sqrt{s_{\mathrm{th}}} \equiv m_h + m_N$ and B_η and B_N are meson and nucleon binding energies, and the momentum dependent term generates additional substantial downward energy shift, since $(\vec{p}_\eta + \vec{p}_N)^2 \neq 0$ unlike the case of the two-body c.m. system. This has significant consequences for the calculated binding energies and widths as will be shown below.

2. The η meson in few-body systems

Few-body calculations of η nuclear clusters have been performed within standard few-body techniques: Faddeev-Yakubovsky equations [9] or variational methods. In ref. [3] the η nuclear cluster wave functions were expanded in a hyperspherical basis. More recent calculations [4, 5] were based on the Stochastic Variational Method (SVM) with a correlated Gaussian basis [10]. Both variational approaches showed sufficient accuracy in the description of η nuclear quasi-bound states and provided almost identical results for ηd, η^3He and η^4He systems.

In our calculations, the nuclear part is described by the Minnesota central *NN* potential [11] or the Argonne AV4' potential [12]. The interaction of η with nucleons of the core is given by a complex two-body energy dependent potential derived from a full chiral coupled-channels model:

$$v_{\eta N}(\delta\sqrt{s},r) = -\frac{4\pi}{2\mu_{\eta N}} b(\delta\sqrt{s})\rho_\Lambda(r), \tag{2.1}$$

where $\delta\sqrt{s} = \sqrt{s} - \sqrt{s_{\rm th}}$, $\rho_\Lambda(r) = (\frac{\Lambda}{2\sqrt{\pi}})^3 exp\left(-\frac{\Lambda^2 r^2}{4}\right)$, and the amplitude $b(\delta\sqrt{s})$ is fitted to phase shifts derived from the ηN scattering amplitude $F_{\eta N}(\delta\sqrt{s})$ in the GW and CS models. The scale parameter Λ is inversely proportional to the range of $V_{\eta N}$ potential. We consider two different values of the scale parameter, $\Lambda = 2$ and 4 fm^{-1} (the choice of the value of Λ is discussed in ref. [3]). It is to be noted that in ref. [5], the *NN* and ηN potentials were constructed within a pionless EFT approach.

The energy argument $\delta\sqrt{s}$ relevant for calculations of η nuclear few-body clusters is expressed in the form [3]:

$$\delta\sqrt{s} = -\frac{B}{A} - \frac{A-1}{A}B_\eta - \xi_N\frac{A-1}{A}\langle T_{NN}\rangle - \xi_\eta\left(\frac{A-1}{A}\right)^2\langle T_\eta\rangle\ , \tag{2.2}$$

where B is the total binding energy of the system, $\xi_{N(\eta)} = m_{N(\eta)}/(m_N + m_\eta)$, T_η is the η kinetic energy in the total c.m. frame and T_{NN} is the pairwise *NN* kinetic energy operator in the *NN* pair c.m. system [3]. The conversion widths are calculated using the expression

$$\Gamma_\eta = -2 < \Psi_{g.s.}|{\rm Im}V_{\eta N}|\Psi_{g.s.} > \tag{2.3}$$

where $|\Psi_{g.s.}>$ stands for the ground state obtained after variation. As was stated already in [3], this approximation is reasonable due to small imaginary contribution $|{\rm Im}V_{\eta N}| \ll |{\rm Re}V_{\eta N}|$.

The results of calculations of η nuclei with $A = 3$ and 4 were discussed in detail in refs. [3, 4, 5]. To summarize, no bound ηNN system was found in the considered two-body interaction models. For ηNNN, a weakly bound state (with η separation energy below 1 MeV) was found for the Minnesota *NN* potential and one particular variant of the ηN potential that reproduced the GW scattering amplitudes. No ηNNN bound states were found using more realistic *NN* interaction model.

In Fig. 2, we demonstrate the self-consistent solution for η^4He, calculated using the AV4' *NN* potential and GW $V_{\eta N}$ potential with $\Lambda = 4$ fm^{-1}. The η^4He bound state energy E and the expectation value $< \delta\sqrt{s} >$ are plotted as a function of the subthreshold energy argument $\delta\sqrt{s}$ of the input potential $V_{\eta N}$. The self-consistency condition is fulfilled by requiring $\delta\sqrt{s} =< \delta\sqrt{s} >$. The corresponding value of $E(< \delta\sqrt{s} >)$ then represents the self-consistent energy of the η nuclear cluster.

