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Extraction of chiral order parameters from $\eta \to 3\pi$ and $\pi\pi$ scattering

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The $\eta \to 3\pi$ decays and $\pi\pi$ scattering are a valuable source of information on low energy QCD. We use a Bayesian approach in the framework of resummed chiral perturbation theory to extract information on the three flavor quark condensate and pseudoscalar decay constant in the chiral limit from these processes, as well as the mass difference of the light quarks. We compare our results with recent χPT and lattice QCD fits and find some tension, as the $\eta \to 3\pi$ data seem to prefer a larger ratio of the chiral order parameters. The results also seem to disfavor a large value of the chiral decay constant, which was found by some recent works.

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The principal order parameters of the spontaneous breaking of chiral symmetry (SBχS) in QCD are the quark condensate and the pseudoscalar decay constant in the chiral limit

$$\Sigma(N_f) = -\langle 0|\bar{q}q|0\rangle|_{m_q\to 0}, \quad F(N_f) = F_P^a|_{m_q\to 0}, \quad ip_\mu F_P^a \; = \; \langle 0|A_\mu^a|P\rangle, \tag{1}$$

where N_f is the number of light quark flavors q and m_q are their masses. A_μ^a are the QCD axial vector currents, while F_P^a the decay constants of the light pseudoscalar mesons P. In chiral perturbation theory (χPT) [1, 2], these order parameters appear at the lowest order of the chiral expansion as low energy constants. A convenient reparametrization of these order parameters can be introduced

$$Z(N_f) = \frac{F(N_f)^2}{F_\pi^2}, \quad X(N_f) = \frac{2\hat{m}\Sigma(N_f)}{F_\pi^2 M_\pi^2}, \tag{2}$$

where we use a common reparametrization of the quark masses

$$\hat{m} = \frac{m_u + m_d}{2}, \quad r = \frac{m_s}{\hat{m}}, \quad R = \frac{m_s - \hat{m}}{m_d - m_u}. \tag{3}$$

Defined in this way, $X(N_f)$ and $Z(N_f)$ are limited to the range $(0,1)$. Standard approach to chiral perturbation series tacitly assumes values of $X(N_f)$ and $Z(N_f)$ not much smaller than one, which means that the leading order terms should dominate the expansion.

Several recent results for the three flavor order parameters are listed in Table 1. As can be seen, some analyses suggest a significant suppression of X(3) and/or Z(3) and thus a non-standard behavior of the spontaneously broken QCD vacuum. It can also be noted that the lattice averaging group FLAG [3] does not report an average for the three flavor chiral order parameters.

phenomenology	$Z(3)$	$X(3)$
NNLO χPT (BE14) [4]	0.59	0.63
NNLO χPT (free fit) [4]	0.48	0.45
lattice QCD	$Z(3)$	$X(3)$
RBC/UKQCD+ReχPT [5]	0.54±0.06	0.38±0.05
RBC/UKQCD+large N_c [6]	0.91±0.08	
MILC 09A [7]	0.72±0.06	0.62±0.07

Table 1: Chosen results for the three flavor order parameters.

We use a Bayesian approach in the framework of resummed chiral perturbation theory [8] to extract information on the three flavor chiral condensate, chiral decay constant and the mass difference of the light quarks. Our experimental input are well known observables connected to $\eta \to 3\pi$ decays and $\pi\pi$ scattering. In the case of the $\eta \to 3\pi$ decays these are the charged and neutral channel decay widths [9] and the charged channel Dalitz plot parameter a [10]

$$\Gamma_{\text{exp}}^{+} = 300 \pm 12 \text{ eV}, \quad \Gamma_{\text{exp}}^{0} = 428 \pm 17 \text{ eV}, \quad a = -1.095 \pm 0.004. \tag{4}$$

For $\pi\pi$ scattering, we use the two lowest order subthreshold parameters in the expansion of the polynomial part of the amplitude [11]

$$\alpha_{\pi\pi}^{\text{exp}} = 1.381 \pm 0.242, \quad \beta_{\pi\pi}^{\text{exp}} = 1.081 \pm 0.023, \quad \rho_{\pi\pi} = -0.14. \tag{5}$$

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$\rho_{\pi\pi}$ is the correlation coefficient between the two parameters.

We assume a reasonable convergence of Green functions connected to these observables and investigate the constraints this assumption can provide for the discussed parameters. The details of the calculations are laid down in [12], explicit formulas can be found in [13].

The assumption of resummed χPT is that by carefully avoiding dangerous manipulations a better converging series can be obtained. In this framework, a chiral expansion of a safe observable is written as $G = G^{(2)} + G^{(4)} + G\,\delta_G^{(6)}$, where $\Delta_G^{(6)} = G\,\delta_G^{(6)}$ is a higher order remainder which contains all terms starting with NNLO. We use an estimate based on general arguments about the convergence of the chiral series [8] $\delta_G^{(4)} \approx \pm 0.3$, $\delta_G^{(6)} \approx \pm 0.1$, where $\delta_G^{(n)}$ collectively denotes all the remainders for the observables we use. We implement this by using a normal distribution with $\mu = 0$ and $\sigma = 0.3$ or $\sigma = 0.1$ for the NLO or NNLO remainders, respectively.

We assume the strange to light quark ratio r to be known and use the lattice QCD average $r = 27.5 \pm 0.4$ [14]. We are then left with three free parameters: $X = X(3)$, $Z = Z(3)$ and R. We use two approaches to deal with the isospin breaking parameter R. In the first one we assume it to be a known quantity and use the N_f=2+1 lattice QCD average $R = 35.8 \pm 2.6$ [14]. Alternatively, we leave R free, or more precisely, assume it to be in a wide range $R \in (0, 80)$.