A precise self-consistent calculation of *p*-shell η nuclear clusters, such as η^6Li, represents highly non-trivial goal. In this report, we present our preliminary results for η^6Li using the central Minnesota V_{NN} and GW $V_{\eta N}$ potentials. This should be regarded as the first step before doing calculations with a more realistic *NN* potential to account for spin dependent force components in the *p* shell. Moreover, we employed only one spin-isospin configuration in the description of the ^{6}Li nuclear core, which yielded binding energy $B(^6{\rm Li}) = 34.66$ MeV. It is reasonable to expect

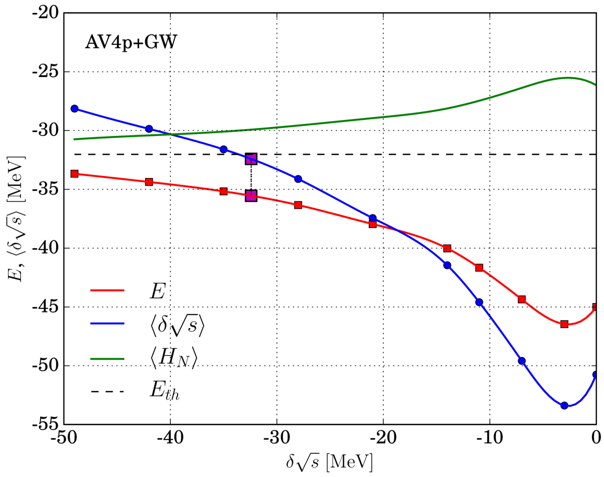

Figure 2: η^4He bound state energy E (red line, squares) and the expectation value $\langle\delta\sqrt{s}\rangle$ (blue line, circles), calculated using the AV4' *NN* potential (denoted here AV4p), as a function of the input energy argument $\delta\sqrt{s}$ of the ηN potential GW with $\Lambda = 4$ fm^{-1}. The dotted vertical line marks the self-consistent output values of $\langle\delta\sqrt{s}\rangle$ and E. The black dashed line denotes the ^{4}He g.s. energy which serves as threshold for bound η. The green curve shows the expectation value $< H_N >$ of the nuclear core energy. Figure adapted from ref. [4].

that taking into account all possible configurations in ^{6}Li will further increase the binding. [1] A full account will be given elsewhere in due course.

The results of the SVM calculations of η binding energies B_η and widths Γ_η in η^3H, η^4He, and η^6Li are summarized in Fig. 3. Moreover, the figure illustrates the extent of the dependence of B_η and Γ_η on the parameter Λ.

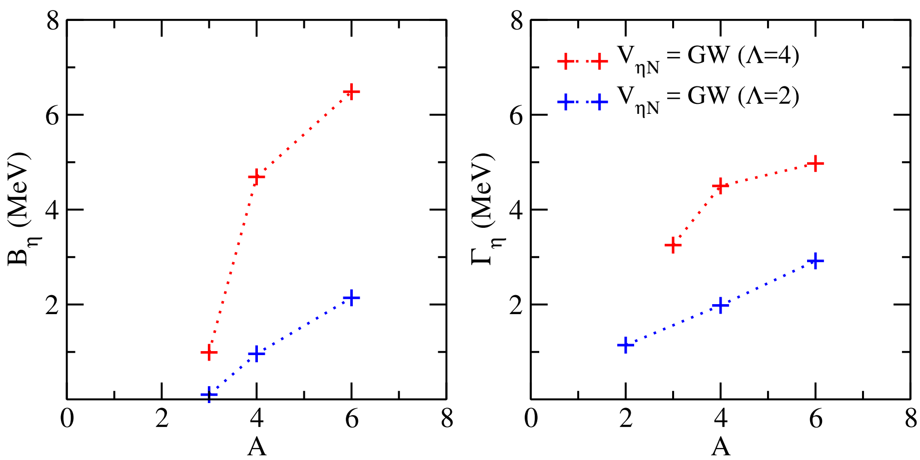

Figure 3: Binding energies B_η (left) and widths Γ_η (right) of $1s$ η quasi-bound states in few-body nuclear systems calcualted using the Minnesota *NN* potential and the ηN potential GW with $\Lambda = 2$ and 4 fm^{-1}.

[1] In ref. [13], a value of $B(^6\text{Li}) = 36.51$ MeV was quoted for the SVM calculation with the Minnesota potential when more spin-isospin configurations were considered.

3. The η meson in many-body systems

The binding energies B_η and widths Γ_η of η quasi-bound states in nuclear many-body systems are determined by solving self-consistently the Klein-Gordon equation

$$[\nabla^2 + \tilde{\omega}_\eta^2 - m_\eta^2 - \Pi_\eta(\omega_\eta, \rho)]\,\psi = 0\,, \tag{3.1}$$

where $\tilde{\omega}_\eta = \omega_\eta - \mathrm{i}\Gamma_\eta/2$ is complex energy of η, $\omega_\eta = m_\eta - B_\eta$. The self-energy operator $\Pi_\eta(\sqrt{s}, \rho) \equiv 2\omega_\eta V_\eta = -(\sqrt{s}/E_N)4\pi F_{\eta N}(\sqrt{s}, \rho)\rho$ is constructed self-consistently using the relevant in-medium ηN scattering amplitude $F_{\eta N}(\sqrt{s})$ and RMF density of the core nucleus.

Modifications of the free-space amplitudes GW due to Pauli blocking in the medium are accounted for by using the multiple scattering approach [14]. In the chirally inspired meson-baryon interaction models CS and GR, Pauli blocking restricts integration domain in the in-medium Green's function which enters the underlying Lippmann-Schwinger (Bethe-Salpeter) equations [7]. Morever, hadron self-energy insertions reflecting in-medium modifications of hadron masses could be included in the in-medium Green's function, as well.

The energy argument in the scattering amplitude $F_{\eta N}(\sqrt{s})$ is approximated as [1]

$$\delta\sqrt{s} = \sqrt{s} - \sqrt{s_{\mathrm{th}}} \approx -B_N\frac{\rho}{\bar{\rho}} - \xi_N B_\eta \frac{\rho}{\rho_0} - \xi_N T_N(\frac{\rho}{\rho_0})^{2/3} - \xi_\eta \frac{\sqrt{s}}{\omega_\eta E_N} 2\pi \mathrm{Re}\, F_{\eta N}(\sqrt{s}, \rho)\rho\,, \tag{3.2}$$

where $\bar{\rho}$ is the average nuclear density, $T_N = 23.0$ MeV at ρ_0, and $B_N \approx 8.5$ MeV is the average nucleon binding energy. It is to be stressed that all terms in Eq. 3.2 are negative definite and thus provide substantial downward energy shift. Since $\mathrm{Re}F_{\eta N}(\sqrt{s})$ and B_η appear as arguments in the expression for $\delta\sqrt{s}$ (Eq. 3.2), which in turn serves as an argument for the self-energy Π_η in Eq. 3.1, a self-consistency scheme is required in calculations. [2]

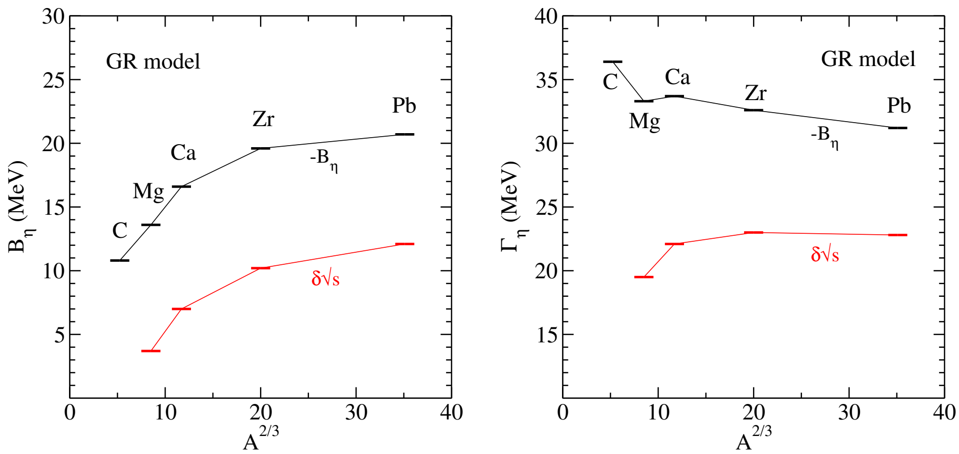

Figure 4: Binding energies (left) and widths (right) of the $1s$ η nuclear states in selected nuclei calculated using the GR ηN scattering amplitude [8] with different procedures for subthreshold energy shift $\delta\sqrt{s}$.

[2]A slightly different form of $\delta\sqrt{s}$ has been used in recent calculations [15, 16], see the contribution of A. Gal in these proceedings.