Our results show the $\eta \to 3\pi$ decays to be sensitive to the values of the three flavor chiral order parameters. As can be seen in the left panel of Fig.1, when assuming $R = 35.8 \pm 2.6$, there is some tension with available results. The $\eta \to 3\pi$ data seem to prefer a larger value of the ratio of the chiral order parameters than recent χPT and lattice fits (Table 1). We get $Y = X/Z = 1.44 \pm 0.32$. The results also appear to disfavor large values of $Z > 0.78$ (2σ CL), which corresponds to $F_0 < 81$MeV.

Adding information from $\pi\pi$ scattering, shown in the right panel of Fig.1, does not change the picture appreciably, possibly due to significant experimental error on the value of these parameters.

The results with R left as a free parameter are shown in Fig.2. The uncertainties are large and thus it's hard to constrain R without additional information on the chiral order parameters and the remainders. Even in this case a part of the parameter space can be excluded at 2σ CL though.

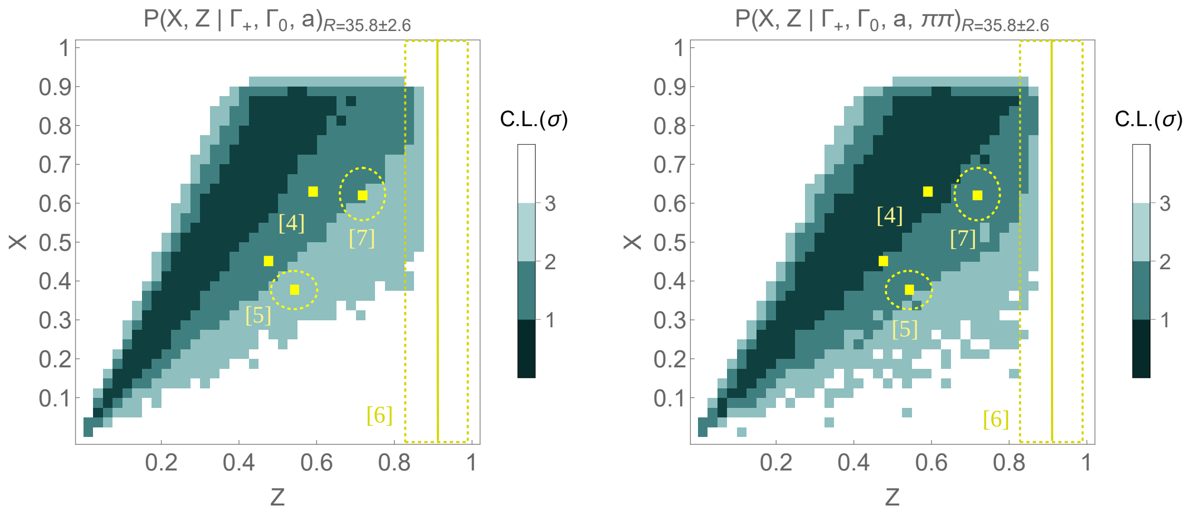

Figure 1: Probability density $P(X, Z|\text{data})$ for $R = 35.8 \pm 2.6$. Data points: results quoted in Table 1. Left: $\eta \to 3\pi$ data. Right: $\eta \to 3\pi$ and $\pi\pi$ scattering data.

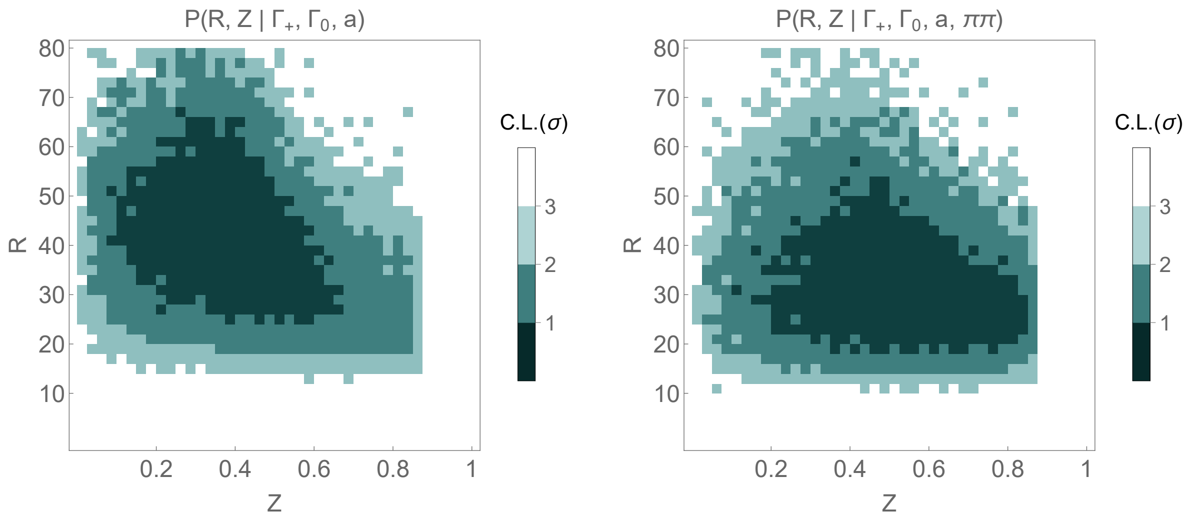

Figure 2: Probability density $P(R,Z|\text{data})$ for R free, X integrated out.
Left: $\eta \to 3\pi$ data. Right: $\eta \to 3\pi$ and $\pi\pi$ scattering data.

The large uncertainty in the extracted value of R indicates that the dependence of R on the values of the chiral order parameters and the higher order remainders is strong. This could be an important information for those determinations of the difference of the light quark masses which use an input from χPT and thus implicitly depend on these uncertainties, such as methods employing a dispersive representation.

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