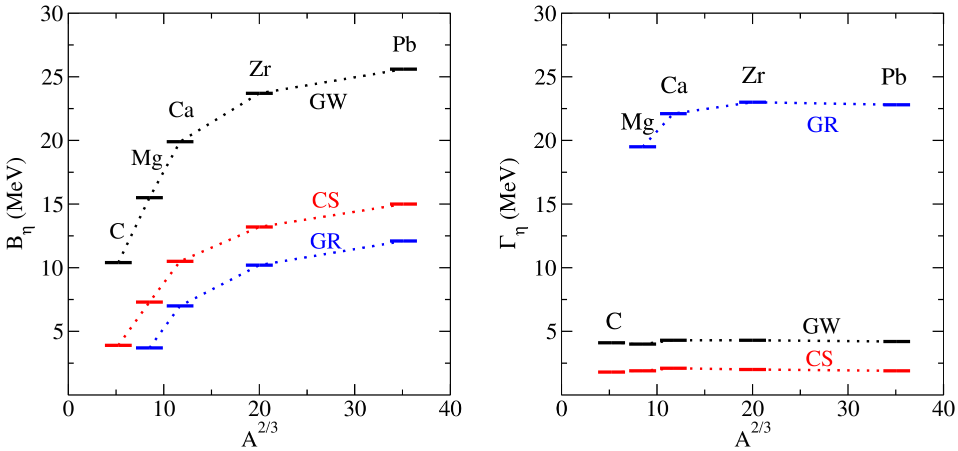

Figure 5: Binding energies (left) and widths (right) of $1s$ η nuclear states in selected nuclei accross the periodic table calculated self consistently using the GW, GR, and GR ηN scattering amplitudes.

It is instructive to compare our self-consistency procedure based on $\delta\sqrt{s}$ of Eq. 3.2, with a self-consistency requirement $\delta\sqrt{s} = -B_\eta$ applied in Ref. [17]. This comparison is presented in Fig. 4 for the in-medium GR amplitude. Our self-consistency formula in Eq. 3.2 (marked $\delta\sqrt{s}$) reduces considerably binding energies and widths of the η meson in nuclei with respect to the calculations of ref. [17] that used $\delta\sqrt{s} = -B_\eta$ (marked $-B_\eta$). However, even the reduced GR widths are still rather large, which suggests that it would be extremely difficult to resolve η nuclear states in this case.

The model dependence of the ηN amplitudes, shown in Fig. 1, has an impact on the calculations of η nuclear quasi-bound states. This is illustrated in Fig. 5 were we present binding energies B_η and widths Γ_η calculated for $1s$ η nuclear states in selected nuclei using the GW, CS and GR models. In the left panel, the hierarchy of the three curves for the η binding energies reflects the strength of the Re$F_{\eta N}(\sqrt{s})$ amplitudes below threshold (compare Fig. 1). For each ηN interaction model the binding energy increases with A and tends to saturate for large values of A.

The right panel demonstrates substantial differences between the η absorption widths Γ_η. While the CS and GW models produce relatively small widths (2 to 4 MeV), almost constant across the periodic table, the GR model yields much larger widths of order 20 MeV which increase with A.

4. Conclusions

In this contribution we briefly reviewed our calculations of η nuclear quasi-bound states accross the periodic table. We applied ηN scattering amplitudes derived from recent meson-baryon coupled-channel interaction models. We demonstrated that the strong energy dependence of scattering amplitudes calls for proper self-consistent treatment. The corresponding ηN amplitudes relevant for calculations of η nuclear states are substantially weaker than the ηN scattering lengths. As a result our calculated η bound states energies and widths are considerably smaller than those obtained in other comparable calculations.

In few-body calculations we explored whether the η meson binds in light nuclei. We found no ηNN bound state. Our results suggest that the onset of η^3He binding occurs for the models providing the ηN scattering length Re$a_{\eta N} \sim 1$ fm. The binding η^4He requires Re$a_{\eta N} \geq 0.7$ fm. It is to be noted that the searches for η^4He bound states performed with the WASA-at-COSY facility have not revealed any signal for a narrow η nuclear state [18].

Small conversion widths in heavier η nuclei obtained in calculations using the CS and GW amplitudes might encourage experimental searches for η nuclear bound states [3] It is to be stressed, however, that the size of the widths Γ_η and binding energies B_η is strongly model dependent. Other models produce either substantially larger widths or even do not generate any η nuclear bound state in a given nucleus.

Acknowledgments

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[3] Additional contributions to the widths due to $\eta N \to \pi\pi N$ and $\eta NN \to NN$ processes, disregarded in our calculations, are estimated to add a few MeV to the total η nuclear widths.